

(G, n)-Tupled Coincidence Point Theorems in the Fuzzy Metric Spaces

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Abstract: The purpose of this work is to introduce a new concepts of (G, n) – tupled fixed point and (G, n) – tupled coincidence point, also we prove the existence and uniqueness of (G, n) – tupled coincidence point in fuzzy metric space .the main results obtained in this paper extend and improve of many results well_ known in the fuzzy metric spaces.

Keywords: fuzzy metric spaces, continuous t – norm, fixed point, upper semi – continuous, equicontinuous.

1. Introduction

One of the main theorems in the fixed point theory is Banach contraction theorem [1]. Many authors studied the generalization of this theorem on the complete partial ordered metric space was given by Ran and Reurings [2]. Bhaskar and Lakshmikantham [3] introduced the concept of coupled fixed point. Later on, Lakshmikantham and Ćirić [4] introduced concept of coincidence point which is generalization of fixed point. The coupled fixed point theorems are studied for type of contraction mapping see [5,6]. On other hand, the theory of fuzzy set was introduced by Zadeh [7]. Kramosil and Michalek [8] introduced the fuzzy metric space by generalized the concept of probabilistic metric space to fuzzy situation.

In this paper, we introduced a concepts of (G, n)- tupled coincidence point, (G, n) – tupled coincidence point and (G, n) – commute. Also, we establish the existence and uniqueness of (G, n) – tupled coincidence point theorems in fuzzy metric space with complete partial ordered.

Now, we recall some of the definitions used in this paper

Definition 1. [3]

A binary operation $*$: $[0,1]^2 \rightarrow [0,1]$ is called a continuous t – norm if the following conditions are satisfy:

- $*$ is a associative and commutative.
- $a * 1 = a \quad \forall a \in [0,1]$.
- $a * b \leq c * d$ whenever $a \leq c$ & $b \leq d, \forall a, b, c, d \in [0,1]$.
- $*$ is continuous.

Definition 2. [4]

A triple $(X, M, *)$ is called fuzzy metric space if $X \neq \emptyset$, $*$ is continuous t – norm and $M: X \times X \times (0, \infty) \rightarrow [0,1]$ is a fuzzy set the satisfying the following conditions:

- $M_{(x,y,t)} > 0$
- $M_{(x,y,t)} = 1$ iff $x = y$
- $M_{(x,y,t)} = M_{(y,x,t)}$
- $M_{(x,y, \cdot)}: (0, \infty) \rightarrow [0,1]$ is continuous.
- $M_{(x,z,t+s)} \geq M_{(x,y,t)} * M_{(y,z,s)} \quad \forall t, s > 0$

We will add the condition $\lim_{t \rightarrow \infty} M_{(x,y,t)} = 1 \quad \forall x, y \in X$

Lemma 3. [3]

In any fuzzy metric space $(X, M, *)$, where $*$ is a continuous t – norm of H – type. If there exist $\emptyset \in \Phi$ such that $M_{(x,y,\emptyset(t))} \leq M_{(x,y,t)}, \forall t > 0$ then $x = y$.

Definition 4. [9]

For any $a \in [0,1]$, the sequence $\langle *^n a \rangle_{n=1}^{\infty}$ be defined by:

$*^1 a = a$ and $*^n a = (*^{n-1} a) * a$. Then a t – norm $*$ is said to be of H – type if the sequence $\langle *^n a \rangle_{n=1}^{\infty}$ is equicontinuous at $a = 1$.

Definition 5. [10]

Let $(X, M, *)$ be a fuzzy metric space $(X, M, *)$.

- A sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$ if $\lim_{t \rightarrow \infty} M_{(x_n, x, t)} = 1$ for all $t > 0$.
- A sequence $\{x_n\}$ in X is called a Cauchy sequence if for each $0 < \varepsilon < 1$ and $t > 0$, there exists a positive integer n_0 such that $M_{(x_n, x_m, t)} > 1 - \varepsilon$ for each $n, m \geq n_0$

Definition 6.

Let $f_1, f_2, \dots, f_n: X^n \rightarrow X$ are mappings. Any element $(x_1, x_2, \dots, x_n) \in X^n$ is called a (G, n)-tupled fixed point of this mappings if

$$\begin{aligned} f_1(f_2(\dots(f_n(x_1, x_2, \dots, x_n)) \dots)) &= x_1 \\ f_1(f_2(\dots(f_n(x_2, x_3, \dots, x_1)) \dots)) &= x_2 \\ &\vdots \\ f_1(f_2(\dots(f_n(x_n, x_1, \dots, x_{n-1})) \dots)) &= x_n \end{aligned}$$

Definition 7.

Let $f_1, f_2, \dots, f_n: X^n \rightarrow X$ and $g_1, g_2, \dots, g_n: X \rightarrow X$ are mappings. Any element $(x_1, x_2, \dots, x_n) \in X^n$ is called (G, n) – tupled coincidence point of this mapping if

$$\begin{aligned} f_1(f_2(\dots(f_n(x_1, x_2, \dots, x_n)) \dots)) & \\ &= g_1(g_2(\dots(g_n(x_1)) \dots)) \\ f_1(f_2(\dots(f_n(x_2, x_3, \dots, x_1)) \dots)) & \\ &= g_1(g_2(\dots(g_n(x_2)) \dots)) \\ &\vdots \\ &\vdots \end{aligned}$$

$$f_1 \left(f_2 \left(\dots \left(f_n(x_{n,x_1, \dots, x_{n-1}}) \dots \right) \right) \right) \\ = g_1 \left(g_2 \left(\dots \left(g_n(x_n) \dots \right) \right) \right)$$

In this paper, we consider Φ is the set of all mappings $\Phi: [0, \infty) \rightarrow [0, \infty)$ such that:

- Φ is non-decreasing.
- Φ is upper semi-continuous from the right.
- $\sum_{n=0}^{\infty} \Phi^n(t) < \infty$; $\forall t > 0$ where $\Phi^{n+1}(t) = \Phi(\Phi^n(t))$, $n \in \mathbb{N}$.

$$M[f_1 \left(f_2 \left(\dots \left(f_n(x_{1,x_2, \dots, x_n}) \dots \right) \right) \right), f_1 \left(f_2 \left(\dots \left(f_n(y_{1,y_2, \dots, y_n}) \dots \right) \right) \right), \Phi(t)] \geq \\ M \left[g_1 \left(g_2 \left(\dots \left(g_n(x_1) \dots \right) \right) \right), g_1 \left(g_2 \left(\dots \left(g_n(y_1) \dots \right) \right) \right), t \right] * \\ M \left[g_1 \left(g_2 \left(\dots \left(g_n(x_2) \dots \right) \right) \right), g_1 \left(g_2 \left(\dots \left(g_n(y_2) \dots \right) \right) \right), t \right] * \dots * \\ M \left[g_1 \left(g_2 \left(\dots \left(g_n(x_n) \dots \right) \right) \right), g_1 \left(g_2 \left(\dots \left(g_n(y_n) \dots \right) \right) \right), t \right] (1)$$

Where $t > 0$ and $x_i, y_i \in X \quad \forall i = 1, 2, \dots, n$

If $g_1 \left(g_2 \left(\dots \left(g_n(x) \dots \right) \right) \right)$ is complete subspace of X containing $f_1 \left(f_2 \left(\dots \left(f_n(x^n) \dots \right) \right) \right)$, then there exists a unique (G, n) -tupled coincidence fixed point of compose the mappings of A and B .

Proof:

Consider $x_0^1, x_0^2, \dots, x_1^n \in X$,
 $g_1 \left(g_2 \left(\dots \left(g_n(x) \dots \right) \right) \right)$
 $f_1 \left(f_2 \left(\dots \left(f_n(x^n) \dots \right) \right) \right)$,
 $x_1^1, x_1^2, \dots, x_1^n \in X$ such that
 $g_1 \left(g_2 \left(\dots \left(g_n(x_1^1) \dots \right) \right) \right) \\ = f_1 \left(f_2 \left(\dots \left(f_n(x_0^1, x_0^2, \dots, x_0^n) \dots \right) \right) \right)$
 $g_1 \left(g_2 \left(\dots \left(g_n(x_1^2) \dots \right) \right) \right) \\ = f_1 \left(f_2 \left(\dots \left(f_n(x_0^2, x_0^3, \dots, x_0^n, x_0^1) \dots \right) \right) \right)$
 \vdots
 $g_1 \left(g_2 \left(\dots \left(g_n(x_1^n) \dots \right) \right) \right) \\ = f_1 \left(f_2 \left(\dots \left(f_n(x_0^n, x_0^1, \dots, x_0^{n-1}) \dots \right) \right) \right)$

Also,

$$g_1 \left(g_2 \left(\dots \left(g_n(x_2^1) \dots \right) \right) \right) \\ = f_1 \left(f_2 \left(\dots \left(f_n(x_1^1, x_1^2, \dots, x_1^n) \dots \right) \right) \right)$$

$$g_1 \left(g_2 \left(\dots \left(g_n(x_2^2) \dots \right) \right) \right) \\ = f_1 \left(f_2 \left(\dots \left(f_n(x_1^2, x_1^3, \dots, x_1^n, x_1^1) \dots \right) \right) \right)$$

$$\vdots$$

$$M \left[g_1 \left(g_2 \left(\dots \left(g_n(x_0^1) \dots \right) \right) \right), g_1 \left(g_2 \left(\dots \left(g_n(x_1^1) \dots \right) \right) \right), t_0 \right] \geq 1 - \mu$$

$$M \left[g_1 \left(g_2 \left(\dots \left(g_n(x_0^2) \dots \right) \right) \right), g_1 \left(g_2 \left(\dots \left(g_n(x_1^2) \dots \right) \right) \right), t_0 \right] \geq 1 - \mu$$

∴(2)

$$\vdots$$

$$M \left[g_1 \left(g_2 \left(\dots \left(g_n(x_0^n) \dots \right) \right) \right), g_1 \left(g_2 \left(\dots \left(g_n(x_1^n) \dots \right) \right) \right), t_0 \right] \geq 1 - \mu$$

By using (1), we get

$$M \left[g_1 \left(g_2 \left(\dots \left(g_n(x_1^1) \dots \right) \right) \right), g_1 \left(g_2 \left(\dots \left(g_n(x_2^1) \dots \right) \right) \right), \Phi(t_0) \right] =$$

2. Main Results

Theorem (1):

Let A and B are two families of mappings such that $A = \{f_1, f_2, \dots, f_n: X^n \rightarrow X\}$, $B = \{g_1, g_2, \dots, g_n: X \rightarrow X\}$ and let $(X, M, *)$ be a fuzzy metric space such that $*$ is a t -norm of H -type. Suppose that $\Phi \in \Phi$ satisfying

$$g_1 \left(g_2 \left(\dots \left(g_n(x_2^n) \dots \right) \right) \right) \\ = f_1 \left(f_2 \left(\dots \left(f_n(x_1^n, x_1^1, \dots, x_1^{n-1}) \dots \right) \right) \right)$$

In general, we can construct the sequences,

$$< g_1 \left(g_2 \left(\dots \left(g_n(x_k^1) \dots \right) \right) \right) >, \\ < g_1 \left(g_2 \left(\dots \left(g_n(x_k^2) \dots \right) \right) \right) >, \dots, \text{ and } < \\ g_1 \left(g_2 \left(\dots \left(g_n(x_k^n) \dots \right) \right) \right) > \text{ as}$$

$$g_1 \left(g_2 \left(\dots \left(g_n(x_k^1) \dots \right) \right) \right) \\ = f_1 \left(f_2 \left(\dots \left(f_n(x_{k-1}^1, x_{k-1}^2, \dots, x_{k-1}^n) \dots \right) \right) \right)$$

$$g_1 \left(g_2 \left(\dots \left(g_n(x_k^2) \dots \right) \right) \right) \\ = f_1 \left(f_2 \left(\dots \left(f_n(x_{k-1}^2, x_{k-1}^3, \dots, x_{k-1}^n, x_{k-1}^1) \dots \right) \right) \right)$$

$$\vdots$$

$$g_1 \left(g_2 \left(\dots \left(g_n(x_k^n) \dots \right) \right) \right) \\ = f_1 \left(f_2 \left(\dots \left(f_n(x_{k-1}^n, x_{k-1}^1, \dots, x_{k-1}^{n-1}) \dots \right) \right) \right)$$

We want to show that the above sequences are Cauchy sequences in $(X, M, *)$, since $*$ is t -norm of H -type, this implies

$\forall \delta > 0 \exists \mu > 0$ such that :
 $(1 - \mu) * (1 - \mu) * \dots * (1 - \mu) \geq 1 - \delta, \forall n \in \mathbb{N}$. on other hand. For all $x, y \in X$, $M(x, y, \cdot)$ is continuous and $\lim_{t \rightarrow \infty} M(x, y, t) = 1$ then there exists $t_0 > 0$ such that.

$$\begin{aligned}
 & M \left[f_1 \left(f_2 \left(\dots \left(f_n(x_0^1, x_0^2, \dots, x_0^n) \right) \dots \right) \right), f_1 \left(f_2 \left(\dots \left(f_n(x_1^1, x_1^2, \dots, x_1^n) \right) \dots \right) \right), \emptyset_{(t_0)} \right] \\
 & \geq M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_0^1) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^1) \right) \dots \right) \right), t_0 \right] * \\
 & M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_0^2) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^2) \right) \dots \right) \right), t_0 \right] * \dots * \\
 & M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_0^n) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^n) \right) \dots \right) \right), t_0 \right]
 \end{aligned}$$

Also,

$$\begin{aligned}
 & M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^2) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_2^2) \right) \dots \right) \right), \emptyset_{(t_0)} \right] = \\
 & M \left[f_1 \left(f_2 \left(\dots \left(f_n(x_0^2, x_0^3, \dots, x_0^n, x_0^1) \right) \dots \right) \right), f_1 \left(f_2 \left(\dots \left(f_n(x_1^2, x_1^3, \dots, x_1^n, x_0^1) \right) \dots \right) \right), \emptyset_{(t_0)} \right] \\
 & \geq M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_0^2) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^2) \right) \dots \right) \right), t_0 \right] * \\
 & M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_0^3) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^3) \right) \dots \right) \right), t_0 \right] * \dots * \\
 & M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_0^n) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^n) \right) \dots \right) \right), t_0 \right]
 \end{aligned}$$

We continue this process in the same way

$$\begin{aligned}
 & M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^n) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_2^n) \right) \dots \right) \right), \emptyset_{(t_0)} \right] = \\
 & M \left[f_1 \left(f_2 \left(\dots \left(f_n(x_0^n, x_0^1, \dots, x_0^{n-1}) \right) \dots \right) \right), f_1 \left(f_2 \left(\dots \left(f_n(x_1^n, x_1^1, \dots, x_1^{n-1}) \right) \dots \right) \right), t_0 \right] \\
 & \geq M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_0^n) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^n) \right) \dots \right) \right), t_0 \right] * \\
 & M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_0^1) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^1) \right) \dots \right) \right), t_0 \right] * \dots * \\
 & M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_0^{n-1}) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^{n-1}) \right) \dots \right) \right), t_0 \right]
 \end{aligned}$$

As the same way and by using above inequalities,

$$\begin{aligned}
 & M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_2^1) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_3^1) \right) \dots \right) \right), \emptyset^2_{(t_0)} \right] = \\
 & M \left[f_1 \left(f_2 \left(\dots \left(f_n(x_1^1, x_1^2, \dots, x_1^n) \right) \dots \right) \right), f_1 \left(f_2 \left(\dots \left(f_n(x_2^1, x_2^2, \dots, x_2^n) \right) \dots \right) \right), \emptyset^2_{(t_0)} \right] \\
 & \geq M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^1) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_2^1) \right) \dots \right) \right), \emptyset_{(t_0)} \right] * \\
 & M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^2) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_2^2) \right) \dots \right) \right), \emptyset_{(t_0)} \right] * \dots * \\
 & M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^n) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_2^n) \right) \dots \right) \right), \emptyset_{(t_0)} \right] \\
 & \geq M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_0^1) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^1) \right) \dots \right) \right), t_0 \right]^n * \\
 & M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_0^2) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^2) \right) \dots \right) \right), t_0 \right]^n * \dots * \\
 & M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_0^n) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^n) \right) \dots \right) \right), t_0 \right]^n
 \end{aligned}$$

- $$\begin{aligned}
 & M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_2^2) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_3^2) \right) \dots \right) \right), \emptyset^2_{(t_0)} \right] = \\
 & M \left[f_1 \left(f_2 \left(\dots \left(f_n(x_1^2, x_1^3, \dots, x_1^n, x_1^1) \right) \dots \right) \right), f_1 \left(f_2 \left(\dots \left(f_n(x_2^2, x_2^3, \dots, x_2^n, x_1^1) \right) \dots \right) \right), \emptyset^2_{(t_0)} \right] \\
 & \geq M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^2) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_2^2) \right) \dots \right) \right), \emptyset_{(t_0)} \right] * \\
 & M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^3) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_2^3) \right) \dots \right) \right), \emptyset_{(t_0)} \right] * \dots * \\
 & M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^n) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_2^n) \right) \dots \right) \right), \emptyset_{(t_0)} \right] \\
 & \geq M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_0^n) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^n) \right) \dots \right) \right), t_0 \right]^n * \\
 & M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_0^1) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^1) \right) \dots \right) \right), t_0 \right]^n * \dots * \\
 & M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_0^{n-1}) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^{n-1}) \right) \dots \right) \right), t_0 \right]^n
 \end{aligned}$$

Continue this process, we get

$$\begin{aligned}
 & M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_2^n) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_3^n) \right) \dots \right) \right), \emptyset^2_{(t_0)} \right] = \\
 & M \left[f_1 \left(f_2 \left(\dots \left(f_n(x_2^n, x_2^1, \dots, x_2^{n-1}) \right) \dots \right) \right), f_1 \left(f_2 \left(\dots \left(f_n(x_3^n, x_3^1, \dots, x_3^n) \right) \dots \right) \right), \emptyset^2_{(t_0)} \right]
 \end{aligned}$$

$$\begin{aligned} &\geq M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_2^n) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_3^n) \right) \dots \right) \right), \emptyset_{(t_0)} \right] * \\ M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_2^1) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_3^1) \right) \dots \right) \right), \emptyset_{(t_0)} \right] * \dots * \\ M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_2^{n-1}) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_3^{n-1}) \right) \dots \right) \right), \emptyset_{(t_0)} \right] \\ &\geq M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_0^1) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^1) \right) \dots \right) \right), t_0 \right]^n * \\ M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_0^2) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^2) \right) \dots \right) \right), t_0 \right]^n * \dots * \\ &M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_0^n) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^n) \right) \dots \right) \right), t_0 \right]^n \end{aligned}$$

Similarly

- $$\begin{aligned} M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_{k-1}) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_{k+1}) \right) \dots \right) \right), \emptyset^k_{(t_0)} \right] = \\ M \left[f_1 \left(f_2 \left(\dots \left(f_n(x_{k-1}^1, x_{k-1}^2, \dots, x_{k-1}^n) \right) \dots \right) \right), f_1 \left(f_2 \left(\dots \left(f_n(x_k^1, x_k^2, \dots, x_k^n) \right) \dots \right) \right), \emptyset^k_{(t_0)} \right] \\ \geq M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_{k-1}^1) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_k^1) \right) \dots \right) \right), \emptyset^{k-1}_{(t_0)} \right] * \\ M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_{k-1}^2) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_k^2) \right) \dots \right) \right), \emptyset^{k-1}_{(t_0)} \right] * \dots * \\ M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_{k-1}^n) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_k^n) \right) \dots \right) \right), \emptyset^{k-1}_{(t_0)} \right] \\ \vdots \\ \geq M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_0^1) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^1) \right) \dots \right) \right), t_0 \right]^{n^{k-1}} * \\ M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_0^2) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^2) \right) \dots \right) \right), t_0 \right]^{n^{k-1}} * \dots * \\ M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_0^n) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^n) \right) \dots \right) \right), t_0 \right]^{n^{k-1}} \end{aligned}$$

Also,

$$\begin{aligned} M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_k^2) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_{k+1}^2) \right) \dots \right) \right), \emptyset^k_{(t_0)} \right] = \\ M \left\{ f_1 \left(f_2 \left(\dots \left(f_n(x_{k-1}^2, x_{k-1}^3, \dots, x_{k-1}^n, x_{k-1}^1) \right) \dots \right) \right), \right. \\ \left. f_1 \left(f_2 \left(\dots \left(f_n(x_k^2, x_k^3, \dots, x_k^n, x_k^1) \right) \dots \right) \right), \emptyset^k_{(t_0)} \right\} \\ \geq M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_{k-1}^2) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_k^2) \right) \dots \right) \right), \emptyset^{k-1}_{(t_0)} \right] * \\ M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_{k-1}^3) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_k^3) \right) \dots \right) \right), \emptyset^{k-1}_{(t_0)} \right] * \dots * \\ M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_{k-1}^1) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_k^1) \right) \dots \right) \right), \emptyset^{k-1}_{(t_0)} \right] \\ \vdots \\ \geq M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_0^2) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^2) \right) \dots \right) \right), t_0 \right]^{n^{k-1}} * \\ M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_0^3) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^3) \right) \dots \right) \right), t_0 \right]^{n^{k-1}} * \dots * \\ M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_0^1) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^1) \right) \dots \right) \right), t_0 \right]^{n^{k-1}} \end{aligned}$$

Continue this process, as the same way we get

$$\begin{aligned} M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_k^n) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_{k+1}^n) \right) \dots \right) \right), \emptyset^k_{(t_0)} \right] \\ \geq M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_0^n) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^n) \right) \dots \right) \right), t_0 \right]^{n^{k-1}} * \\ M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_0^1) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^1) \right) \dots \right) \right), t_0 \right]^{n^{k-1}} * \dots * \\ M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_0^{n-1}) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^{n-1}) \right) \dots \right) \right), t_0 \right]^{n^{k-1}} \end{aligned}$$

Now, by using above inequalities and for each $n_0 \leq n < m$, we have

$$M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_k^n) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_m^n) \right) \dots \right) \right), \sum_{k=n_0}^{\infty} \emptyset^k_{(t_0)} \right]$$

$$\begin{aligned}
 &\geq M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_k^n) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_m^n) \right) \dots \right) \right), \sum_{k=n_0}^{m-1} \phi^{k+1}(t_0) \right] \\
 &\quad \geq M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_k^n) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_{k+1}^n) \right) \dots \right) \right), \phi^k(t_0) \right] * \\
 &\quad \quad M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_{k+1}^n) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_{k+2}^n) \right) \dots \right) \right), \phi^{k+1}(t_0) \right] * \dots * \\
 &\quad \quad M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_{m-1}^n) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_m^n) \right) \dots \right) \right), \phi^{m-1}(t_0) \right] \\
 &\quad \quad \quad \vdots \\
 &\quad \quad \quad \vdots \\
 &\quad \geq M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_0^n) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^n) \right) \dots \right) \right), t_0 \right]^{n^{k-1}} * \\
 &\quad \quad M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_0^1) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^1) \right) \dots \right) \right), t_0 \right]^{n^{k-1}} * \dots * \\
 &\quad M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_0^{n-1}) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^{n-1}) \right) \dots \right) \right), t_0 \right]^{n^{k-1}} * \\
 &\quad \quad M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_0^n) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^n) \right) \dots \right) \right), t_0 \right]^{n^k} * \\
 &\quad \quad M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_0^1) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^1) \right) \dots \right) \right), t_0 \right]^{n^k} * \dots * \\
 &\quad M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_0^{n-1}) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^{n-1}) \right) \dots \right) \right), t_0 \right]^{n^k} * \dots * \\
 &\quad \quad M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_0^n) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^n) \right) \dots \right) \right), t_0 \right]^{n^{m-2}} * \\
 &\quad \quad M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_0^1) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^1) \right) \dots \right) \right), t_0 \right]^{n^{m-2}} * \dots * \\
 &\quad M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_0^{n-1}) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^{n-1}) \right) \dots \right) \right), t_0 \right]^{n^{m-2}}
 \end{aligned}$$

Let $l = \max\{n^{k-1}, n^k, n^{m-2}\}$

$$\begin{aligned}
 &\geq M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_0^n) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^n) \right) \dots \right) \right), t_0 \right]^l * \\
 &\quad M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_0^1) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^1) \right) \dots \right) \right), t_0 \right]^l * \dots * \\
 &\quad M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_0^{n-1}) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^{n-1}) \right) \dots \right) \right), t_0 \right]^l * \\
 &\quad \quad M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_0^n) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^n) \right) \dots \right) \right), t_0 \right]^l * \\
 &\quad \quad M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_0^1) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^1) \right) \dots \right) \right), t_0 \right]^l * \dots * \\
 &\quad \quad M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_0^n) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^n) \right) \dots \right) \right), t_0 \right]^l * \\
 &\quad \quad M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_0^1) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^1) \right) \dots \right) \right), t_0 \right]^l * \dots * \\
 &\quad M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_0^{n-1}) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^{n-1}) \right) \dots \right) \right), t_0 \right]^l \\
 &\quad > M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_0^n) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^n) \right) \dots \right) \right), t_0 \right]^{ml} * \\
 &\quad \quad M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_0^1) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^1) \right) \dots \right) \right), t_0 \right]^{ml} * \dots * \\
 &\quad M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_0^{n-1}) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_1^{n-1}) \right) \dots \right) \right), t_0 \right]^{ml} \\
 &\quad \quad \geq (1 - \mu) * (1 - \mu) * \dots * (1 - \mu) \geq (1 - \Delta)
 \end{aligned}$$

And hence,

$$M \left[g_1 \left(g_2 \left(\dots \dots \left(g_n(x_k^n) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \dots \left(g_n(x_m^n) \right) \dots \right) \right), t \right] > (1 - \Delta)$$

So, $\langle g_1 \left(g_2 \left(\dots \dots \left(g_n(x_k^n) \right) \dots \right) \right) \rangle$ is Cauchy sequence.

As the same way, we get

$$\langle g_1 \left(g_2 \left(\dots \dots \left(g_n(x_k^1) \right) \dots \right) \right) \rangle, \langle g_1 \left(g_2 \left(\dots \dots \left(g_n(x_k^2) \right) \dots \right) \right) \rangle \text{ and}$$

$\langle g_1 \left(g_2 \left(\dots \dots \left(g_n(x_k^{n-1}) \right) \dots \right) \right) \rangle$ are Cauchy sequences

Now, to prove that the mappings in A and B have (G, n) -tuple coincidence fixed point.

Since $g_1(g_2(\dots(g_n(x))\dots))$ is complete subspace of X then there exists $x_1, x_2, \dots, x_n \in g_1(g_2(\dots(g_n(x))\dots))$ and $a_1, a_2, \dots, a_n \in X$ such that

$$\lim_{k \rightarrow \infty} g_1(g_2(\dots(g_n(x_{k-1}))\dots)) = \lim_{k \rightarrow \infty} f_1(f_2(\dots(f_n(x_{k-1}^1, \dots, x_{k-1}^n))\dots)) \rightarrow g_1(g_2(\dots(g_n(a_1))\dots)) = x_1$$

$$\lim_{k \rightarrow \infty} g_1(g_2(\dots(g_n(x_{k-1}^2))\dots)) = \lim_{k \rightarrow \infty} f_1(f_2(\dots(f_n(x_{k-1}^2, \dots, x_{k-1}^1))\dots)) \rightarrow g_1(g_2(\dots(g_n(a_2))\dots)) = x_2$$

⋮

$$\lim_{k \rightarrow \infty} g_1(g_2(\dots(g_n(x_{k-1}^n))\dots)) = \lim_{k \rightarrow \infty} f_1(f_2(\dots(f_n(x_{k-1}^n, \dots, x_{k-1}^{n-1}))\dots)) \rightarrow g_1(g_2(\dots(g_n(a_n))\dots)) = x_n$$

$$M[f_1(f_2(\dots(f_n(x_{k-1}^1, x_{k-1}^2, \dots, x_{k-1}^n))\dots)), f_1(f_2(\dots(f_n(a_1, a_2, \dots, a_n))\dots)), \phi(t)]$$

$$\geq M[g_1(g_2(\dots(g_n(x_{k-1}^1))\dots)), g_1(g_2(\dots(g_n(a_1))\dots)), t] *$$

$$M[g_1(g_2(\dots(g_n(x_{k-1}^2))\dots)), g_1(g_2(\dots(g_n(a_2))\dots)), t] * \dots *$$

$$M[g_1(g_2(\dots(g_n(x_{k-1}^n))\dots)), g_1(g_2(\dots(g_n(a_n))\dots)), t]$$

As $n \rightarrow \infty$ and by continuity of M , we get

$$M[g_1(g_2(\dots(g_n(a_1))\dots)), f_1(f_2(\dots(f_n(a_1, a_2, \dots, a_n))\dots)), \phi(t)] = 1$$

Also, $M[f_1(f_2(\dots(f_n(x_{k-1}^2, x_{k-1}^3, \dots, x_{k-1}^1))\dots)), f_1(f_2(\dots(f_n(a_1, a_2, \dots, a_n))\dots)), \phi(t)]$

$$\geq M[g_1(g_2(\dots(g_n(x_{k-1}^2))\dots)), g_1(g_2(\dots(g_n(a_2))\dots)), t] *$$

$$M[g_1(g_2(\dots(g_n(x_{k-1}^3))\dots)), g_1(g_2(\dots(g_n(a_3))\dots)), t] * \dots *$$

$$M[g_1(g_2(\dots(g_n(x_{k-1}^1))\dots)), g_1(g_2(\dots(g_n(a_1))\dots)), t]$$

As $n \rightarrow \infty$,

$$M[g_1(g_2(\dots(g_n(a_2))\dots)), f_1(f_2(\dots(f_n(a_2, a_3, \dots, a_n))\dots)), \phi(t)] = 1$$

Continuity

$$M[g_1(g_2(\dots(g_n(x_{k-1}^n, x_{k-1}^1, \dots, x_{k-1}^{n-1}))\dots)), f_1(f_2(\dots(f_n(a_n, a_1, \dots, a_{n-1}))\dots)), \phi(t)]$$

$$\geq M[g_1(g_2(\dots(g_n(x_{k-1}^n))\dots)), g_1(g_2(\dots(g_n(a_n))\dots)), t] *$$

$$M[g_1(g_2(\dots(g_n(x_{k-1}^1))\dots)), g_1(g_2(\dots(g_n(a_1))\dots)), t] * \dots *$$

$$M[g_1(g_2(\dots(g_n(x_{k-1}^{n-1}))\dots)), g_1(g_2(\dots(g_n(a_{n-1}))\dots)), t]$$

As $n \rightarrow \infty$, we get

$$M[g_1(g_2(\dots(g_n(a_n))\dots)), f_1(f_2(\dots(f_n(a_n, a_1, \dots, a_{n-1}))\dots)), \phi(t)] = 1$$

$$\Rightarrow g_1(g_2(\dots(g_n(a_1))\dots)) = f_1(f_2(\dots(f_n(a_1, a_2, \dots, a_n))\dots)) = x_1$$

$$g_1(g_2(\dots(g_n(a_2))\dots)) = f_1(f_2(\dots(f_n(a_2, a_3, \dots, a_1))\dots)) = x_2$$

⋮

$$g_1(g_2(\dots(g_n(a_n))\dots)) = f_1(f_2(\dots(f_n(a_n, \dots, a_{n-1}))\dots)) = x_n$$

Therefore, (a_1, a_2, \dots, a_n) is (G, n) -tupled coincidence point of compose the mappings of A and B .

Corollary(2)

Let $(X, M, *)$ be a fuzzy metric space .Under the same assumptions of theorem(1) but

$$M[f_1(f_2(\dots(f_n(x_1, x_2, \dots, x_n))\dots)), f_1(f_2(\dots(f_n(y_1, y_2, \dots, y_n))\dots)), kt] \geq$$

$$M[g_1(g_2(\dots(g_n(x_1))\dots)), g_1(g_2(\dots(g_n(y_1))\dots)), t] *$$

$$M[g_1(g_2(\dots(g_n(x_2))\dots)), g_1(g_2(\dots(g_n(y_2))\dots)), t] * \dots *$$

$$M[g_1(g_2(\dots(g_n(x_n))\dots)), g_1(g_2(\dots(g_n(y_n))\dots)), t] \dots \dots (1)$$

Where $k \in (0,1)$, $t > 0$ and $x_i, y_i \in X \quad \forall i = 1, 2, \dots, n$. Then there exists a unique (G, n) -tupled coincidence point of compose the mappings of A and B .

Corollary(3)

Let $(X, M, *)$ be a fuzzy metric space .Under the same assumptions of theorem(1) but

$$M \left[f_1 \left(f_2 \left(\dots \left(f_n(x_1, x_2, \dots, x_n) \right) \dots \right) \right), f_1 \left(f_2 \left(\dots \left(f_n(y_1, y_2, \dots, y_n) \right) \dots \right) \right), \emptyset(t) \right] \\ \geq M \left[g_1 \left(g_2 \left(\dots \left(g_n(x_1) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \left(g_n(y_1) \right) \dots \right) \right), t \right]^{a_1} * \dots *$$

$$M \left[g_1 \left(g_2 \left(\dots \left(g_n(x_2) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \left(g_n(y_2) \right) \dots \right) \right), t \right]^{a_2} * \dots *$$

$$M \left[g_1 \left(g_2 \left(\dots \left(g_n(x_n) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \left(g_n(y_n) \right) \dots \right) \right), t \right]^{a_n}$$

(1)

Where $\sum_{i=1}^n a_i \leq 1$, $t > 0$ and $x_i, y_i \in X \quad \forall i = 1, 2, \dots, n$. Then there exists a unique (G, n) -tupled coincidence point of compose the mappings of A and B .

Corollary(4)

Let $(X, M, *)$ be a fuzzy metric space .Under the same assumptions of theorem(1) but

$$M \left[f_1 \left(f_2 \left(\dots \left(f_n(x_1, x_2, \dots, x_n) \right) \dots \right) \right), f_1 \left(f_2 \left(\dots \left(f_n(y_1, y_2, \dots, y_n) \right) \dots \right) \right), kt \right] \\ \geq M \left[g_1 \left(g_2 \left(\dots \left(g_n(x_1) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \left(g_n(y_1) \right) \dots \right) \right), t \right]^{a_1} * \dots *$$

$$M \left[g_1 \left(g_2 \left(\dots \left(g_n(x_2) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \left(g_n(y_2) \right) \dots \right) \right), t \right]^{a_2} * \dots *$$

$$M \left[g_1 \left(g_2 \left(\dots \left(g_n(x_n) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \left(g_n(y_n) \right) \dots \right) \right), t \right]^{a_n} \quad (1)$$

Where $\sum_{i=1}^n a_i \leq 1$, $k \in (0, 1)$, and $x_i, y_i \in X \quad \forall i = 1, 2, \dots, n$. Then there exists a unique (G, n) -tupled coincidence point of compose the mappings of A and B .

Corollary (5)

Let $(X, M, *)$ be a fuzzy metric space .Under the same assumptions of theorem(1) but

$$M \left[f_1 \left(f_2 \left(\dots \left(f_n(x_1, x_2, \dots, x_n) \right) \dots \right) \right), f_1 \left(f_2 \left(\dots \left(f_n(y_1, y_2, \dots, y_n) \right) \dots \right) \right), kt \right] \\ \geq M[x_1, y_1, t] * M[x_2, y_2, t] * \dots * M[x_n, y_n, t]$$

Where $k \in (0, 1)$, $t > 0$ and $x_i, y_i \in X \quad \forall i = 1, 2, \dots, n$. Then there exists a unique (G, n) - tupled fixed point of compose the mappings in A

Corollary (6)

Let $(X, M, *)$ be a fuzzy metric space .Under the same assumptions of theorem(1) but

$$M \left[f_1 \left(f_2 \left(\dots \left(f_n(x_1, x_2, \dots, x_n) \right) \dots \right) \right), f_1 \left(f_2 \left(\dots \left(f_n(y_1, y_2, \dots, y_n) \right) \dots \right) \right), \emptyset(t) \right] \\ \geq M \left[g_1 \left(g_2 \left(\dots \left(g_n(x_1) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \left(g_n(y_1) \right) \dots \right) \right), t \right]^{a_1} * \dots * \\ M \left[g_1 \left(g_2 \left(\dots \left(g_n(x_2) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \left(g_n(y_2) \right) \dots \right) \right), t \right]^{a_2} * \dots * \\ * M \left[g_1 \left(g_2 \left(\dots \left(g_n(x_n) \right) \dots \right) \right), g_1 \left(g_2 \left(\dots \left(g_n(y_n) \right) \dots \right) \right), t \right]^{a_n}$$

Where $\sum_{i=1}^n a_i \leq 1$, $t > 0$ and $x_i, y_i \in X \quad \forall i = 1, 2, \dots, n$. Then there exists a unique (G, n) - tupled fixed point of compose the mappings in A .

Corollary (7)

Let $(X, M, *)$ be a fuzzy metric space .Under the same assumptions of theorem(1) but

$$M \left[f_1 \left(f_2 \left(\dots \left(f_n(x_1, x_2, \dots, x_n) \right) \dots \right) \right), f_1 \left(f_2 \left(\dots \left(f_n(y_1, y_2, \dots, y_n) \right) \dots \right) \right), kt \right] \\ \geq M[x_1, y_1, t]^{a_1} * M[x_2, y_2, t]^{a_2} * \dots * M[x_n, y_n, t]^{a_n}$$

Where $\sum_{i=1}^n a_i \leq 1$, $k \in (0, 1)$, and $x_i, y_i \in X \quad \forall i = 1, 2, \dots, n$. Then there exists a unique (G, n) - tupled fixed of compose the mappings in A .

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