Mathematical Modeling and Optimization of Clutch Pedal Mechanism to Reduce Pedal Effort

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Abstract: *Clutch pedal mechanism is used in all automobiles to engage and disengage power from engine to the transmission. Driving comfort is one of the most important factors affecting automobile comfort. Clutch pedal force and pedal motion stroke mainly affect the driving comfort. High clutch pedal effort is a customer dis-satisfier as this leads to high fatigue of operator and lesser productivity. So to reduce the clutch pedal effort with minimal cost, the linkages can be optimized by doing Design of Experiments (DOE) on different setting of linkages. Analysis (ANSYS workbench) model is generally used to calculate the pedal forces which are sometimes time consuming. An MATLAB based tool (mathematical model) was designed to calculate the pedal forces. The output of this is used in DOE to optimize the linkages. The results are also verified with output of analysis model. This helps in saving time and engineering cost.*

Keywords: Pedal effort, Design of Experiments, optimization, ANSYS Workbench, Driving comfort.

1. Introduction

A clutch is a one of the part of the drive train component used to transfer torque in different types of machinery. Driving comfort is one of the most important factors affecting automobile comfort. Clutch pedal force and pedal motion stroke mainly affect the driving comfort. High clutch pedal effort is a customer dis-satisfier as this leads to high fatigue of operator and lesser productivity. We have to frequently use clutch while driving in heavy traffic, thus pedal effort required to engage and disengage the clutch should as minimum as possible. Most of the drivers are generally facing the problems about knee pain due high pedal effort. This happened when clutch, brake and accelerator pedal mechanism are not properly designed. Now adays latest technologies like turbocharging, wet clutch etc are incorporated in drive train system which increases engine power etc. but it will also increase clamping load of clutch. Increase in clamping load of the clutch tends to increase in clutch pedal effort [1].

Oral Vatan, Serdar Ozkkan have presented design and custom analysis of clutch pedal for low floor manual transmission midibus vehicle. Their focus of research is on driving comfort and ergonomic study of clutch pedal i.e. height of clutch actuation point to cabin floor mat, lateral position acc. to steering column, size and angle of pedal pad, maximum pedal travel, working angle, pedal force and pedal ratio, packaging and modulation requirements. They design clutch pedal by assuming maximum force of 500N [custom anlaysis] [2].

C.R.Mehta, M.M.Pandey have presented design of different actuation system in tractor and strength limits to operate these controls because it is very important from comfort and safety operation point of view. Therefore, strength parameters of 105 agricultural workers (75 male and 30 female) were measured on "strength measurement setup" and recommended that, the average push strength of clutch pedal

for male and female workers was found to be 248.2 and 171.0 N respectively. According to the I.S. 10703 maximum actuating force for clutch pedal should not exceed 280N [analytical study of strength] [3].

Dong-Woo Kim, Myeong-Woo Cho have performed patented work on Application of Design of Experiment Method for Thrust Force Minimization in Step-feed Micro Drilling [4]

Erickson et al have performed a patented work on improving drive mechanism efficiency of cycle by changing diameter of drive sprocket. Efficiency of drive mechanism or mechanical leverage increases with increase in velocity ratio [5].

2. Objective

As stated earlier high clutch pedal effort is a customer dis-satisfier as this leads to high fatigue of operator and lesser productivity. Though integrated latest technologies like wet clutch increases torque carrying capacity but, it will also increase clamping load and hence clutch pedal effort. Keeping above perspective in view the objective of research is:
1) Mathematical modeling of mechanism to find pedal effort in MATLAB software.
2) Optimization of mechanism to reduce pedal effort by conducting Design of Experiments on different settings of linkages.

3. Mathematical Modeling

A mathematical model may help to explain a system and to study the effects of different components, and to make predictions about behavior. A mathematical model usually describes a system by a set of variables and a set of equations that establish relationships between the variables. Variables may be of many types; real or integer numbers, boolean values or strings, for example. The variables represent some

properties of the system, for example, measured system outputs often in the form of signals, timing data, counters, and event occurrence (yes/no). The actual model is the set of functions that describe the relations between the different variables.

In mathematical modeling, different mathematical equations are used to calculate the output of the mechanism (pedal effort required by driver to engage and disengage the clutch) from the given input variables. Mathematical model includes, calculation of Mechanical Advantage, spring force, spring deflection, pedal effort of the mechanism. Fig. 1 shows line diagram of clutch pedal mechanism. MATLAB software is used to calculate pedal effort of the mechanism. Following formulae are used in the mathematical formulations:
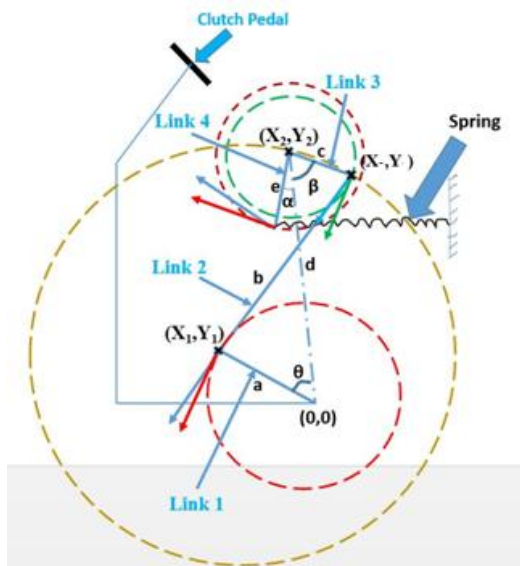

Figure 1

i) $x = R * \theta$
Where, x = Pedal Travel in mm.
R = effective radius of pedal in mm.
θ = Pedal angular displacement in rad.

ii) Two circle intersection equation :

$$X = \left(\frac{X_2 + X_1}{2}\right) + \left[\frac{(X_2 - X_1)*(r_2^2 - r_3^2)}{2 * D^2}\right] + \left(\frac{Y_2 - Y_1}{2}\right)$$
$$* \sqrt{\left[(r_2 + r_3)^2 - D^2\right] * \left[D^2 - (r_2 - r_3)^2\right]}$$

$$Y = \left(\frac{Y_2 + Y_1}{2}\right) + \left[\frac{(Y_2 - Y_1)*(r_2^2 - r_3^2)}{2 * D^2}\right] + \left(\frac{X_2 - X_1}{2}\right)$$
$$* \sqrt{\left[(r_2 + r_3)^2 - D^2\right] * \left[D^2 - (r_2 - r_3)^2\right]}$$

Where, $X_1 = -(a_1 * \sin\theta)$
$Y_1 = (a_1 * \cos\theta)$
$X_2 = 0$
$Y_2 = d$
$D = \sqrt{(X_2 - X_1)^2 - (Y_2 - Y_1)^2}$
r_2 = Radius of link '2'
r_3 = Radius of link '3'

iii) Length of perpendicular from a point to a line to find mechanical advantage of mechanism :
$y = m * x + c$
$c = y - m * x$ ……………………… (1)
Where, m = Slope of line = $\frac{Y - Y_1}{X - X_1}$
c = intercept

Length of perpendicular from a point to line = $\frac{A * M + B * N + c}{\sqrt{A^2 + B^2}}$
Where, A and B are coefficients of x and y in equation (1)
M and N are coordinates of a point from which length of perpendicular is measured

iv) Mechanical Advantage of mechanism = $\frac{Input perpendicular length}{Output perpendicular length}$

v) Spring force = Stiffness * Deflection
Spring force = $K * (e * \alpha)$

vi) Pedal Effort = $\frac{Spring force}{Mechanical Advantage}$

The program is prepared in MATLAB with the following input values are used to calculate pedal effort of mechanism. Input variables:- $\Theta = 44^0$, $a = 150$mm, $c = 40$mm, $\beta = 75^0$, $e = 64$mm, $\alpha = 1.0^0$

The output result obtained by the MATLAB program:-
Pedal Effort= 22.65Kg.

4. Design of Experiments of mechanism

Design of Experiments as a methodology is used for systematically applying statistics to experimentation. It consists of a series of tests in which purposeful changes are made to the input variables (factors) of a product or process so that one may observe and identify the reasons for these changes in the output response. DOE provides a quick and cost-effective method to understand and optimize products and processes.

Goal of experiments:
- Experiments help us in understanding the behavior of a (mechanical) system
- Data collected by systematic variation of influencing factors helps us to quantitatively describe the underlying phenomenon or phenomena.

DOE (Design of Experiments) provides a dominant means to get advance improvements in product quality and process efficiency. From the manufacturing fields point of view, DOE can reduce the number of necessary experiments when the number of factors affecting experimental results is under considerations. DOE can illustrate how to carry out the smallest number of experiments while keeping the most important information. The most important procedure of the DOE is determining the independent variable values at which a restricted number of experiments will be conducted [2].

4.1 Degree of Freedom in DOE

Degree of freedom (DOF) is a common term used in engineering and science. However, there is no visible interpretation of DOF applied to experimental data. Regarding statistical analysis of experimental data, DOF provides an indication of the amount of information contained in a data set. In DOE processes, DOF is applied to characterize four separate items as follows:

(1) DOF of a factor = number of levels of the factor – 1 DOF of a column = number of levels of the column – 1 DOF of an array = total of all column DOFs for the array DOF of an experiment = total number of results of all trials – 1

DOF is the minimal number of relationship between levels of factors or interactions to improve process characteristics. The type of orthogonal array used in DOE can be selected by the DOF. When determining factors and levels, the orthogonal array has to be selected. In this case, the DOF is taken into account as a reference for selecting a certain type of orthogonal array. Determining the number of factors and levels, a suitable orthogonal array can be selected by the total DOF of the experiment, because the total DOF of factors and levels used in an orthogonal array is already determined [2].

4.2 Analysis of Variance

ANOVA (Analysis of Variance) is a statistical method that is used to identify factors significantly affecting the experimental results. ANOVA consists of (1) summing squares of characteristic values of experimental data (2) balanced variance; (3) adding this total sum into the sums of squares for all factors used in the experiment; (4) calculating balanced variances through the sums of squares for all factors (5) calculating the variance ratio F_0 by dividing each balanced variance by the error variance; and (6) identifying factors significantly influence experimental results by analyzing the error variance. This process can be done by constructing an ANOVA table [1].

4.3 Selection of Factors

Selection of probable parameters to be varied is the first task before conducting the experiments. A series of experiments was performed [1] on the Stationary Hook Hopper Feeder to identify the effect of various parameters that influence the output response.

The one factor at a time (OFAT) experiments gave satisfactory results when only one factor is changed keeping others constant. But the results obtained could not be used successfully to set the parameters for mechanism optimization. The reason for this can be recognized to the fact that the OFAT approach fails to represent the effect caused by the interaction of various factors on the mechanism performance [2]. Interaction is defined as the failure of one factor to produce the same effect on the response at different levels of another factor.

Therefore the requirement for a better statistical model was for optimization of feed rate and its accurate calculation. In the current work this aim has been done using factorial design of the experiments. Such statistical method provides an efficient method to analyze the effect of interaction on the output response of the process that too in limited number of experimental runs. The reason for this can be recognized to the fact that the OFAT approach fails to represent the effect caused by the interaction of various factors on the feeder performance [2]. Interaction is defined as the failure of one factor to produce the same effect on the response at different levels of another factor.

Table 1

Factors	Low Level	High Level
Ө (deg)	40	80
a (mm)	80	150
c (mm)	45	70
β (deg)	75	89
e (mm)	62	64
α (deg)	1	7

As shown in the fig. 1 of clutch pedal mechanism, the factors which affects the output of the mechanism (pedal effort) are 'θ, a, c, e, β, α'. Each factors are having two levels (low, high) within which it works. The levels of the mechanism are decided based on availability of space. The 'Table 1' shows the six factors with its corresponding levels.

Design of Experiments of the clutch pedal mechanism is done in "MINI-TAB" software. In the DOE there are '2^6' experiments i.e. total '64' runs with full factorial DOE. So, instead of using full factorial DOE, fractional Factorial DOE is used in project. Due to this change, the number of runs will get reduced to '16' so, the time required in the analysis will get reduced. Table 2 shows the Design of Experiments of clutch pedal mechanism with '16' runs.

Table 2

	theta	a	C	beta	e	alpha	pedal effort
1	80	150	70	88	64	7	9.12
2	40	80	45	75	62	1	11.10
3	80	150	45	75	62	7	15.20
4	80	80	70	75	62	7	5.07
5	40	150	45	88	64	1	16.80
6	40	80	70	75	64	7	7.30
7	80	80	70	88	62	1	5.20
8	80	80	45	75	64	1	9.30
9	40	80	70	88	64	1	7.40
10	80	150	70	75	64	1	8.00
11	40	80	45	88	62	7	11.50
12	40	150	45	75	64	7	20.30
13	40	150	70	88	62	7	12.70
14	40	150	70	75	62	1	11.40
15	80	150	45	88	62	1	13.42
16	80	80	45	88	64	7	9.50

The purpose of the experimentation is to establish a statistical model to predict the output feed rate and its successful optimization using 2k factorial design.

4.4 Analysis

Minitab® is an excellent statistical package that assists in data analysis. Various plots like Cube plot, Interaction plot and Main Effects plot are obtained to examine effects of factors on output. Pareto plot and Normal plot of the standardized effects are obtained to compare the significance of each effect. Analysis of Variance (ANOVA) table is build for identifying the significant factors affecting the output response.

A. Effect of Factors on Pedal effort:
It is important to know how the system behaves when variation is brought upon by varying only one parameter keeping the others constant. A main effect occurs when the mean response changes across the levels of a factor. The main effect graphs (Fig 2) can be used to compare the relative strength of the effects across factors. It can be asserted from the graph that the "theta, a and c" have positive effects while rest all factors has negative effect on the output pedal effort.
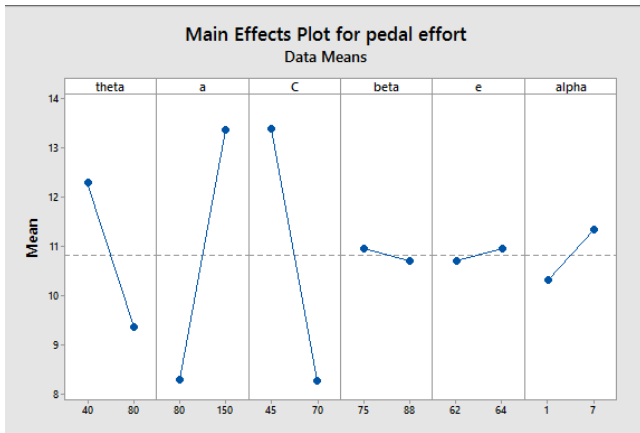
Figure 2

B. Significance of Various Factors

The analysis of Table 3 shows that the factors "theta, a and c"are highly significant. All those effects have very small P-values. Since the P-value of all the factors except "theta, a and c"are greater than the chosen value of α=0.05 for the analysis, it has a negligible effect on the output pedal effort.

Table 3

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	13	259.256	19.943	111.70	0.009
Linear	6	247.262	41.210	230.83	0.004
theta	1	35.076	35.076	196.47	0.005
a	1	102.870	102.870	576.20	0.002
beta	1	0.258	0.258	1.44	0.353
C	1	104.704	104.704	586.47	0.002
e	1	0.284	0.284	1.59	0.335
alpha	1	4.070	4.070	22.80	0.041
2-Way Interaction	7	11.995	1.714	9.60	0.098
theta*a	1	3.267	3.267	18.30	0.051
theta*beta	1	0.117	0.117	0.66	0.503
theta*C	1	0.047	0.047	0.26	0.658
theta*e	1	4.070	4.070	22.80	0.041
theta*alpha	1	0.284	0.284	1.59	0.335
a*beta	1	0.851	0.851	4.77	0.161
a*alpha	1	3.358	3.358	18.81	0.049
Error	2	0.357	0.179		
Total	15	259.613			

The Pareto Chart of the Effects (Fig. 3) and the Normal Plot of Standardized Effects (Fig. 4) also help to ascertain the magnitude and the importance of an effect. Pareto chart shows the absolute value of the effects and represent a reference line on the chart at t-value limit, where t is the (1 - α/2) quantile of a t-distribution with degrees of freedom equal to the degrees of freedom (24) for the error term. Any effect that extends within this reference line is statistically. Insignificant. . The effect of "A (theta), B (a), C (c)" has the highest standardized effect on the pedal effort. Other factors and their interactions are statistically insignificant effect on the pedal effoer Hence, these terms should not be considered for the empirical relation. The insignificance of these factors can also be reasserted from the normal plot, in which, the points that do not fall near the fitted line are important. The factors having negligible effect on the output response tend to be smaller and are centered around zero.

C. Regression Analysis

Regression analysis is used to investigate and model the relationship between a response variable and one or more predictors. The Minitab software is used to calculate regression model. While performing fit regression model you should follow the path.
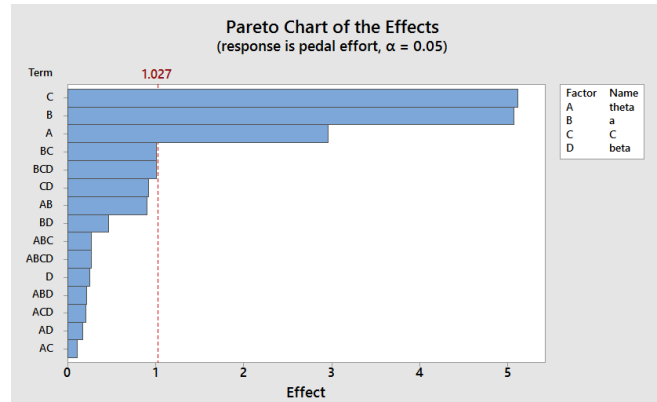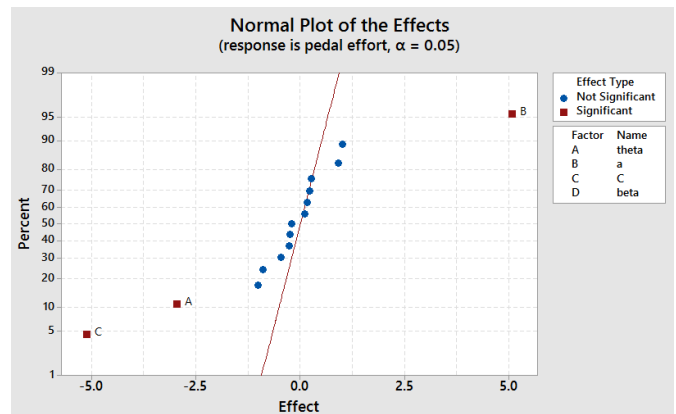

Figure 2

Figure 4

Add interaction terms and polynomial terms to your model. By default, the model contains only the predictor variables that you entered in the main dialog box. The histogram shows general characteristics of the data, including typical values, spread or variation, and shape unusual values in the data. Long tails in the plot may indicate skewness in the data. If one or two bars are far from the others, those points may be outliers. Normal probability plot of residuals shows the straight line indicating the residuals are normally distributed. Residuals versus fits this plot should show a random pattern of residuals on both sides of '0'. A positive correlation is indicated by a clustering of residuals with the same sign. A negative correlation is indicated by rapid changes in the signs of consecutive residuals.

The basis of this approach is the assumption of a simplified linear model for the optimisation parameter η given by equation (3).

$$\eta = b_0 + b_1 x_1 + b_2 x_2 + b_3 x_3 \qquad (3)$$

Where x_1, x_2, x_3 etc, are the factors on which η depends and b_0, b_1, b_2, b_3 etc, represent the 'true' values of the corresponding unknowns. From the results of an experiment comprising a finite number of trials, one can arrive at sample estimates of the coefficients, b, which are then usually fitted into a linear regression equation (3) of the type.

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5. Response Optimization

Response optimization is used to recognize the combination of variable settings that mutually optimize a single response or a set of responses. This is useful when we need to assess the impact of multiple variables on a response. Minitab calculates an individual desirability for each response and weights each by the importance you assign it. These values are combined to find the composite, or overall, desirability of the multi-response system. An optimal solution occurs where composite desirability obtains its maximum. Using an optimization plot, you can adjust the variable settings and determine how the changes affect the response.

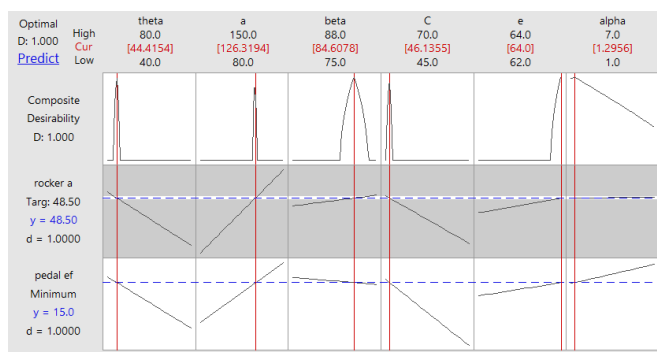## 6. Results and Discussions

A. Optimized Result

Figure 5

The optimization plot shows (Fig. 5) the effect of each factor (rows) on the responses or composite desirability (colums). The vertical red lines on the graph represent the current factor settings. The numbers displayed at the top of a column show the current factor level settings (in red). The horizontal blue lines and numbers represent the responses for the current factor level.

Pedal effort obtained with this optimized setting of the mechanism is:

Mechanism setting: 1) Ө (deg) = 44.41
        2) a (mm) = 126.31
        3) c (mm) = 46.13
        4) β (deg) = 84.60
        5) e (mm) = 64
        6) α (deg) = 1.29
Pedal Effort at 110mm pedal travel = **15.26Kg**

B. Experimental Result
Actual clutch pedal mechanism is fabricated and tested on Load cell. The results obtained by the measurements are as follows:
Mechanism setting: 1) Ө (deg) = 44.41
        2) a (mm) = 126.31
        3) c (mm) = 46.13
        4) β (deg) = 84.60
        5) e (mm) = 64
        6) α (deg) = 1.29
Pedal Effort at 110mm pedal travel = **17.49Kg**

7. Conclusion

i) Analysis (ANSYS workbench) model is generally used to calculate the pedal forces which are sometimes time consuming. So, the analysis model can be replaced with MATLAB based tool (mathematical model). This helps in saving time and engineering cost.

ii) A reliable statistical model based on full factorial experiment design has been developed which can be used for the optimization of clutch pedal mechanism to reduce the pedal effort. The model is significant to explain 89% of variability in new data. The pedal effort obtained by optimization of the linkages is **15.26Kg at 110mm** pedal travel.

iii) The pedal effort obtained by experimentation is **17.49Kg at 110mm** pedal travel. There is **12%** error between analytical and experimental results.

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