(G. n)–Tupled Common Point Theorems in the Fuzzy Metric Spaces

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Abstract: ***In this paper, a new type of $(G.n)$ – tupled common fixed point and $(G.n)$ – commute mapping in Fuzzy metric space are introduced and studied, we also discuss the existence and uniqueness for $(G.n)$ – tupled common fixed point of mappings having $(G.n)$ – commute. A $(G.n)$ – tuplet common fixed point theorems for this type of mappings are established.***

Keywords: fuzzy metric spaces , commute mapping, continuous t – norm, Cauchy sequence, common fixed point, equicontinuous.

1.Introduction and Preliminaries:

Fuzzy set was defined by Zadeh [1]. Kramosil and Michalek [2] introduced Fuzzy metric space, Georege and Veermain [3] modified the motion of Fuzzy metric space with the help of continuous t-norms. Many reserchers have obtained common fixed point theorems for mappings. Panit [4] introduced the new concept of reciprocally continuous mapping and established some common fixed point in Fuzzy metric space can be viewed in [5].

Now we will give the following concepts:

Definition (1.1): Let $\mathcal{R}_1: \mathrm{E}^n \to \mathrm{E}$, $\mathcal{R}_2, \dots \dots, \mathcal{R}_n: \mathrm{E}^n \to \mathrm{E}^n$

and $\mathcal{K}_1, \mathcal{K}_2, \dots \dots, \mathcal{K}_n: \mathrm{E} \to \mathrm{E}$ are mappings. A point $(x_1, x_2, \dots \dots, x_n) \in \mathrm{E}^n$ is called $(G.n)$ – tupled coincidence point of this mapping if

- $\mathcal{R}_1\left(\mathcal{R}_2\left(\dots \dots \left(\mathcal{R}_{n(x_1,x_2,\dots,x_n)}\right) \dots ..\right)\right) = \mathcal{K}_1\left(\mathcal{K}_2\left(\dots \dots \left(\mathcal{K}_{n(x_1)}\right) \dots ..\right)\right)$
- $\mathcal{R}_1\left(\mathcal{R}_2\left(\dots \dots \left(\mathcal{R}_{n(x_2,x_3,\dots,x_1)}\right) \dots ..\right)\right) = \mathcal{K}_1\left(\mathcal{K}_2\left(\dots \dots \left(\mathcal{K}_{n(x_2)}\right) \dots ..\right)\right)$

⋮

- $\mathcal{R}_1\left(\mathcal{R}_2\left(\dots \dots \left(\mathcal{R}_{n(x_n,x_1,\dots,x_{n-1})}\right) \dots ..\right)\right) = \mathcal{K}_1\left(\mathcal{K}_2\left(\dots \dots \left(\mathcal{K}_{n(x_n)}\right) \dots ..\right)\right)$

Definition(1.2): Let $\mathcal{R}_1: \mathrm{E}^n \to \mathrm{E}, \mathcal{R}_2, \dots \dots, \mathcal{R}_n: \mathrm{E}^n \to \mathrm{E}^n$ and $\mathcal{K}_1, \mathcal{K}_2, \dots \dots, \mathcal{K}_n: \mathrm{E} \to \mathrm{E}$ are mappings. A point $(x_1, x_2, \dots \dots, x_n)$ is called $(G.n)$–tupled common fixed point if

- $\mathcal{R}_1\left(\mathcal{R}_2\left(\dots \dots \left(\mathcal{R}_{n(x_1,x_2,\dots,x_n)}\right) \dots ..\right)\right) = \mathcal{K}_1\left(\mathcal{K}_2\left(\dots \dots \left(\mathcal{K}_{n(x_1)}\right) \dots ..\right)\right) = x_1$
- $\mathcal{R}_1\left(\mathcal{R}_2\left(\dots \dots \left(\mathcal{R}_{n(x_2,x_3,\dots,x_1)}\right) \dots ..\right)\right) = \mathcal{K}_1\left(\mathcal{K}_2\left(\dots \dots \left(\mathcal{K}_{n(x_2)}\right) \dots ..\right)\right) = x_2$

⋮

- $\mathcal{R}_1\left(\mathcal{R}_2\left(\dots \dots \left(\mathcal{R}_{n(x_n,x_1,\dots,x_{n-1})}\right) \dots \right)\right) = \mathcal{K}_1\left(\mathcal{K}_2\left(\dots \dots \left(\mathcal{K}_{n(x_n)}\right) \dots ..\right)\right) = x_n.$

Definition(1.3): Let $\mathcal{R}_1: \mathrm{E}^n \to \mathrm{E}, \mathcal{R}_2, \dots \dots,$

$\mathcal{R}_n: \mathrm{E}^n \to \mathrm{E}^n and$ $\mathcal{K}_1, \mathcal{K}_2, \dots \dots, \mathcal{K}_n: \mathrm{E} \to \mathrm{E}$ are mappings. We say that these mappings are $(G.n)$ – commute at the point $(x_1, x_2, \dots \dots, x_n)$ if

$$\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_n\left[\mathcal{R}_1\left(\mathcal{R}_2\left(\dots\left(\mathcal{R}_{n(x_1,x_2,\dots,x_n)}\right)..\right)\right)\right]\right)\dots..\right)\right)$$

$$= \mathcal{R}_1\left(\mathcal{R}_2\left(\dots\mathcal{R}_n\begin{pmatrix} \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_1)}\right)..\right)\right) \\ ,\ \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_2)}\right)..\right)\right), \\ \dots,\ \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_n)}\right)..\right)\right) \end{pmatrix}\right)\right).$$

In this paper ,we consider Ψ is the set of all mappings $\mathcal{G}: [0, \infty) \to [0, \infty)$ such that:

i. $\mathcal{G}$ is non – decreasing.
ii. $\mathcal{G}$ is upper semi – continuous from the right.
iii. $\sum_{n=0}^{\infty} \mathcal{G}^n{}_{(t)} < \infty,\ \forall t > 0$ s.t $\mathcal{G}^{n+1}{}_{(t)} = \mathcal{G}\left(\mathcal{G}^n{}_{(t)}\right), n \in N$.

2. Main results:

In this section, we establish our main results

Theorem (2.1): Let A and B are two families of mappings such that, $A = \{\mathcal{R}_1, \mathcal{R}_2, \dots, \mathcal{R}_n$ $s.t$ $\mathcal{R}_1: \mathrm{E}^n \to \mathrm{E}, \mathcal{R}_2, \dots \dots, \mathcal{R}_n: \mathrm{E}^n \to \mathrm{E}^n\}, B = \{\mathcal{K}_1, \mathcal{K}_2, \dots \dots, \mathcal{K}_n: \mathrm{E} \to \mathrm{E}\}$ and let $(\mathrm{E}, \mathcal{T}, ⊛)$ be a fuzzy metric space such that ⊛ is a t – norm of H – type. Suppose that $\mathcal{G} \in \Psi$ satisfying: $\mathcal{T}[\mathcal{R}_1\left(\mathcal{R}_2\left(\dots\left(\mathcal{R}_{n(x_1,x_2,\dots,x_n)}\right)..\right)\right),$

$$\mathcal{R}_1\left(\mathcal{R}_2\left(\dots\left(\mathcal{R}_{n(y_1,y_2,\dots,y_n)}\right)\dots\right)\right), \mathcal{G}_{(t)}] \geq$$

$$\mathcal{T}\begin{bmatrix} \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\ \left(\mathcal{K}_{n(x_1)}\right)\dots..\right)\right), \\ \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\ \left(\mathcal{K}_{n(y_1)}\right)\dots..\right)\right), t \end{bmatrix} ⊛$$

$$\mathcal{T}\begin{bmatrix} \mathcal{K}_1\left(\ \mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_2)}\right)\dots..\right)\right) \\ ,\ \mathcal{K}_1\left(\ \mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(y_2)}\right)\dots..\right)\right), t \end{bmatrix} ⊛ \dots ⊛$$

$$\mathcal{T}\begin{bmatrix} \mathcal{K}_1\left(\ \mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_n)}\right)\dots..\right)\right) \\ ,\ \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(y_n)}\right)\dots..\right)\right), t \end{bmatrix} \qquad (2.1)$$

where $t > 0$ and $x_i, y_i \in E$, for all $i = 1,2, \dots\dots, n$. If $\mathcal{K}_1(\mathcal{K}_2(\dots\dots (\mathcal{K}_n) \dots\dots))$ is complete subspace of E containing $\mathcal{R}_1(\mathcal{R}_2(\dots\dots(\mathcal{R}_n) \dots\dots))$ and these mappings in A and B are $(G.n)$ – commute mappings. Then A and B have a unique $(G.n)$ – tupled coincidence fixed point of compose the mappings of $A$and B.

Proof: Consider $x_0{}^1, x_0{}^2, \dots\dots, x_1{}^n \in E$, since $\mathcal{K}_1(\mathcal{K}_2(\dots(\mathcal{K}_n)..))$containing $\mathcal{R}_1(\mathcal{R}_2(\dots(\mathcal{R}_n) \dots))$, that there exists $x_1{}^1, x_1{}^2, \dots\dots, x_1{}^n \in E$ such that

$$\mathcal{K}_1\left(\mathcal{K}_2\left(\dots \left(\mathcal{K}_{n(x_1{}^1)}\right) \dots\right)\right) = \mathcal{K}_1\left(\mathcal{R}_2\left(\dots\left(\mathcal{R}_{n(x_0{}^1,x_0{}^2,\dots,x_0{}^n)}\right) \dots\right)\right),$$

$$\mathcal{K}_1\left(\mathcal{K}_2\left(\dots \left(\mathcal{K}_{n(x_1{}^2)}\right)..\right)\right) = \mathcal{R}_1\left(\mathcal{R}_2\left(\dots\left(\mathcal{R}_{n(x_0{}^2,x_0{}^3,\dots,x_0{}^n,x_0{}^1)}\right)..\right)\right) \quad ,\dots$$

$\mathcal{K}_1\left(\mathcal{K}_2(\dots \left(\mathcal{K}_{n(x_1{}^n)}\right) \dots)\right) = \mathcal{R}_1\left(\mathcal{R}_2\left(\dots\left(\mathcal{R}_{n(x_0{}^n,x_0{}^1,\dots,x_0{}^{n-1})}\right) \dots\right)\right)$. Also, $\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_2{}^1)}\right)..\right)\right) = \mathcal{R}_1\left(\mathcal{R}_2\left(\dots\left(\mathcal{R}_{n(x_1{}^1,x_1{}^2,\dots,x_1{}^n)}\right)..\right)\right), \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_2{}^2)}\right)..\right)\right) = \mathcal{R}_1\left(\mathcal{R}_2\left(\dots\left(\mathcal{R}_{n(x_1{}^2,x_1{}^3,\dots,x_1{}^n,x_1{}^1)}\right) \dots.\right)\right),..,$

$$\mathcal{K}_1\left(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_2{}^n)}) \dots..)\right) = \mathcal{R}_1\left(\mathcal{R}_2\left(\dots\dots\left(\mathcal{R}_{n(x_1{}^n,x_1{}^1,\dots,x_1{}^{n-1})}\right) \dots..\right)\right)$$

In general, we can construct the sequences

$< \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_k{}^1)}\right) \dots..\right)\right) >, < \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_k{}^2)}\right) \dots..\right)\right) >, \dots, and < \mathcal{K}_1\left(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_k{}^n)}) \dots..)\right) >$ such that,

$$\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_k{}^1)}\right) \dots\right)\right)$$
$$= \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_{k-1}{}^1,x_{k-1}{}^2,\dots,x_{k-1}{}^n)}\right)..\right)\right),$$

$$\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_k{}^2)}\right) \dots\right)\right)$$
$$= \mathcal{R}_1\left(\mathcal{R}_2\left(\dots\left(\mathcal{R}_{n(x_{k-1}{}^2,x_{k-1}{}^3,\dots,x_1{}^n,x_{k-1}{}^1)}\right) \dots\right)\right) \dots..$$

$$\mathcal{K}_1\left(\mathcal{K}_2(\dots(\mathcal{K}_{n(x_k{}^n)}) \dots)\right)$$
$$= \mathcal{R}_1\left(\mathcal{R}_2\left(\dots\left(\mathcal{R}_{n(x_{k-1}{}^n,x_{k-1}{}^1,\dots,x_{k-1}{}^{n-1})}\right) \dots\right)\right)$$

We want to show that the above sequences are Cauchy sequences in $(E, \mathcal{T}, \circledast)$, since $\circledast$is t – norm of H – type.This implies, $\forall\ \lambda > 0\ \exists\ \mu > 0$ such that : $(1-\mu) \circledast (1-\mu) \circledast \dots \circledast (1-\mu) \geq 1 - \lambda,\ \forall n \in N$. on the other hand, for all $x, y \in E$, $\mathcal{T}(x, y, t)$ is continuous and $\lim_{t\to\infty}(x, y, t) = 1$, then there exists $t_\circ > 0$ such that.

$$\mathcal{T}\left[\begin{matrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_0{}^1)}\right) \dots\right)\right), \\ \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_1{}^1)}\right) \dots\right)\right), t_0\end{matrix}\right] \geq 1 - \mu$$

$$\mathcal{T}\left[\begin{matrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_0{}^2)}\right) \dots\right)\right), \\ \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_1{}^2)}\right) \dots\right)\right), t_0\end{matrix}\right] \geq 1 - \mu \ \dots\dots$$

$$\mathcal{T}\left[\begin{matrix}\mathcal{K}_1\left(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_0{}^n)}) \dots)\right), \\ \mathcal{K}_1\left(\mathcal{K}_2\ (\dots\dots(\mathcal{K}_{n(x_1{}^n)}) \dots)\right), t_0\end{matrix}\right] \geq 1 - \mu \qquad (2.2)$$

By using (2.1), we get:

$$\mathcal{T}\left[\begin{matrix}\mathcal{R}_1\left(\mathcal{R}_2\left(\dots\left(\mathcal{R}_{n(x_0{}^1,x_0{}^2,\dots,x_0{}^n)}\right) \dots\right)\right) \\ ,\mathcal{R}_1\left(\mathcal{R}_2\left(\dots\left(\mathcal{R}_{n(x_1{}^1,x_1{}^2,\dots,x_1{}^n)}\right) \dots\right)\right), \mathcal{G}_{(t_0)}\end{matrix}\right] \geq$$
$$\mathcal{T}\left[\begin{matrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_0{}^1)}\right) \dots\right)\right) \\ ,\mathcal{K}_1\left(\mathcal{K}_2\left(\dots \left(\mathcal{K}_{n(x_1{}^1)}\right) \dots\right)\right), t_0\end{matrix}\right] \circledast$$
$$\mathcal{T}\left[\begin{matrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_0{}^2)}\right) \dots\right)\right), \\ \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_1{}^2)}\right) \dots\right)\right), t_0\end{matrix}\right] \circledast$$
$$\dots\dots\ \mathcal{T}\left[\begin{matrix}\mathcal{K}_1\left(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_0{}^n)}) \dots)\right), \\ \mathcal{K}_1\left(\mathcal{K}_2\ (\dots\dots(\mathcal{K}_{n(x_1{}^n)}) \dots)\right), t_0\end{matrix}\right].$$

Also

$$\mathcal{T}\left[\begin{matrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\dots \left(\mathcal{K}_{n(x_1{}^2)}\right) \dots\right)\right), \\ \mathcal{K}_1\left(\mathcal{K}_2\left(\dots \left(\mathcal{K}_{n(x_2{}^2)}\right) \dots\right)\right), \mathcal{G}_{(t_0)}\end{matrix}\right] =$$
$$\mathcal{T}\left[\begin{matrix}\mathcal{R}_1\left(\mathcal{R}_2\left(\dots\left(\mathcal{R}_{n(x_0{}^2,x_0{}^3,\dots,x_0{}^n,x_0{}^1)}\right) \dots\right)\right) \\ ,\mathcal{R}_1\left(\mathcal{R}_2\left(\dots\left(\mathcal{R}_{n(x_1{}^2,x_1{}^3,\dots,x_1{}^n,x_0{}^1)}\right) \dots\right)\right), \mathcal{G}_{(t_0)}\end{matrix}\right] \geq$$
$$\mathcal{T}\left[\begin{matrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots \left(\mathcal{K}_{n(x_0{}^2)}\right) \dots\right)\right), \\ \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots \left(\mathcal{S}_{n(x_1{}^2)}\right) \dots\right)\right), t_0\end{matrix}\right] \circledast$$
$$\mathcal{T}\left[\begin{matrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_0{}^3)}\right) \dots\right)\right), \\ \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_1{}^3)}\right) \dots\right)\right), t_0\end{matrix}\right] \circledast \dots \circledast$$
$$\mathcal{T}\left[\begin{matrix}\mathcal{K}_1\left(\mathcal{K}_2(\dots(\mathcal{K}_{n(x_0{}^n)}) \dots)\right), \\ \mathcal{K}_1\left(\mathcal{K}_2(\dots(\mathcal{K}_{n(x_1{}^n)}) \dots)\right), t_0\end{matrix}\right], \text{continue}$$

$$\left[\begin{matrix}\mathcal{K}_1\left(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_1{}^n)}) \dots)\right), \\ \mathcal{K}_1\left(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_2{}^n)}) \dots)\right), \mathcal{G}_{(t_0)}\end{matrix}\right]$$
$$= \mathcal{T}\left[\begin{matrix}\mathcal{R}_1\left(\mathcal{R}_2\left(\dots\left(\mathcal{R}_{n(x_0{}^n,x_0{}^1,\dots,x_0{}^{n-1})}\right) \dots\right)\right), \\ \mathcal{R}_1\left(\mathcal{R}_2\left(\dots\left(\mathcal{R}_{n(x_1{}^n,x_1{}^1,\dots,x_1{}^{n-1})}\right) \dots\right)\right), t_0\end{matrix}\right]$$

$$\mathcal{T} \geq \mathcal{T}\begin{bmatrix} \mathcal{K}_1(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_0{}^n)})\dots)), \\ \mathcal{K}_1(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_1{}^n)})\dots)), t_0 \end{bmatrix} \circledast \mathcal{T}\begin{bmatrix} \mathcal{K}_1(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_0{}^1)})\dots)), \\ \mathcal{K}_1(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_1{}^1)})\dots)), t_0 \end{bmatrix} \circledast \dots\dots \circledast \mathcal{T}\begin{bmatrix} \mathcal{K}_1(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_0{}^{n-1})})\dots)), \\ \mathcal{K}_1(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_1{}^{n-1})})\dots)), t_0 \end{bmatrix}$$

As the same way and by using above inequalities, we get

$$\mathcal{T}\begin{bmatrix} \mathcal{K}_1(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_2{}^1)})\dots)), \\ \mathcal{K}_1(\mathcal{K}_2(\dots(\mathcal{K}_{n(x_3{}^1)})\dots)), \mathcal{G}^2{}_{(t_0)} \end{bmatrix} = \mathcal{T}\begin{bmatrix} \mathcal{R}_1(\mathcal{R}_2(\dots(\mathcal{R}_{n(x_1{}^1, x_1{}^2, \dots, x_1{}^n)})\dots)), \\ \mathcal{R}_1(\mathcal{R}_2(\dots(\mathcal{R}_{n(x_2{}^1, x_2{}^2, \dots, x_2{}^n)})\dots)), \mathcal{G}^2{}_{(t_0)} \end{bmatrix}$$

$$\geq \mathcal{T}\begin{bmatrix} \mathcal{K}_1(\mathcal{K}_2(\dots(\mathcal{K}_{n(x_1{}^1)})\dots)), \\ \mathcal{K}_1(\mathcal{K}_2(\dots(\mathcal{K}_{n(x_2{}^1)})\dots)), \mathcal{G}_{(t_0)} \end{bmatrix} \circledast \mathcal{T}\begin{bmatrix} \mathcal{K}_1(\mathcal{K}_2(\dots(\mathcal{K}_{n(x_1{}^2)})\dots)) \\ , \mathcal{K}_1(\mathcal{K}_2(\dots(\mathcal{K}_{n(x_2{}^2)})\dots)), \mathcal{G}_{(t_0)} \end{bmatrix} \circledast \dots \circledast \mathcal{T}\begin{bmatrix} \mathcal{K}_1(\mathcal{K}_2(\dots(\mathcal{K}_{n(x_1{}^n)})\dots)) \\ , \mathcal{K}_1(\mathcal{K}_2(\dots(\mathcal{K}_{n(x_2{}^n)})\dots)), \mathcal{G}_{(t_0)} \end{bmatrix}$$

$$\geq \mathcal{T}\begin{bmatrix} \mathcal{K}_1(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_0{}^1)})\dots)), \\ \mathcal{K}_1(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_1{}^1)})\dots)), t_0 \end{bmatrix}^n \circledast \mathcal{T}\begin{bmatrix} \mathcal{K}_1(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_0{}^2)})\dots)), \\ \mathcal{K}_1(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_1{}^2)})\dots)), t_0 \end{bmatrix}^n \circledast \dots\dots \circledast \mathcal{T}\begin{bmatrix} \mathcal{K}_1(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_0{}^n)})\dots)), \\ \mathcal{K}_1(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_1{}^n)})\dots)), t_0 \end{bmatrix}^n$$

$$\begin{bmatrix} \mathcal{K}_1(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_2{}^2)})\dots)), \\ \mathcal{K}_1(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_3{}^2)})\dots)), \mathcal{G}^2{}_{(t_0)} \end{bmatrix} = \mathcal{T}\begin{bmatrix} \mathcal{R}_1(\mathcal{R}_2(\dots(\mathcal{R}_{n(x_1{}^2, x_1{}^3, \dots, x_1{}^n, x_1{}^1)})\dots)), \\ \mathcal{R}_1(\mathcal{R}_2(\dots(\mathcal{R}_{n(x_2{}^2, x_2{}^3, \dots, x_2{}^n, x_1{}^1)})\dots)), \mathcal{G}^2{}_{(t_0)} \end{bmatrix}$$

$$\geq \mathcal{T}\begin{bmatrix} \mathcal{K}_1(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_1{}^2)})\dots)), \\ \mathcal{K}_1(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_2{}^2)})\dots)), \mathcal{G}_{(t_0)} \end{bmatrix} \mathcal{T} \circledast \mathcal{T}\begin{bmatrix} \mathcal{K}_1(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_1{}^3)})\dots)), \\ \mathcal{K}_1(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_2{}^3)})\dots)), \mathcal{G}_{(t_0)} \end{bmatrix} \circledast \dots \circledast$$

$$\mathcal{T}\begin{bmatrix} \mathcal{K}_1(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_1{}^n)})\dots)), \\ \mathcal{K}_1(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_2{}^n)})\dots)), \mathcal{G}_{(t_0)} \end{bmatrix} \geq \mathcal{T}\begin{bmatrix} \mathcal{K}_1(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_0{}^n)})\dots)), \\ \mathcal{K}_1(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_1{}^n)})\dots)), t_0 \end{bmatrix}^n \circledast \mathcal{T}\begin{bmatrix} \mathcal{K}_1(\mathcal{K}_2(\dots(\mathcal{K}_{n(x_0{}^1)})\dots)), \\ \mathcal{K}_1(\mathcal{K}_2(\dots(\mathcal{K}_{n(x_1{}^1)})\dots)), t_0 \end{bmatrix}^n \circledast \dots\dots \circledast \mathcal{T}\begin{bmatrix} \mathcal{K}_1(\mathcal{K}_2(\dots(\mathcal{K}_{n(x_0{}^{n-1})})\dots)), \\ \mathcal{K}_1(\mathcal{K}_2(\dots(\mathcal{K}_{n(x_1{}^{n-1})})\dots)), t_0 \end{bmatrix}^n, \text{continue}$$

$$\mathcal{T}\begin{bmatrix} \mathcal{K}_1(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_2{}^n)})\dots)), \\ \mathcal{K}_1(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_3{}^n)})\dots)), \mathcal{G}^2{}_{(t_0)} \end{bmatrix} = \mathcal{T}\begin{bmatrix} \mathcal{R}_1(\mathcal{R}_2(\dots(\mathcal{R}_{n(x_2{}^n, x_2{}^1, \dots, x_2{}^{n-1})})\dots)), \\ \mathcal{R}_1(\mathcal{R}_2(\dots(\mathcal{R}_{n(x_3{}^n, x_3{}^1, \dots, x_3{}^n)})\dots)), \mathcal{G}^2{}_{(t_0)} \end{bmatrix}$$

$$\geq \mathcal{T}\begin{bmatrix} \mathcal{K}_1(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_2{}^n)})\dots)), \\ \mathcal{K}_1(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_3{}^n)})\dots)), \mathcal{G}_{(t_0)} \end{bmatrix} \circledast \mathcal{T}\begin{bmatrix} \mathcal{K}_1(\mathcal{K}_2(\dots(\mathcal{K}_{n(x_2{}^1)})\dots)), \\ \mathcal{K}_1(\mathcal{K}_2(\dots(\mathcal{K}_{n(x_3{}^1)})\dots)), \mathcal{G}_{(t_0)} \end{bmatrix} \circledast \dots \circledast \mathcal{T}\begin{bmatrix} \mathcal{K}_1(\mathcal{K}_2(\dots(\mathcal{K}_{n(x_2{}^{n-1})})\dots)), \\ \mathcal{K}_1(\mathcal{K}_2(\dots(\mathcal{K}_{n(x_3{}^{n-1})})\dots)), \mathcal{G}_{(t_0)} \end{bmatrix}$$

$$\geq \mathcal{T}\begin{bmatrix} \mathcal{K}_1(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_0{}^1)})\dots)), \\ \mathcal{K}_1(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_1{}^1)})\dots)), t_0 \end{bmatrix}^n \circledast \mathcal{T}\begin{bmatrix} \mathcal{K}_1(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_0{}^2)})\dots)), \\ \mathcal{K}_1(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_1{}^2)})\dots)), t_0 \end{bmatrix}^n \circledast \dots\dots \circledast \mathcal{T}\begin{bmatrix} \mathcal{K}_1(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_0{}^n)})\dots)), \\ \mathcal{K}_1(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_1{}^n)})\dots)), t_0 \end{bmatrix}^n$$

Similarly, $\mathcal{T}\begin{bmatrix} \mathcal{K}_1(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_k{}^1)})\dots)), \\ \mathcal{K}_1(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_{k+1}{}^1)})\dots)), \mathcal{G}^k{}_{(t_0)} \end{bmatrix} =$

$$\mathcal{T}\begin{bmatrix} \mathcal{R}_1(\mathcal{R}_2(\dots(\mathcal{R}_{n(x_{k-1}{}^1, x_{k-1}{}^2, \dots, x_{k-1}{}^n)})\dots)) \\ , \mathcal{R}_1(\mathcal{R}_2(\dots(\mathcal{R}_{n(x_k{}^1, x_k{}^2, \dots, x_k{}^n)})\dots)), \mathcal{G}^k{}_{(t_0)} \end{bmatrix} \geq \mathcal{T}\begin{bmatrix} \mathcal{K}_1(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_{k-1}{}^1)})\dots)), \\ \mathcal{K}_1(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_k{}^1)})\dots)), \mathcal{G}^{k-1}{}_{(t_0)} \end{bmatrix} \circledast \mathcal{T}\begin{bmatrix} \mathcal{K}_1(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_{k-1}{}^2)})\dots)) \\ , \mathcal{K}_1(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_k{}^2)})\dots)), \mathcal{G}^{k-1}{}_{(t_0)} \end{bmatrix} \circledast \dots \circledast$$

$$\mathcal{T}\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_{k-1}{}^{n})})\dots)\right),\\ \mathcal{K}_1\left(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_k{}^{n})})\dots)\right),\mathcal{G}^{k-1}{}_{(t_0)}\end{bmatrix}\dots.\geq$$

$$\mathcal{T}\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_0{}^{1})}\right)\dots\right)\right),\\ \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_1{}^{1})}\right)\dots\right)\right),t_0\end{bmatrix}^{n^{k-1}}\circledast$$

$$\mathcal{T}\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_0{}^{2})}\right)\dots\right)\right),\\ \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_1{}^{2})}\right)\dots\right)\right),t_0\end{bmatrix}^{n^{k-1}}\circledast\dots\dots\circledast$$

$$M\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_0{}^{n})})\dots)\right),\\ \mathcal{K}_1\left(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_1{}^{n})})\dots)\right),t_0\end{bmatrix}^{n^{k-1}}$$

Also,

$$\mathcal{T}\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_k{}^{2})}\right)\dots\right)\right),\\ \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_{k+1}{}^{2})}\right)\dots\right)\right),\mathcal{G}^{k}{}_{(t_0)}\end{bmatrix}$$

$$=\Big[\mathcal{R}_1\left(\mathcal{R}_2\left(\dots\left(\mathcal{R}_{n(x_{k-1}{}^{2},x_{k-1}{}^{3},\dots,x_{k-1}{}^{n},x_{k-1}{}^{1})}\right)\dots\right)\right),$$

$$\mathcal{R}_1\left(\mathcal{R}_2\left(\dots\left(\mathcal{R}_{n(x_k{}^{2},x_k{}^{3},\dots,x_k{}^{n},x_k{}^{1})}\right)\dots\right)\right),\mathcal{G}^{k}{}_{(t_0)}\Big]$$

$$\geq\mathcal{T}\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_{k-1}{}^{2})}\right)\dots\right)\right),\\ \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_k{}^{2})}\right)\dots\right)\right),\mathcal{G}^{k-1}{}_{(t_0)}\end{bmatrix}$$

$$\circledast\,\mathcal{T}\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_{k-1}{}^{3})}\right)\dots\right)\right),\\ \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_k{}^{3})}\right)\dots\right)\right),\mathcal{G}^{k-1}{}_{(t_0)}\end{bmatrix}\circledast\dots$$

$$\circledast\,\mathcal{T}\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_{k-1}{}^{1})}\right)\dots\right)\right),\\ \mathcal{R}_1\left(\mathcal{R}_2\left(\dots\dots\left(\mathcal{R}_{n(x_k{}^{1})}\right)\dots\right)\right),\mathcal{G}^{k-1}{}_{(t_0)}\end{bmatrix}\quad\dots.$$

$$\geq\mathcal{T}\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_0{}^{2})}\right)\dots\right)\right),\\ \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_1{}^{2})}\right)\dots\right)\right),t_0\end{bmatrix}^{n^{k-1}}$$

$$\circledast\,\mathcal{T}\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_0{}^{3})}\right)\dots\right)\right),\\ \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_1{}^{3})}\right)\dots\right)\right),t_0\end{bmatrix}^{n^{k-1}}\circledast\dots\dots$$

$$\circledast\,\mathcal{T}\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_0{}^{1})}\right)\dots\right)\right),\\ \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_1{}^{1})}\right)\dots\right)\right),t_0\end{bmatrix}^{n^{k-1}}$$

Continue, $\mathcal{T}\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_k{}^{n})})\dots)\right),\\ \mathcal{K}_1\left(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_{k+1}{}^{n})})\dots)\right),\mathcal{G}^{k}{}_{(t_0)}\end{bmatrix}\geq$

$$\mathcal{T}\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_0{}^{n})})\dots)\right),\\ \mathcal{K}_1\left(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_1{}^{n})})\dots)\right),t_0\end{bmatrix}^{n^{k-1}}\circledast$$

$$\mathcal{T}\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_0{}^{1})}\right)\dots\right)\right),\\ \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_1{}^{1})}\right)\dots\right)\right),t_0\end{bmatrix}^{n^{k-1}}\circledast\dots\dots\circledast$$

$$\mathcal{T}\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_0{}^{n-1})}\right)\dots\right)\right),\\ \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_1{}^{n-1})}\right)\dots\right)\right),t_0\end{bmatrix}^{n^{k-1}}$$

Now,if $l=\max\{n^{k-1},n^{k},n^{m-2}\}$ then by using above inequalities, then

$$\mathcal{T}\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_k{}^{n})})\dots)\right),\\ \mathcal{K}_1\left(\mathcal{K}_2(\dots(\mathcal{K}_{n(x_m{}^{n})})\dots)\right),\sum_{k=n_0}^{\infty}\mathcal{G}^{k}{}_{(t_0)}\end{bmatrix}$$

$$\geq\mathcal{T}\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_k{}^{n})})\dots)\right),\\ \mathcal{K}_1\left(\mathcal{K}_2(\dots(\mathcal{K}_{n(x_m{}^{n})})\dots)\right),\sum_{k=n_0}^{m-1}\mathcal{G}^{k+1}{}_{(t_0)}\end{bmatrix}$$

$$\geq\mathcal{T}\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2(\dots(\mathcal{K}_{n(x_k{}^{n})})\dots)\right),\\ \mathcal{K}_1\left(\mathcal{K}_2(\dots(\mathcal{K}_{n(x_{k+1}{}^{n})})\dots)\right),\mathcal{G}^{k}{}_{(t_0)}\end{bmatrix}$$

$$\circledast\;\mathcal{T}\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2(\dots(\mathcal{K}_{n(x_{k+1}{}^{n})})\dots)\right),\\ \mathcal{K}_1\left(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_{k+2}{}^{n})})\dots)\right),\mathcal{G}^{k+1}{}_{(t_0)}\end{bmatrix}\circledast\dots$$

$$\circledast\,\mathcal{T}\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_{m-1}{}^{n})})\dots)\right),\\ \mathcal{K}_1\left(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_m{}^{n})})\dots)\right),\mathcal{G}^{m-1}{}_{(t_0)}\end{bmatrix}$$

$$\geq\mathcal{T}\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_0{}^{n})})\dots)\right)\\ ,\mathcal{K}_1\left(\mathcal{K}_2(\dots\dots(\mathcal{K}_{n(x_1{}^{n})})\dots)\right),t_0\end{bmatrix}^{n^{k-1}}$$

$$\circledast\,\mathcal{T}\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_0{}^{1})}\right)\dots\right)\right),\\ \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_1{}^{1})}\right)\dots\right)\right),t_0\end{bmatrix}^{n^{k-1}}\circledast\dots\dots$$

$$\circledast\,\mathcal{T}\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_0{}^{n-1})}\right)\dots\right)\right),\\ \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_1{}^{n-1})}\right)\dots\right)\right),t_0\end{bmatrix}^{n^{k-1}}$$

$$\circledast\,\mathcal{T}\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2(\dots(\mathcal{K}_{n(x_0{}^{n})})\dots)\right)\\ ,\mathcal{K}_1\left(\mathcal{K}_2(\dots(\mathcal{K}_{n(x_1{}^{n})})\dots)\right),t_0\end{bmatrix}^{n^{k}}$$

$$\circledast\,\mathcal{T}\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_0{}^{1})}\right)\dots\right)\right)\\ ,\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_1{}^{1})}\right)\dots\right)\right),t_0\end{bmatrix}^{n^{k}}\circledast\dots\dots$$

$$\circledast\,\mathcal{T}\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_0{}^{n-1})}\right)\dots\right)\right),\\ \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_1{}^{n-1})}\right)\dots\right)\right),t_0\end{bmatrix}^{n^{k}}$$

$$\circledast\dots$$

$$\circledast\,\mathcal{T}\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2(\dots(\mathcal{K}_{n(x_0{}^{n})})\dots)\right)\\ ,\mathcal{K}_1\left(\mathcal{K}_2(\dots(\mathcal{K}_{n(x_1{}^{n})})\dots)\right),t_0\end{bmatrix}^{n^{m-2}}$$

$$\circledast\, \mathcal{T}\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\ldots\left(\mathcal{K}_{n(x_0{}^1)}\right)\ldots\right)\right), \\ \mathcal{K}_1\left(\mathcal{K}_2\left(\ldots\left(\mathcal{K}_{n(x_1{}^1)}\right)\ldots\right)\right), t_0\end{bmatrix}^{n^{m-2}} \circledast \ldots$$

$$\circledast\, \mathcal{T}\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\ldots\ldots\left(\mathcal{K}_{n(x_0{}^{n-1})}\right)\ldots\right)\right), \\ \mathcal{K}_1\left(\mathcal{K}_2\left(\ldots\ldots\left(\mathcal{K}_{n(x_1{}^{n-1})}\right)\ldots\right)\right), t_0\end{bmatrix}^{n^{m-2}}$$

$$\circledast\, \mathcal{T}\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\ldots\left(\mathcal{K}_{n(x_0{}^1)}\right)\ldots\right)\right) \\ , \mathcal{K}_1\left(\mathcal{K}_2\left(\ldots\left(\mathcal{K}_{n(x_1{}^1)}\right)\ldots\right)\right), t_0\end{bmatrix}^{l} \circledast \ldots\ldots$$

$$\circledast\, \mathcal{T}\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\ldots\ldots\left(\mathcal{K}_{n(x_0{}^{n-1})}\right)\ldots\right)\right), \\ \mathcal{K}_1\left(\mathcal{K}_2\left(\ldots\ldots\left(\mathcal{K}_{n(x_1{}^{n-1})}\right)\ldots\right)\right), t_0\end{bmatrix}^{l}$$

$$\geq \mathcal{T}\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\ldots\left(\mathcal{K}_{n(x_0{}^n)}\right)\ldots\right)\right), \\ \mathcal{K}_1\left(\mathcal{K}_2\left(\ldots\left(\mathcal{K}_{n(x_1{}^n)}\right)\ldots\right)\right), t_0\end{bmatrix}^{l} \circledast,$$

$$\circledast\, \mathcal{T}\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\ldots\left(\mathcal{K}_{n(x_0{}^n)}\right)\ldots\right)\right), \\ \mathcal{K}_1\left(\mathcal{K}_2\left(\ldots\left(\mathcal{K}_{n(x_1{}^n)}\right)\ldots\right)\right), t_0\end{bmatrix}^{l}$$

$$\circledast\, \mathcal{T}\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\ldots\left(\mathcal{K}_{n(x_0{}^1)}\right)\ldots\right)\right), \\ \mathcal{K}_1\left(\mathcal{K}_2\left(\ldots\left(\mathcal{K}_{n(x_1{}^1)}\right)\ldots\right)\right), t_0\end{bmatrix}^{l}$$

$$\circledast \ldots\ldots \circledast\, \mathcal{T}\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\ldots\left(\mathcal{K}_{n(x_0{}^n)}\right)\ldots\right)\right), \\ \mathcal{K}_1\left(\mathcal{K}_2\left(\ldots\left(\mathcal{K}_{n(x_1{}^n)}\right)\ldots\right)\right), t_0\end{bmatrix}^{l}$$

$$> \mathcal{T}\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\ldots\ldots\left(\mathcal{K}_{n(x_0{}^n)}\right)\ldots\right)\right), \\ \mathcal{K}_1\left(\mathcal{K}_2\left(\ldots\ldots\left(\mathcal{K}_{n(x_1{}^n)}\right)\ldots\right)\right), t_0\end{bmatrix}^{ml}$$

$$\circledast\, \mathcal{T}\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\ldots\left(\mathcal{K}_{n(x_0{}^1)}\right)\ldots\right)\right), \\ \mathcal{K}_1\left(\mathcal{K}_2\left(\ldots\left(\mathcal{K}_{n(x_1{}^1)}\right)\ldots\right)\right), t_0\end{bmatrix}^{ml}$$

$$\circledast \ldots\ldots$$

$$\circledast\, \mathcal{T}\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\ldots\ldots\left(\mathcal{K}_{n(x_0{}^{n-1})}\right)\ldots\right)\right), \\ \mathcal{K}_1\left(\mathcal{K}_2\left(\ldots\ldots\left(\mathcal{K}_{n(x_1{}^{n-1})}\right)\ldots\right)\right), t_0\end{bmatrix}^{ml}$$

$$\geq (1-\mu) \circledast (1-\mu) \circledast \ldots\ldots \circledast (1-\mu)$$
$$\geq (1-\lambda)$$

And hence, $\mathcal{T}\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\ldots\ldots\left(\mathcal{K}_{n(x_k{}^n)}\right)\ldots\right)\right), \\ \mathcal{K}_1\left(\mathcal{K}_2\left(\ldots\ldots\left(\mathcal{K}_{n(x_m{}^n)}\right)\ldots\right), t\right)\end{bmatrix} > (1-\lambda)$

So, $< \mathcal{K}_1\left(\mathcal{K}_2\left(\ldots\ldots\left(\mathcal{K}_{n(x_k{}^n)}\right)\ldots\right)\right) >$ is Cauchy sequence.

As the same way, we get, $< \mathcal{K}_1\left(\mathcal{K}_2\left(\ldots\ldots\left(\mathcal{K}_{n(x_k{}^1)}\right)\ldots\right)\right) >$,

$< \mathcal{K}_1\left(\mathcal{K}_2\left(\ldots\left(\mathcal{K}_{n(x_k{}^2)}\right)\ldots\right)\right) >, <$
$\mathcal{K}_1\left(\mathcal{K}_2\left(\ldots\left(\mathcal{K}_{n(x_k{}^{n-1})}\right)\ldots\right)\right) >$ are Cauchy sequences.
Now,since $\mathcal{S}_1\left(\mathcal{S}_2\left(\ldots\ldots\left(\mathcal{S}_{n(X)}\right)\ldots\right)\right)$ is complete subspace of E then there exists $x_1, x_2, \ldots, x_n \in \mathcal{K}_1\left(\mathcal{K}_2\left(\ldots\ldots\left(\mathcal{K}_{n(X)}\right)\ldots\right)\right)$ and $a_1, a_2, \ldots\ldots, a_n \in$ E such that,

$$\lim_{k\to\infty} \mathcal{K}_1\left(\mathcal{K}_2\left(\ldots\ldots\left(\mathcal{K}_{n(x_k{}^1)}\right)\ldots\right)\right) = \lim_{k\to\infty} \mathcal{R}_1\left(\mathcal{R}_2\left(\ldots\left(\mathcal{R}_{n(x_{k-1}{}^1,\ldots,x_{k-1}{}^n)}\right)\ldots\right)\right) \to \mathcal{K}_1\left(\mathcal{K}_2\left(\ldots\ldots\left(\mathcal{K}_{n(a_1)}\right)\ldots\right)\right) = x_1$$

$$\lim_{k\to\infty} \mathcal{K}_1\left(\mathcal{K}_2\left(\ldots\left(\mathcal{K}_{n(x_k{}^2)}\right)\ldots\right)\right) = \lim_{k\to\infty} \mathcal{R}_1\left(\mathcal{R}_2\left(\ldots\left(\mathcal{R}_{n(x_{k-1}{}^2,\ldots,x_{k-1}{}^1)}\right)\ldots\right)\right) \to \mathcal{K}_1\left(\mathcal{K}_2\left(\ldots\ldots\left(\mathcal{K}_{n(a_2)}\right)\ldots\right)\right) = x_2$$

$$\lim_{k\to\infty} \mathcal{K}_1\left(\mathcal{K}_2\left(\ldots\left(\mathcal{K}_{n(x_k{}^n)}\right)\ldots\right)\right) = \lim_{k\to\infty} \mathcal{R}_1\left(\mathcal{R}_2\left(\ldots\left(\mathcal{R}_{n(x_{k-1}{}^n,\ldots,x_{k-1}{}^{n-1})}\right)\ldots\right)\right) \to \mathcal{K}_1\left(\mathcal{K}_2\left(\ldots\ldots\left(\mathcal{K}_{n(a_n)}\right)\ldots\right)\right) = x_n$$

Now, $$\mathcal{T}\begin{bmatrix}\mathcal{R}_1\left(\mathcal{R}_2\left(\ldots\left(\mathcal{R}_{n(x_{k-1}{}^1,x_{k-1}{}^2,\ldots,x_{k-1}{}^n)}\right)\ldots\right)\right), \\ \mathcal{R}_1\left(\mathcal{R}_2\left(\ldots\left(\mathcal{R}_{n(a_1,a_2,\ldots,a_n)}\right)\ldots\right)\right), \mathcal{G}_{(t)}\end{bmatrix} \geq$$

$$\mathcal{T}\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\ldots\ldots\left(\mathcal{K}_{n(x_{k-1}{}^1)}\right)\ldots\right)\right), \\ \mathcal{K}_1\left(\mathcal{K}_2\left(\ldots\ldots\left(\mathcal{K}_{n(a_1)}\right)\ldots\right)\right), t\end{bmatrix} \circledast$$

$$\mathcal{T}\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\ldots\left(\mathcal{K}_{n(x_{k-1}{}^2)}\right)\ldots\right)\right), \\ \mathcal{K}_1\left(\mathcal{K}_2\left(\ldots\left(\mathcal{K}_{n(a_2)}\right)\ldots\right)\right), t\end{bmatrix} \circledast \ldots \circledast$$

$$\mathcal{T}\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\ldots\ldots\left(\mathcal{K}_{n(x_{k-1}{}^n)}\right)\ldots\right)\right), \\ \mathcal{K}_1\left(\mathcal{K}_2\left(\ldots\ldots\left(\mathcal{K}_{n(a_n)}\right)\ldots\right)\right), t\end{bmatrix}$$

As $n \to \infty$ and by continuity of M, we get

$$\mathcal{T}\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\ldots\ldots\left(\mathcal{K}_{n(a_1)}\right)\ldots\right)\right), \\ \mathcal{K}_1\left(\mathcal{K}_2\left(\ldots\left(\mathcal{K}_{n(a_1,a_2,\ldots,a_n)}\right)\ldots\right)\right) \\ , \mathcal{G}_{(t)}\end{bmatrix} = 1$$

Also, $$\mathcal{T}\begin{bmatrix}\mathcal{R}_1\left(\mathcal{R}_2\left(\ldots\left(\mathcal{R}_{n(x_{k-1}{}^2,x_{k-1}{}^3,\ldots,x_{k-1}{}^1)}\right)\ldots\right)\right) \\ , \mathcal{R}_1\left(\mathcal{R}_2\left(\ldots\left(\mathcal{R}_{n(a_1,a_2,\ldots,a_n)}\right)\ldots\right)\right), \mathcal{G}_{(t)}\end{bmatrix} \geq$$

$$\mathcal{T}\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\ldots\ldots\left(\mathcal{K}_{n(x_{k-1}{}^2)}\right)\ldots\right)\right), \\ \mathcal{K}_1\left(\mathcal{K}_2\left(\ldots\ldots\left(\mathcal{K}_{n(a_2)}\right)\ldots\right)\right), t\end{bmatrix} \circledast$$

$$\mathcal{T}\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\ldots\left(\mathcal{K}_{n(x_{k-1}{}^3)}\right)\ldots\right)\right), \\ \mathcal{K}_1\left(\mathcal{K}_2\left(\ldots\left(\mathcal{K}_{n(a_3)}\right)\ldots\right)\right), t\end{bmatrix} \circledast \ldots \circledast$$

$$\mathcal{T}\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\ldots\ldots\left(\mathcal{K}_{n(x_{k-1}{}^1)}\right)\ldots\right)\right) \\ , \mathcal{K}_1\left(\mathcal{K}_2\left(\ldots\ldots\left(\mathcal{K}_{n(a_1)}\right)\ldots\right)\right), t\end{bmatrix}$$

$$\mathcal{T}\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\ldots\ldots\left(\mathcal{K}_{n(a_2)}\right)\ldots\right)\right), \\ \mathcal{R}_1\left(\mathcal{R}_2\left(\ldots\left(\mathcal{R}_{n(a_2,a_3,\ldots,a_n)}\right)\ldots\right)\right) \\ , \mathcal{G}_{(t)}\end{bmatrix} = 1, \text{ as s } n \to \infty.$$

Continuity,

$$\mathcal{T}\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_{k-1}{}^n, x_{k-1}{}^1,\dots,x_{k-1}{}^{n-1})}\right)\dots\right)\right),\\ \mathcal{R}_1\left(\mathcal{R}_2\left(\dots\left(\mathcal{R}_{n(a_n,a_1,\dots,a_{n-1})}\right)\dots\right)\right),\mathcal{G}_{(t)}\end{bmatrix}\geq$$

$$\mathcal{T}\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_{k-1}{}^n)}\right)\dots\right)\right)\\ ,\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(a_n)}\right)\dots\right)\right),t\end{bmatrix}\circledast$$

$$\mathcal{T}\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_{k-1}{}^1)}\right)\dots\right)\right)\\ ,\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(a_1)}\right)\dots\right)\right),t\end{bmatrix}\circledast\dots\circledast$$

$$\mathcal{T}\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_{k-1}{}^{n-1})}\right)\dots\right)\right),\\ \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(a_{n-1})}\right)\dots\right)\right),t\end{bmatrix}$$

$$\text{As } n\to\infty, \mathcal{T}\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(a_n)}\right)\dots\right)\right),\\ \mathcal{R}_1\left(\mathcal{R}_2\left(\dots\left(\mathcal{R}_{n(a_n,a_1,\dots,a_{n-1})}\right)\dots\right)\right)\\ ,\mathcal{G}_{(t)}\end{bmatrix}=1$$

$$\Longrightarrow \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(a_1)}\right)\dots\right)\right)$$
$$=\mathcal{R}_1\left(\mathcal{R}_2\left(\dots\left(\mathcal{R}_{n(a_1,a_2,\dots,a_n)}\right)\dots\right)\right)=x_1$$

$$\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(a_2)}\right)\dots\right)\right)=\mathcal{R}_1\left(\mathcal{R}_2\left(\dots\left(\mathcal{R}_{n(a_2,a_3,\dots,a_1)}\right)\dots\right)\right)$$
$$=x_2$$

$$\vdots$$

$$\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(a_n)}\right)\dots\right)\right)=\mathcal{R}_1\left(\mathcal{R}_2\left(\dots\left(\mathcal{R}_{n(a_n,\dots,a_{n-1})}\right)\dots\right)\right)$$
$$=x_n$$

Therefore, $(a_1, a_2, \dots\dots, a_n)$ is an n – tupled coincidence point.
■

Theorem (2.2): Let A *and* B are two families of mappings such that $A=\left\{\begin{matrix}\mathcal{R}_1,\mathcal{R}_2,\dots,\mathcal{R}_n \ s.t\ \mathcal{R}_1: \mathrm{E}^n\to \mathrm{E},\\ \mathcal{R}_2,\dots\dots, \mathcal{R}_n: \mathrm{E}^n\to \mathrm{E}^n\end{matrix}\right\}, B=$ $\mathcal{K}_1,\mathcal{K}_2,\dots\dots,\mathcal{K}_n:\mathrm{E}\to\mathrm{E}\}$ and let $(\mathrm{E},\mathcal{T},\circledast)$ be a fuzzy metric space. If the same conditions in theorems (2.1) are satisfied. Then A *and* B have a unique $(G.n)$ – tupled common fixed point of compose the mappings of $A$and B.

Proof: By theorem (2.1), the families A *and* B have a unique $(G.n)$ – tupled coincidence point. But the mappings lies in A and B are $(G.n)$ – commute, this implies

- $$\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_n\left[\mathcal{R}_1\left(\mathcal{R}_2\left(\dots\left(\mathcal{R}_{n(a_1,a_2,\dots,a_n)}\right)\dots\right)\right)\right]\right)\dots\right)\right)=$$

$$\mathcal{R}_1\left(\mathcal{R}_2\left(\dots\left(\mathcal{R}_n\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(a_1)}\right)\dots\right)\right),\\ \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(a_2)}\right)\dots\right)\right),\\ \dots,\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(a_n)}\right)\dots\right)\right)\end{bmatrix}\right)\dots\right)\right)$$

- $$\mathcal{K}_1\left(\left(\dots\left(\mathcal{K}_n\left[\mathcal{R}_1\left(\mathcal{R}_2\left(\dots\left(\mathcal{R}_{n(a_2,a_3,\dots,a_1)}\right)\dots\right)\right)\right]\right)\dots\right)\right)$$

$$=\mathcal{R}_1\left(\mathcal{R}_2\left(\dots\left(\mathcal{R}_n\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(a_2)}\right)\dots\right)\right)\\ ,\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(a_3)}\right)\dots\right)\right),\\ \dots,\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(a_1)}\right)\dots\right)\right)\end{bmatrix}\right)\dots\right)\right)$$

Continue,

$$\mathcal{K}_1\left(\left(\dots\left(\mathcal{K}_n\left[\mathcal{R}_1\left(\mathcal{R}_2\left(\dots\left(\mathcal{R}_{n(a_n,a_1,\dots,a_{n-1})}\right)\dots\right)\right)\right]\right)\dots\right)\right)$$

$$=\mathcal{R}_1\left(\mathcal{R}_2\left(\dots\left(\mathcal{R}_n\begin{bmatrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(a_n)}\right)\dots\right)\right),\\ \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(a_1)}\right)\dots\right)\right),\\ \dots,\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(a_{n-1})}\right)\dots\right)\right)\end{bmatrix}\right)\dots\right)\right)$$

By above inquisitions, we have

- $\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_1)}\right)\dots\right)\right)=$ $\mathcal{R}_1\left(\mathcal{R}_2\left(\dots\left(\mathcal{R}_{n(x_1,x_2,\dots,x_n)}\right)\dots\right)\right)$
- $\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_2)}\right)\dots\right)\right)=$ $\mathcal{R}_1\left(\mathcal{R}_2\left(\dots\left(\mathcal{R}_{n(x_2,x_3,\dots,x_1)}\right)\dots\right)\right)$ ……
- $\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_n)}\right)\dots\right)\right)=$ $\mathcal{R}_1\left(\mathcal{R}_2\left(\dots\left(\mathcal{R}_{n(x_n,x_1,\dots,x_{n-1})}\right)\dots\right)\right)$ (2.3)

Now, since $\circledast$ is a t – norm of H – type, we have

$(1-\mu)\circledast\dots\dots\circledast(1-\mu)\geq(1-\lambda)$. But $\mathcal{T}(x,y,.)$ is continuous and $\lim_{t\to\infty}\mathcal{T}_{(x,y,t)}=1$, $\forall\, x,y\in \mathrm{E}$ then there exits $t_0>0$ such that, $\mathcal{T}\left(\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_1)}\right)\dots\right)\right),x_2,t_0\right)\geq 1-\mu$

$$\mathcal{T}\left(\mathcal{S}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_2)}\right)\dots\right)\right),x_3,t_0\right)\geq 1-\mu \quad \dots..$$

$$\mathcal{T}\left(\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_n)}\right)\dots\right)\right),x_1,t_0\right)\geq 1-\mu$$

On other hand, , $\mathcal{T}\begin{pmatrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}\mathcal{K}_{n(x_1)}\right)\dots\right)\right),\\ \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_k{}^2)}\right)\dots\right)\right),\mathcal{G}_{(t_0)}\end{pmatrix}=$

$$\mathcal{T}\begin{pmatrix}\mathcal{R}_1\left(\mathcal{R}_2\left(\dots\left(\mathcal{R}_{n(x_1,x_2,\dots,x_n)}\right)\dots\right)\right),\\ \mathcal{R}_1\left(\mathcal{R}_2\left(\dots\left(\mathcal{R}_{n(x_{k-1}{}^2,x_{k-1}{}^3,\dots,x_{k-1}{}^1)}\right)\dots\right)\right),\mathcal{G}_{(t_0)}\end{pmatrix}$$

$$\geq\mathcal{T}\begin{pmatrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_1)}\right)\dots\right)\right),\\ \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_{k-1}{}^2)}\right)\dots\right)\right),t_0\end{pmatrix}$$

$$\circledast\mathcal{T}\begin{pmatrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_2)}\right)\dots\right)\right),\\ \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_{k-1}{}^3)}\right)\dots\right)\right),t_0\end{pmatrix}$$

$$\circledast\dots\dots$$

$$\circledast\mathcal{T}\begin{pmatrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_2)}\right)\dots\right)\right),\\ \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_k{}^3)}\right)\dots\right)\right),\mathcal{G}_{(t_0)}\end{pmatrix}$$

$$=$$

$$\mathcal{T}\begin{pmatrix}\mathcal{R}_1\left(\mathcal{R}_2\left(\dots\left(\mathcal{R}_{n(x_2,x_3,\dots,x_1)}\right)\dots\right)\right)\\ ,\mathcal{R}_1\left(\mathcal{R}_2\left(\dots\left(\mathcal{R}_{n(x_{k-1}{}^3,x_{k-1}{}^4,\dots,x_{k-1}{}^2)}\right)\dots\right)\right),\mathcal{G}_{(t_0)}\end{pmatrix}\geq$$

$$\mathcal{T}\begin{pmatrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_2)}\right)\dots\right)\right)\\ ,\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_{k-1}{}^3)}\right)\dots\right)\right),t_0\end{pmatrix}\circledast$$

$$\mathcal{T}\begin{pmatrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_3)}\right)\dots\right)\right),\\ \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_{k-1}{}^4)}\right)\dots\right)\right),t_0\end{pmatrix}\circledast\dots\dots\circledast$$

$$\mathcal{T}\begin{pmatrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_1)}\right)\dots\right)\right),\\ \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_{k-1}{}^2)}\right)\dots\right)\right),t_0\end{pmatrix}$$ Continue,

$$\mathcal{T}\begin{pmatrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_n)}\right)\dots\right)\right),\\ \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_k{}^1)}\right)\dots\right)\right),\mathcal{G}_{(t_0)}\end{pmatrix}=$$

$$\mathcal{T}\begin{pmatrix}\mathcal{R}_1\left(\mathcal{R}_2\left(\dots\left(\mathcal{R}_{n(x_n,x_1,\dots,x_{n-1})}\right)\dots\right)\right)\\ ,\mathcal{R}_1\left(\mathcal{R}_2\left(\dots\left(\mathcal{R}_{n(x_{k-1}{}^1,x_{k-1}{}^2,\dots,x_{k-1}{}^n)}\right)\dots\right)\right)\\ ,\mathcal{G}_{(t_0)}\end{pmatrix}\geq$$

$$\mathcal{T}\begin{pmatrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_n)}\right)\dots\right)\right),\\ \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_{k-1}{}^1)}\right)\dots\right)\right),t_0\end{pmatrix}\circledast$$

$$\mathcal{T}\begin{pmatrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_1)}\right)\dots\right)\right)\\ ,\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_{k-1}{}^2)}\right)\dots\right)\right),t_0\end{pmatrix}\circledast\dots\dots\circledast$$

$$\mathcal{T}\begin{pmatrix}\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_{n-1})}\right)\dots\right)\right)\\ ,_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_{k-1}{}^n)}\right)\dots\right)\right),t_0\end{pmatrix}.$$ As $n\to\infty$

$$\mathcal{T}\left(\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_1)}\right)\dots\right)\right),x_2,\mathcal{G}_{(t_0)}\right)\geq$$
$$\mathcal{T}\left(\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}\mathcal{K}_{n(x_n)}\right)\dots\right)\right),x_1,t_0\right)\circledast$$
$$\mathcal{T}\left(\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_1)}\right)\dots\right)\right),x_2,t_0\right)\circledast\dots\circledast$$
$$\left(\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_{n-1})}\right)\dots\right)\right),x_n,t_0\right)$$

i. $$\mathcal{T}\left(\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_2)}\right)\dots\right)\right),x_3,\mathcal{G}_{(t_0)}\right)\geq$$
$$\mathcal{T}\left(\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_2)}\right)\dots\right)\right),x_3,t_0\right)\circledast$$
$$\mathcal{T}\left(\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_3)}\right)\dots\right)\right),x_4,t_0\right)\circledast\dots\circledast$$
$$\mathcal{T}\left(\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_1)}\right)\dots\right)\right),x_2,t_0\right)$$
$$\vdots$$

ii. $$\mathcal{T}\left(\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_n)}\right)\dots\right)\right),x_1,\mathcal{G}_{(t_0)}\right)\geq$$
$$\mathcal{T}\left(\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_n)}\right)\dots\right)\right),x_1,t_0\right)\circledast$$
$$\mathcal{T}\left(\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_1)}\right)\dots\right)\right),x_2,t_0\right)\circledast\dots\dots\circledast$$
$$\mathcal{T}\left(\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_{n-1})}\right)\dots\right)\right),x_n,t_0\right).$$

By (i), (ii) and (iii)we have

$$\mathcal{T}\left(\mathcal{S}_1\left(\mathcal{S}_2\left(\dots\left(\mathcal{S}_{n(x_1)}\right)\dots\right)\right),x_2,\mathcal{G}_{(t_0)}\right)$$
$$\circledast\,\mathcal{T}\left(\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_2)}\right)\dots\right)\right),x_3,\mathcal{G}_{(t_0)}\right)$$
$$\circledast\dots\dots\circledast\,\mathcal{T}\left(\mathcal{S}_1\left(\mathcal{S}_2\left(\dots\left(\mathcal{S}_{n(n)}\right)\dots\right)\right),x_1,\mathcal{G}_{(t_0)}\right)$$
$$\geq\mathcal{T}\left[\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_n)}\right)\dots\right)\right),x_1,\mathcal{G}_{(t_0)}\right]^n$$
$$\circledast\,\mathcal{T}\left[\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_1)}\right)\dots\right)\right),x_2,\mathcal{G}_{(t_0)}\right]^n\circledast\dots\circledast$$
$$\mathcal{T}\left[\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_{n-1})}\right)\dots\right)\right),x_n,\mathcal{G}_{(t_0)}\right]^n.$$

By induction, $\mathcal{T}\left(\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_1)}\right)\dots\right)\right),x_2,\mathcal{G}^k{}_{(t_0)}\right)\circledast$

$$\mathcal{T}\left(\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_2)}\right)\dots\right)\right),x_3,\mathcal{G}^k{}_{(t_0)}\right)\circledast\dots\dots$$
$$\circledast\,\mathcal{T}\left(\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_n)}\right)\dots\right)\right),x_1,\mathcal{G}^k{}_{(t_0)}\right)$$
$$\geq\mathcal{T}\left[\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_n)}\right)\dots\right)\right),x_1,\mathcal{G}^{k-1}{}_{(t_0)}\right]^n$$
$$\circledast\,\mathcal{T}\left[\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_1)}\right)\dots\right)\right),x_2,\mathcal{G}^{k-1}{}_{(t_0)}\right]^n\circledast\dots$$
$$\circledast\,\mathcal{T}\left[\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_{n-1})}\right)\dots\right)\right),x_n,\mathcal{G}^{k-1}{}_{(t_0)}\right]^n\dots$$

But $\sum_{n=1}^{\infty}\emptyset^n{}_{(t_0)}<t$, $\forall\ n\in N$, we get

$$\mathcal{T}\left[\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_1)}\right)\dots\right)\right),x_2,t\right]^{n^k}\circledast$$
$$\mathcal{T}\left[\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(2)}\right)\dots\right)\right),x_3,t\right]^{n^k}\circledast\dots\circledast$$
$$\mathcal{T}\left[\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_n)}\right)\dots\right)\right),x_1,t\right]^{n^k}\geq$$
$$\mathcal{T}\left(\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_1)}\right)\dots\right)\right),x_2,\textstyle\sum_{k=n_0}^{\infty}\mathcal{G}^k{}_{(t_0)}\right)\circledast$$
$$\mathcal{T}\left(\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_2)}\right)\dots\right)\right),x_3,\textstyle\sum_{k=n_0}^{\infty}\mathcal{G}^k{}_{(t_0)}\right)\dots\circledast$$
$$\mathcal{T}\left(\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_n)}\right)\dots\right)\right),x_1,\textstyle\sum_{k=n_0}^{\infty}\mathcal{G}^k{}_{(t_0)}\right)\geq$$
$$\mathcal{T}\left(\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_1)}\right)\dots\right)\right),x_2,\mathcal{G}^{n_0}{}_{(t_0)}\right)\circledast$$
$$\mathcal{T}\left(\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_2)}\right)\dots\right)\right),x_3,\mathcal{G}^{n_0}{}_{(t_0)}\right)\dots\circledast$$
$$\mathcal{T}\left(\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_n)}\right)\dots\right)\right),x_1,\mathcal{G}^{n_0}{}_{(t_0)}\right)\geq$$
$$\mathcal{T}\left[\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_n)}\right)\dots\right)\right),x_1,t_0\right]^{n^{n_0}}\circledast$$
$$\mathcal{T}\left[\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_1)}\right)\dots\right)\right),x_2,t_0\right]^{n^{n_0}}\circledast\dots\circledast$$
$$\mathcal{T}\left[\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_{n-1})}\right)\dots\right)\right),x_n,t_0\right]^{n^{n_0}}\geq(1-\mu)\circledast(1-\mu)\circledast\dots\circledast(1-\mu)\geq(1-\lambda).$$ Therefore,

$$\mathcal{T}\left(\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_1)}\right)\dots\right)\right),x_2,t\right)$$
$$\circledast\,\mathcal{T}\left(\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_2)}\right)\dots\right)\right),x_3,t\right)$$
$$\circledast\dots\dots$$
$$\circledast\,\mathcal{T}\left(\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_n)}\right)\dots\right)\right),x_1,t\right)$$
$$\geq(1-\lambda)$$

$\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_1)}\right)\dots\right)\right)=x_2$, $\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_2)}\right)\dots\right)\right)=x_3$, …. $\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_n)}\right)\dots\right)\right)=x_1$ (2.4). Now, we will show that $x_1=x_2=\dots\dots=x_n$. As that same way, since $\mathcal{T}(x,y,.)$ is continuous and $\lim_{t\to\infty}\mathcal{T}_{(x,y,t)}=1$, $\forall\,x,y\in\mathrm{E}\Longrightarrow\exists\,t_0>0$ such that $\mathcal{T}_{(x_1,x_2,t_0)}\geq1-\mu$, $\mathcal{T}_{(x_2,x_3,t_0)}\geq1-\mu$ … , $\mathcal{T}_{(x_n,x_1,t_0)}\geq1-\mu$. But, $\sum_{n=1}^{\infty}\mathcal{G}^n{}_{(t_0)}<\infty$, then we have,

- $$\mathcal{T}\begin{pmatrix} \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_k{}^1)}\right)\dots\right)\right), \\ \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_k{}^2)}\right)\dots\right)\right), \mathcal{G}_{(t_0)} \end{pmatrix} =$$

$$\mathcal{T}\begin{pmatrix} \mathcal{R}_1\left(\mathcal{R}_2\left(\dots\left(\mathcal{R}_{n(x_{k-1}{}^1,\dots,x_{k-1}{}^n)}\right)\dots\right)\right) \\ ,\mathcal{R}_1\left(\mathcal{R}_2\left(\dots\left(\mathcal{R}_{n(x_{k-1}{}^2,\dots,x_{k-1}{}^1)}\right)\dots\right)\right), \mathcal{G}_{(t_0)} \end{pmatrix}$$

$$\geq \mathcal{T}\begin{pmatrix} \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_{k-1}{}^1)}\right)\dots\right)\right), \\ \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_{k-1}{}^2)}\right)\dots\right)\right) \end{pmatrix} \circledast \dots\dots \circledast \mathcal{T}\begin{pmatrix} \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_{k-1}{}^n)}\right)\dots\right)\right), \\ \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_{k-1}{}^1)}\right)\dots\right)\right) \end{pmatrix}$$

Also,

- $$\mathcal{T}\begin{pmatrix} \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_k{}^2)}\right)\dots\right)\right), \\ \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_k{}^3)}\right)\dots\right)\right), \mathcal{G}_{(t_0)} \end{pmatrix} =$$

$$\mathcal{T}\begin{pmatrix} \mathcal{R}_1\left(\mathcal{R}_2\left(\dots\left(\mathcal{R}_{n(x_{k-1}{}^2,\dots,x_{k-1}{}^1)}\right)\dots\right)\right) \\ ,\mathcal{R}_1\left(\mathcal{R}_2\left(\dots\left(\mathcal{R}_{n(x_{k-1}{}^3,\dots,x_{k-1}{}^2)}\right)\dots\right)\right), \mathcal{G}_{(t_0)} \end{pmatrix} \geq$$

$$\mathcal{T}\begin{pmatrix} \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_{k-1}{}^2)}\right)\dots\right)\right), \\ \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_{k-1}{}^3)}\right)\dots\right)\right), t_0 \end{pmatrix} \circledast \dots\dots \circledast$$

$$\mathcal{T}\begin{pmatrix} \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_{k-1}{}^1)}\right)\dots\right)\right), \\ \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_{k-1}{}^2)}\right)\dots\right)\right), t_0 \end{pmatrix}$$

Continue,

- $$\mathcal{T}\begin{pmatrix} \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_k{}^n)}\right)\dots\right)\right), \\ \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_k{}^1)}\right)\dots\right)\right), \mathcal{G}_{(t_0)} \end{pmatrix} =$$

$$\mathcal{T}\begin{pmatrix} \mathcal{R}_1\left(\mathcal{R}_2\left(\dots\left(\mathcal{R}_{n(x_{k-1}{}^n,\dots,x_{k-1}{}^{n-1})}\right)\dots\right)\right), \\ \mathcal{R}_1\left(\mathcal{R}_2\left(\dots\left(\mathcal{R}_{n(x_{k-1}{}^1,\dots,x_{k-1}{}^n)}\right)\dots\right)\right), \mathcal{G}_{(t_0)} \end{pmatrix} \geq$$

$$\mathcal{T}\begin{pmatrix} \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\dots\left(\mathcal{K}_{n(x_{k-1}{}^n)}\right)\dots\right)\right), \\ \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_{k-1}{}^1)}\right)\dots\right)\right), t_0 \end{pmatrix} \circledast \dots\dots \circledast$$

$$\mathcal{T}\begin{pmatrix} \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_{k-1}{}^{n-1})}\right)\dots\right)\right), \\ \mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_{k-1}{}^n)}\right)\dots\right)\right), t_0 \end{pmatrix}$$

As $n \to \infty$, we get,

$$\mathcal{T}(x_1, x_2, \mathcal{G}_{(t_0)}) \geq \mathcal{T}(x_1, x_2, t_0) \circledast \mathcal{T}(x_2, x_3, t_0) \circledast \dots\dots \circledast \mathcal{T}(x_n, x_1, t_0)$$

$$\mathcal{T}(x_2, x_3, \mathcal{G}_{(t_0)}) \geq \mathcal{T}(x_2, x_3, t_0) \circledast \mathcal{T}(x_3, x_4, t_0) \circledast \dots\dots \circledast \mathcal{T}(x_1, x_2, t_0)$$ Continue,

$$\mathcal{T}(x_n, x_1, \mathcal{G}_{(t_0)}) \geq \mathcal{T}(x_n, x_1, t_0) \circledast \mathcal{T}(x_1, x_2, t_0) \circledast \dots\dots \circledast \mathcal{T}(x_{n-1}, x_n, t_0)$$
$$\Longrightarrow \mathcal{T}(x_1, x_2, \mathcal{G}_{(t_0)}) \circledast \mathcal{T}(x_2, x_3, \mathcal{G}_{(t_0)}) \circledast \dots\dots \circledast \mathcal{T}(x_n, x_1, \mathcal{G}_{(t_0)})$$
$$\geq [\mathcal{T}(x_1, x_2, t_0)]^n \circledast [\mathcal{T}(x_2, x_3, t_0)]^n \circledast \dots\dots \circledast [\mathcal{T}(x_n, x_1, t_0)]^n$$

By induction,

$$\mathcal{T}\left(x_1, x_2, \mathcal{G}^k{}_{(t_0)}\right) \circledast \mathcal{T}\left(x_2, x_3, \mathcal{G}^k{}_{(t_0)}\right) \circledast \dots\dots \circledast \mathcal{T}\left(x_n, x_1, \mathcal{G}^k{}_{(t_0)}\right)$$
$$\geq \left[\mathcal{T}\left(x_1, x_2, \mathcal{G}^{k-1}{}_{(t_0)}\right)\right]^n \circledast \left[\mathcal{T}\left(x_2, x_3, \mathcal{G}^{k-1}{}_{(t_0)}\right)\right]^n \circledast \dots\dots * \left[\mathcal{T}\left(x_n, x_1, \mathcal{G}^{k-1}{}_{(t_0)}\right)\right]^n \dots$$
$$\geq [\mathcal{T}(x_1, x_2, t_0)]^{n^k} \circledast [\mathcal{T}(x_2, x_3, t_0)]^{n^k} \circledast \dots\dots \circledast [\mathcal{T}(x_n, x_1, t_0)]^{n^k}$$

Now, $\mathcal{T}(x_1, x_2, t) \circledast \mathcal{T}(x_2, x_3, t) \circledast \dots\dots \circledast \mathcal{T}(x_n, x_1, t) \geq \mathcal{T}\left(x_1, x_2, \sum_{k=n_0}^{\infty} \mathcal{G}^k{}_{(t_0)}\right) \circledast \mathcal{T}\left(x_2, x_3, \sum_{k=n_0}^{\infty} \mathcal{G}^k{}_{(t_0)}\right) \circledast \dots\dots \circledast \mathcal{T}\left(x_n, x_1, \sum_{k=n_0}^{\infty} \mathcal{G}^k{}_{(t_0)}\right) \geq \mathcal{T}\left(x_1, x_2, \mathcal{G}^{n_0}{}_{(t_0)}\right) \circledast \mathcal{T}\left(x_2, x_3, \mathcal{G}^{n_0}{}_{(t_0)}\right) \circledast \dots\dots \circledast \mathcal{T}\left(x_n, x_1, \mathcal{G}^{n_0}{}_{(t_0)}\right) \geq [\mathcal{T}(x_1, x_2, t_0)]^{n^{n_0}} \circledast [\mathcal{T}(x_2, x_3, t_0)]^{n^{n_0}} \circledast \dots\dots \circledast [\mathcal{T}(x_n, x_1, t_0)]^{n^{n_0}} \geq (1-\mu) \circledast (1-\mu) \circledast \dots\dots \circledast (1-\mu) \geq 1 - \lambda$

Therefore, $x_1 = x_2 = x_3 = \dots = x_n$. And hence, by (2.3) and (2.4)

$$\mathcal{K}_1\left(\mathcal{K}_2\left(\dots\left(\mathcal{K}_{n(x_1)}\right)..\right)\right) = \mathcal{R}_1\left(\mathcal{R}_2\left(\dots\left(\mathcal{R}_{n(x_1, x_2, \dots, x_n)}\right)..\right)\right) = x_1$$

$$\mathcal{K}_1\left(\mathcal{K}_2\left(\left(\mathcal{K}_{n(x_2)}\right)..\right)\right) = \mathcal{R}_1\left(\mathcal{R}_2\left(\dots\left(\mathcal{R}_{n(x_2, x_3, \dots, x_1)}\right)..\right)\right) = x_2 .$$

And, $\mathcal{K}_1\left(\mathcal{K}_2\left(..\left(\mathcal{K}_{n(x_n)}\right)..\right)\right) = \mathcal{R}_1\left(\mathcal{R}_2\left(..\left(\mathcal{R}_{n(x_n, x_1, \dots, x_{n-1})}\right)..\right)\right) = x_n$ ■

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