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On Shrinkage Estimation of $R_{(s,k)}$ Based on Exponentiated Weibull Distribution

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Abstract: ***In this paper, estimation of a reliability of the multi-component system in stress-strength model $R_{(s,k)}$ is considered, when the stress and strength are independent random variables and follows the Exponentiated Weibull Distribution (EWD) with known first shape parameter θ and, the second shape parameter α is unknown. Different shrinkage estimation methods of $R_{(s,k)}$ for (EWD) are introduced based on maximum likelihood and moment methods. The comparisons among the proposed estimators that depend on the simulation technique are made and mean squared error (MSE) is used as criteria.***

Keywords: Exponentiated Weibull Distribution (EWD), Reliability of multi-component Stress – Strength models $R_{(s,k)}$, Maximum likelihood estimator (MLE), Moment estimator (MOM), Shrinkage Estimator (sh), mean squared error (MSE)

1. Introduction

The stress-strength (S-S) reliability of the system which contains one component is denoted by R=P(Y<X), where Y arise the stress random variable and X arise the strength random variable. And hence "the stress-strength (S-S) reliability is computed as $R = \int_0^\infty f_{\text{Strength}}(x).F_{\text{Stress}}(x)dx$. Its application has spread out into many fields; one of the most developed usual applications focuses on engineering-oriented problems. For example, the common tennis racket is composed of two basic subsystems, the frame, and string. During a tennis match, the random stress a racket is exposed to includes hitting the ball repeated times or falling on the ground accidently;[23]. As well as (S-S) model is used in engineering devices, it has been usually determined, how long rang time of the system will be live. The stress-strength model has been studied by many authors like Ali, Pal, and Woo (2012), they estimated the (S-S) reliability of Generalized Gamma distribution with four parameters; [2]. Hussian in 2013, considered the estimation of the reliability of stress- strength model forgeneralized inverted exponential distribution; [9]. Ghitany et al. (2015), studied estimation of the reliability of stress-strength system from power Lindley distribution; [6]. Najarzadegan et al. (2016), considered the estimation of P (Y<X) for the Levy distribution; [14].

The reliability of the system model; s out of k (s-k) denoted by $R_{(s,k)}$ functioning when at least s ($1 \leq s \leq k$) of components survive was introduced by Bhattacharyya and Johnson (1974). "They developed the reliability multi-component stress-strength system model $R_{(s,k)}$ which including k of a component and identical strength component put up with common stress function if s ($1 \leq s \leq k$) or more of the components simultaneously operate"; [5]. This system works successfully, if at least s out of k components resist the stress. Noted that, if s=1 and s=k corresponded respectively, to parallel and series systems. The mentioned model was used in many applications in physics and engineering such as strength failure and the system collapse; [7].

In 2010, Rao & Kantam studied a system of k multi-component which have independently and identically strength x_1, x_2, …, x_k random variables distributed experiencing the random stress y when stress and strength follow the Log–Logistic distribution; [17]. Srinivasa Rao (2012) estimated the multi-component system of reliability for log-logistic distribution with different shape parameters; [20]. As well as, in the same year, Srinivasa Rao studied estimation for the reliability of multi-component stress-strength model based generalized exponential distribution; [19]. While Hassan & Basheikh (2012) studied reliability estimation of stress- strength model with non-identical component strengths by using the exponentiated Pareto distribution; [7]. In 2016, Srinivasa Rao et al. estimated the multi-component stress-strength reliability of a system when stress and strength follow exponentiated Weibull distribution; [18]. Recently, Hassan (2017) studied the estimation of multi-component of reliability (S-S) system model when each of stress and strength follows Lindley distribution; [8].

Mudholkar and Srivastava(1993) introduced the exponentiated Weibull distribution (EWD) as an extension of the Weibull distribution, the obtained distribution is characterized by bathtub-shaped and model failure rates besides a broader class of monotone failure rates; [12]. Mudholkar et al.(1995) explained also applications of the exponentiated Weibull distribution in reliability and survival; [13]. Some properties and a flood data application of exponentiated Weibull family were studied by Mudholkar and Huston (1996); [11]. Nassar and Eissa (2003) had done some studies on exponentiated Weibull distribution model; [15]. Several authors have been used the (EWD) as a model of many fields like flood data, biological studies also in physical explanation …etc.

The aim of this paper is to estimate the multi-component system reliability of stress-strength model $R_{(s,k)}$ based on exponentiated Weibull distribution with known shape parameter θ and the other shape parameter α will be unknown via different estimation methods like MLE and MOM, as well as some of theshrinkage methods. The comparisons between the proposed estimator methods by using simulation areperformed, depends on a mean squared error as criteria.

Now, The probability density function (p. d. f.) of exponentiated Weibull distribution is as below:

$$f(x,\alpha) = \alpha\theta x^{\theta-1}e^{-x^{\theta}}(1-e^{-x^{\theta}})^{\alpha-1} \qquad x>0,\ \alpha,\theta>0 \qquad (1)$$

while the cumulative distribution function (c. d. f.) is:

$$F(x,\alpha) = (1-e^{-x^{\theta}})^{\alpha} \qquad x>0,\ \alpha,\theta>0 \qquad (2)$$

In order to find a system reliability consisting of k[th] identical components(when at least s out of k function) for the strength X_1, X_2, …, $X_k$which are random variables with EWD (α_1,θ), subjected to a stress Y which is a random variable follows EWD (α_2,θ) depend on Bhattacharyya and Johnson (1974), the reliability of a multi-component stress-strength model $R_{(s,k)}$; [5] can be calculated as $R_{(s,k)}$ =P(at least s of the X_1, X_2,…, $X_k$exceed mine)

$$=\sum_{i=s}^{k}\binom{k}{i}\int_{o}^{\infty}(1-F_x(y))^{i}(F_x(y))^{k-i}dG(y)$$

$=\sum_{i=s}^{k}\binom{k}{i}\int_{o}^{\infty}(1-$
(1−e−yθ)α1)i((1−e−yθ)α1)k−iα2θyθ−1e−yθ(1−e−y θ)α2−1dy

$=\sum_{i=s}^{k}\binom{k}{i}\int_{o}^{\infty}(1-(1-e^{-y^{\theta}})^{\alpha_1})^{i}\alpha_2\theta y^{\theta-1}e^{-y^{\theta}}(1-$
e−yθ)α1k−iα1+α2−1dy

And by some of the simplification, we get

$$R_{(s,k)}=\frac{\alpha_2}{\alpha_1}\sum_{i=s}^{k}\frac{k!}{(k-i)!}\left[\prod_{j=0}^{i}\left(k+\frac{\alpha_2}{\alpha_1}-j\right)\right]^{-1}$$

where k, I and j are integers (3)

2. Estimation methods of $R_{(s,k)}$:

2.1 Maximum Likelihood Estimator (MLE)

Suppose that k of components are put on life-testing experiment, in this case, we consider that $x_1,x_2,\dots,x_n$are a random sample of size n follows EWD (α_1,θ), then $x_{(1)}<x_{(2)}<\dots<x_{(n)}$ represent the order random sample of x and $y_1,y_2,\dots,y_m$be a random sample of size m follows EWD (α_2,θ), and $y_{(1)}<y_{(2)}<\dots<y_{(m)}$ refers to the order random sample of y. Then the likelihood function of the mentioned system will be:

$$l\equiv L(\alpha_1,\alpha_2,\theta;x,y)=\prod_{i=1}^{n}f(x_i)\prod_{j=1}^{m}g(y_j)$$

$=\prod_{i=1}^{n}\alpha_1\theta x_i^{\theta-1}e^{-x_i^{\theta}}(1-$
e−xiθ)α1−1j=1mα2θyjθ−1e−yjθ(1−e−yjθ)α2−1

$=\alpha_1^{\,n}\theta^{n}\prod_{i=1}^{n}x_i^{\theta-1}e^{-\sum_{i=1}^{n}x_i^{\theta}}\prod_{i=1}^{n}(1-$
e−xiθ)α1−1α2mθmj=1myjθ−1e−j=1myjθj=1m(1−e −yjθ)α2−1

Take logarithm for both sides, we get:

$$\ln(l) = n\ln\alpha_1 + n\ln\theta + (\theta-1)\sum_{i=1}^{n}\ln x_i^{\theta-1} - \sum_{i=1}^{n}x_i^{\theta} + (\alpha_1-1)\sum_{i=1}^{n}(1-e^{-x_i^{\theta}})\, m\ln\alpha_2 + m\ln\theta + (\theta-1)\sum_{j=1}^{m}\ln y_j^{\theta-1} - \sum_{j=1}^{m}y_j^{\theta} + (\alpha_2-1)\sum_{j=1}^{m}(1-e^{-y_j^{\theta}})$$

Derive the above equation with respect to the unknown shape parameters α_i (i=1,2) and equating the result to zero, we get:

$$\frac{d\mathrm{Ln}(l)}{d\alpha_1} = \frac{n}{\alpha_1} + \sum_{i=1}^{n} ln\left(1-e^{-x_i^{\theta}}\right) = 0$$

$$\frac{d\mathrm{Ln}(l)}{d\alpha_2} = \frac{m}{\alpha_2} + \sum_{j=1}^{m} ln\left(1-e^{-y_j^{\theta}}\right) = 0$$

Thus, the maximum likelihood estimator of the parameter α_i (i=1,2) will be as follows:

$$\hat{\alpha}_{1_{mle}} = \frac{-n}{\sum_{i=1}^{n} ln\left(1-e^{-x_i^{\theta}}\right)} \quad (4)$$

$$\hat{\alpha}_{2_{mle}} = \frac{-m}{\sum_{j=1}^{m} ln\left(1-e^{-y_j^{\theta}}\right)} \quad (5)$$

By substituting $\hat{\alpha}_{i_{mle}}$ (i=1,2) in equation (3) we get the reliability estimation for $R_{(s,k)}$ model via Maximum Likelihood method as below:

$$\hat{R}_{(s,k)_{mle}} = \frac{\hat{\alpha}_{2_{mle}}}{\hat{\alpha}_{1_{mle}}}\sum_{i=s}^{k}\frac{k!}{(k-i)!}\left[\prod_{j=0}^{i}\left(k+\frac{\hat{\alpha}_{2_{mle}}}{\hat{\alpha}_{1_{mle}}}-j\right)\right]^{-1} (6)$$

2.2 Moment Method (MOM) $R_{(s,k)}$:

Let x_1,x_2, …,$x_n$be a random sample of size $n$for strength$X$ follows EW (α_1,θ), and y_1 , y_2, …, $y_m$be a random sample of size m for stress Y follows EW (α_2,θ). Let $\bar{X}$ and$\bar{Y}$ are the means of samples of strength and stress respectively, then the population moments of X, Y are given by; see[1 6].

$E(X^r)$=

$$\begin{cases} \alpha\sum_{j=0}^{\alpha-1}\binom{\alpha-1}{j}(-1)^{j}(j+1)^{-\frac{r}{\theta}-1}\Gamma\left(\frac{r}{\theta}+1\right) & ,if\,\alpha\in N \\ \alpha\sum_{j=0}^{\alpha-1}\frac{\alpha-1P_j}{j!}(-1)^{j}(j+1)^{-\frac{r}{\theta}-1}\Gamma\left(\frac{r}{\theta}+1\right) & ,if\ \alpha\notin N \end{cases} \quad \text{for}$$

r=1,2,3….

Where αP_j=α (α-1) (α-2)….. (α-j+1) and N is the set of natural number

Therefore, the population means of X and Y are respectively as below:

E(X)=

$$\begin{cases} \alpha_1\sum_{j=0}^{\alpha_1-1}\binom{\alpha_1-1}{j}(-1)^{j}(j+1)^{-\frac{1}{\theta}-1}\Gamma\left(\frac{1}{\theta}+1\right) & ,if\,\alpha_1\in N \\ \alpha_1\sum_{j=0}^{\alpha_1-1}\frac{\alpha_1-1^{P_j}}{j!}(-1)^{j}(j+1)^{-\frac{1}{\theta}-1}\Gamma\left(\frac{1}{\theta}+1\right) & ,if\ \alpha_1\notin N \end{cases}$$

… (7)

And,

E(Y)

$$= \begin{cases} \alpha_2\sum_{j=0}^{\alpha_2-1}\binom{\alpha_2-1}{j}(-1)^{j}(j+1)^{-\frac{1}{\theta}-1}\Gamma\left(\frac{1}{\theta}+1\right) & ,if\,\alpha_2\in N \\ \alpha_2\sum_{j=0}^{\alpha_2-1}\frac{\alpha_2-1^{P_j}}{j!}(-1)^{j}(j+1)^{-\frac{1}{\theta}-1}\Gamma\left(\frac{1}{\theta}+1\right) & ,if\ \alpha_2\notin N \end{cases}$$

… (8)

Equating the samples mean with the corresponding populations mean for bath X and Y as follows:-

$$\bar{X} = \frac{\sum_{j=1}^{n}x_j}{n} = \alpha_1\sum_{j=0}^{\alpha_1-1}\binom{\alpha_1-1}{j}(-1)^{j}(j+1)^{-\frac{1}{\theta}-1}\Gamma\left(\frac{1}{\theta}+1\right)$$

$$\bar{Y} = \frac{\sum_{j=1}^{m}y_j}{m}=\alpha_2\sum_{j=0}^{\alpha_2-1}\binom{\alpha_2-1}{j}(-1)^{j}(j+1)^{-\frac{1}{\theta}-1}\Gamma\left(\frac{1}{\theta}+1\right)$$

By simplification, we obtain the estimation of unknown shape parameters α_i (i=1,2) using moment method as follows:

$$\hat{\alpha}_{1_{mOm}} = \frac{\bar{X}}{\sum_{j=0}^{\alpha_1-1}\binom{\alpha_1-1}{j}(-1)^j (j+1)^{-\frac{1}{\theta}-1}\Gamma\left(\frac{1}{\theta}+1\right)} \quad (9)$$

$$\hat{\alpha}_{2_{mOm}} = \frac{\bar{Y}}{\sum_{j=0}^{\alpha_2-1}\binom{\alpha_2-1}{j}(-1)^j (j+1)^{-\frac{1}{\theta}-1}\Gamma\left(\frac{1}{\theta}+1\right)} \quad (10)$$

Substitution $\hat{\alpha}_{i_{mOm}}$ (i=1,2) in equation (3), we conclude the reliability estimation for $R_{(s,k)}$ model via moment method as below:

$$\hat{R}_{(s,k)_{mom}} = \frac{\hat{\alpha}_{2_{mom}}}{\hat{\alpha}_{1_{mom}}} \sum_{i=s}^{k} \frac{k!}{(k-i)!} \left[\prod_{j=0}^{i} \left(k + \frac{\hat{\alpha}_{2_{mom}}}{\hat{\alpha}_{1_{mom}}} - j\right) \right]^{-1}$$

…(11)

2.3 Shrinkage Estimation Method (Sh):

Thompson in 1968 has suggested the problem of shrink a usual estimator $\hat{\alpha}$ of the parameter α to prior information α_0 using shrinkage weight factor $\emptyset(\hat{\alpha})$, such that $0 \le \emptyset(\hat{\alpha}) \le 1$. Thompson says that "We are estimating α and we believe α_0 is closed to the true value of α or we fear that α_0 may be near the true value of α, that is mean something bad happens if $\alpha_0 \approx \alpha$ and we do not use α_0". Thus, the form of shrinkage estimator of α say $\hat{\alpha}_{sh}$ will be: [1][3][4][21][22]

$$\hat{\alpha}_{sh} = \emptyset(\hat{\alpha})\hat{\alpha} + (1 - \emptyset(\hat{\alpha}))\hat{\alpha}_0 \quad (12)$$

In this work, we apply the unbiased estimator $\hat{\alpha}_{ub}$ as a usual estimator and the moment estimator as a prior estimation of α in equation (12) above. Where $\emptyset(\hat{\alpha})$ denote the shrinkage weight factor as we mentioned above such that $0 \le \emptyset(\hat{\alpha}) \le 1$, which may be a function of $\hat{\alpha}_{ub}$; a function of sample size (n,m) or may be constant or can be found by minimizing the mean square error of $\hat{\alpha}_{sh}$.Thus, the shrinkage estimator for the shape parameter α of EWD will be as follows:

$$\hat{\alpha}_{sh} = \emptyset(\hat{\alpha})\hat{\alpha}_{ub} + (1 - \emptyset(\hat{\alpha}))\hat{\alpha}_{mom} \quad (13)$$

Note that,

$$\hat{\alpha}_{1_{ub}} = \frac{n-1}{n}\hat{\alpha}_{1_{mle}} = \frac{n-1}{-\sum_{i=1}^{n} ln\left(1-e^{-x_i^{\theta}}\right)},$$

Hence,

$E(\hat{\alpha}_{1_{ub}})=\alpha_1$ and $Var(\hat{\alpha}_{1_{ub}})=\frac{\alpha_1^2}{n-2}$

And,

$$\hat{\alpha}_{2_{ub}} = \frac{m-1}{m}\hat{\alpha}_{2_{mle}} = \frac{m-1}{-\sum_{j=1}^{m} ln\left(1-e^{-y_j^{\theta}}\right)}$$

Implies,

$E(\hat{\alpha}_{2_{ub}})=\alpha_2$ and $Var(\hat{\alpha}_{2_{ub}})=\frac{\alpha_2^2}{m-2}$.

2.3.1 The shrinkage weight function (sh1):

In this subsection, we consider the shrinkage weight factor as a function of sizes n and m respectively.

i.e. $\emptyset_1(\hat{\alpha}_1) = e^{-n}$, and $\emptyset_2(\hat{\alpha}_2) = e^{-m}$

Where n, and m is defined in subsection (2.1), therefore the shrinkage estimator of α_i which is defined in equation (13) upon above shrinkage weight function will be:

$$\hat{\alpha}_{i_{sh1}} = \emptyset_i(\hat{\alpha}_i)\hat{\alpha}_{i_{ub}} + (1 - \emptyset_i(\hat{\alpha}_i))\hat{\alpha}_{i_{mom}} \quad \text{for } i=1,2$$

…(14)

substitute equation (14) in equation (3) we obtain the shrinkage reliability estimation of $R_{(s,k)}$ model as follows:

$$\hat{R}_{(s,k)_{sh1}} = \frac{\hat{\alpha}_{2_{sh1}}}{\hat{\alpha}_{1_{sh1}}} \sum_{i=s}^{k} \frac{k!}{(k-i)!} \left[\prod_{j=0}^{i} \left(k + \frac{\hat{\alpha}_{2_{sh1}}}{\hat{\alpha}_{1_{sh1}}} - j\right) \right]^{-1}$$

… (15)

2.3.2 Constant shrinkage weight factor (sh2):

We suggest in this subsection a constant shrinkage weight factor $\varphi_i(\hat{\alpha}_i) = 0.3$; ($i$=1,2). Therefore, the shrinkage estimator using specific constant weight factor will be as follows

$$\hat{\alpha}_{i_{sh2}} = \varphi_i(\hat{\alpha}_i)\hat{\alpha}_{i_{ub}} + (1 - \varphi_i(\hat{\alpha}_i))\hat{\alpha}_{i_{mom}} \quad \text{for } i=1,2$$

…(16)

Substitute equation (16) in equation (3) to find shrinkage estimation of $R_{(s,k)}$ using the above constant shrinkage weight factor as follows:

$$\hat{R}_{(s,k)_{sh2}} = \frac{\hat{\alpha}_{2_{sh2}}}{\hat{\alpha}_{1_{sh2}}} \sum_{i=s}^{k} \frac{k!}{(k-i)!} \left[\prod_{j=0}^{i} \left(k + \frac{\hat{\alpha}_{2_{sh2}}}{\hat{\alpha}_{1_{sh2}}} - j\right) \right]^{-1}$$

…(17)

2.3.3 Modified Thompson type shrinkage weight function (th)

In this subsection, we introduce and modify the shrinkage weight factor consider by Thompson in 1968 as below.

$$\gamma(\hat{\alpha}_i) = \frac{\left(\hat{\alpha}_{i_{ub}} - \hat{\alpha}_{i_{mom}}\right)^2}{\left(\hat{\alpha}_{i_{ub}} - \hat{\alpha}_{i_{mom}}\right)^2 + Var\left(\hat{\alpha}_{i_{ub}}\right)}(0.01) \quad \text{for } i=1,2$$

…(18)

Therefore, the shrinkage estimator of α_i(i=1,2) by using above modified shrinkage weight factor will be:

$$\hat{\alpha}_{i_{th}} = \gamma(\hat{\alpha}_i)\hat{\alpha}_{i_{ub}} + (1 - \gamma(\hat{\alpha}_i))\hat{\alpha}_{i_{mom}} \quad \text{for } i=1,2$$

… (19)

Substitute equation (19) in equation (3), we conclude the reliability estimation of R(s,k) based on modified Thompson type shrinkage weight factor as follows:

$$\hat{R}_{(s,k)_{th}} = \frac{\hat{\alpha}_{2_{th}}}{\hat{\alpha}_{1_{th}}} \sum_{i=s}^{k} \frac{k!}{(k-i)!} \left[\prod_{j=0}^{i} \left(k + \frac{\hat{\alpha}_{2_{th}}}{\hat{\alpha}_{1_{th}}} - j\right) \right]^{-1} \quad (20)$$

3. Simulation Study

In this section, numerical results were studied to compare the performance of the different estimators of reliability which is obtained in section 2, using different sample size =(10, 30, 50 and 100), based on 1000 replicationvia MSE criteria. For this purpose, Mote Carlo simulation was used as the following steps[10]:

Procedure:

Step1: Generate the random sample which follows the continuous uniform distribution defined on the interval (0,1) as u_1,u_2,…, u_n.

Step 2: Generate the random sample which follows the continuous uniform distribution defined on the interval (0, 1) as v_1, v_2,…, v_m.

Step 3: Transform the uniform random samples in step1 to random samples follows EWD, applying the theorem that using the inverse cumulative probability distribution function (c. d. f.) as below shown:

$$F(x) = (1 - e^{-x_i^{\theta}})^{\alpha_1}$$

$$U_i = (1 - e^{-x_i^{\theta}})^{\alpha_1}$$

$$x_i = [-\ln(1 - U_i^{\frac{1}{\alpha_1}})]^{\frac{1}{\theta}}$$

And, calculate V_j from step2 by the same method to obtain the random variable y_j:

$$y_j = [-\ln(1 - V_j^{\frac{1}{\alpha_2}})]^{\frac{1}{\theta}}$$

Step4: Recall the $R_{(s,k)}$ as in equation (3).

Step5: Compute the maximum likelihood estimator of $R_{(s,k)}$ using equation (6).

Step6: apply the moment method on $R_{(s,k)}$using equation (11).

Step7: Calculate the three shrinkage estimators of $R_{(s,k)}$ using equations (15), (17) and (20).

Step8: Based on (L=1000) replication, the MSE for all proposed estimation methods of $R_{(s,k)}$ is utilized as follows:

$$\mathbf{MSE} = \frac{1}{L}\sum_{i=1}^{L}(\hat{R}_{(s,k)_i} - R_{(s,k)})^2$$

Where $\hat{R}_{(s,k)}$ refers the proposed estimators of real value of reliability $R_{(s,k)}$. Note that in this paper, we consider s=1 with k=3, s=2 with k=3 and s=2 with k=4, and all the results are put it in the tables (1-12) below.

Table 1. $R_{(s,k)}$ and $\hat{R}_{(s,k)}$ when s=1 with k=3 , α_1=2, α_2=4 and θ=3

(n,m)	$R_{(s,k)}$	$\hat{R}_{(s,k)mle}$	$\hat{R}_{(s,k)sh\,1}$	$\hat{R}_{(s,k)sh\,2}$	$\hat{R}_{(s,k)th}$
(10,10)	0.60000	0.59337	0.59884	0.59772	0.59883
(10,30)	0.60000	0.81560	0.59900	0.65036	0.60059
(10,50)	0.60000	0.87986	0.59815	0.65975	0.60005
(10,100)	0.60000	0.93673	0.59816	0.66882	0.60030
(30,10)	0.60000	0.33093	0.60003	0.47906	0.59595
(30,30)	0.60000	0.59527	0.59929	0.59820	0.59927
(30,50)	0.60000	0.71448	0.59972	0.63040	0.60065
(30,100)	0.60000	0.83386	0.59977	0.654860	0.60146
(50,10)	0.60000	0.22795	0.59956	0.39735	0.59075
(50,30)	0.60000	0.46937	0.59968	0.55332	0.59830
(50,50)	0.60000	0.59858	0.59957	0.59935	0.59956
(50,100)	0.60000	0.75026	0.59988	0.63846	0.60108
(100,10)	0.60000	0.12734	0.59944	0.28056	0.57942
(100,30)	0.60000	0.31071	0.60037	0.46773	0.59506
(100,50)	0.60000	0.42899	0.60019	0.53569	0.59796
(100,100)	0.60000	0.59846	0.59992	0.59952	0.59991

Table 2: MSE for $\hat{R}_{(s,k)}$when s=1 with k=3 , α_1=2, α_2=4, θ=3, and $R_{(s,k)}$= 0.60000

(n,m)	$\hat{R}_{(s,k)mle}$	$\hat{R}_{(s,k)sh\,1}$	$\hat{R}_{(s,k)sh\,2}$	$\hat{R}_{(s,k)th}$	Best
(10,10)	0.01134	0.00059	0.00240	0.00061	Sh1
(10,30)	0.04951	0.00046	0.00381	0.00047	Sh1
(10,50)	0.07956	0.00042	0.00469	0.00043	Sh1
(10,100)	0.11378	0.00041	0.00587	0.00042	Sh1
(30,10)	0.07872	0.00033	0.01768	0.00040	Sh1
(30,30)	0.00396	0.00020	0.00079	0.00021	Sh1
(30,50)	0.01522	0.00017	0.00144	0.00018	Sh1
(30,100)	0.05554	0.00014	0.00342	0.00015	Sh1
(50,10)	0.14227	0.00028	0.044682	0.00046	Sh1
(50,30)	0.02026	0.00015	0.00299	0.00016	Sh1
(50,50)	0.00225	0.000115	0.00045	0.000119	Sh1
(50,100)	0.02364	0.00009	0.00177	0.00010	Sh1
(100,10)	0.22466	0.00023	0.10490	0.00085	Sh1
(100,30)	0.08558	0.00010	0.01849	0.00014	Sh1
(100,50)	0.03104	0.00007	0.00465	0.00008	Sh1
(100,100)	0.00110	0.00005	0.00021	0.00006	Sh1

Table 3: $R_{(s,k)}$ and $\hat{R}_{(s,k)}$ when s=2 with k=3 , α_1=2, α_2=4 and θ=3

(n,m)	$R_{(s,k)}$	$\hat{R}_{(s,k)mle}$	$\hat{R}_{(s,k)sh\,1}$	$\hat{R}_{(s,k)sh\,2}$	$\hat{R}_{(s,k)th}$
(10,10)	*0.30000*	0.30617	0.29913	0.29993	0.29914
(10,30)	*0.30000*	0.61554	0.29933	0.36156	0.30114
(10,50)	*0.30000*	0.73585	0.29907	0.37489	0.30123
(10,100)	*0.30000*	0.85049	0.29873	0.38489	0.30113
(30,10)	*0.30000*	0.08613	0.29906	0.18288	0.29446
(30,30)	*0.30000*	0.30381	0.29972	0.30048	0.29973
(30,50)	*0.30000*	0.44627	0.29977	0.33490	0.30081
(30,100)	*0.30000*	0.63920	0.29917	0.36448	0.30106
(50,10)	*0.30000*	0.04005	0.30002	0.12493	0.29044
(50,30)	*0.30000*	0.18073	0.30019	0.25280	0.29870
(50,50)	*0.30000*	0.30255	0.29999	0.30042	0.30000
(50,100)	*0.30000*	0.50188	0.30004	0.34565	0.30139
(100,10)	*0.30000*	0.01258	0.30028	0.06132	0.27874
(100,30)	*0.30000*	0.07360	0.30038	0.17401	0.29449
(100,50)	*0.30000*	0.14472	0.29991	0.23304	0.29742
(100,100)	*0.30000*	0.30002	0.29975	0.29968	0.29975

Table 4: MSE for $\hat{R}_{(s,k)}$ when s=2 with k=3 , α_1=2, α_2=4, θ=3, and $R_{(s,k)}$= 0.30000

(n,m)	$\hat{R}_{(s,k)mle}$	$\hat{R}_{(s,k)sh\,1}$	$\hat{R}_{(s,k)sh\,2}$	$\hat{R}_{(s,k)th}$	Best
(10,10)	0.01375	0.00072	0.00298	0.00075	Sh1
(10,30)	0.10818	0.00055	0.00578	0.00057	Sh1
(10,50)	0.19475	0.00047	0.00738	0.00049	Sh1
(10,100)	0.30113	0.00050	0.00904	0.00051	Sh1
(30,10)	0.04776	0.00041	0.01583	0.00050	Sh1
(30,30)	0.00485	0.00025	0.00099	0.00026	Sh1
(30,50)	0.02574	0.00020	0.00195	0.00021	Sh1
(30,100)	0.11780	0.00017	0.00479	0.00018	Sh1
(50,10)	0.06804	0.00035	0.03208	0.00054	Sh1
(50,30)	0.01644	0.00019	0.00307	0.00020	Sh1
(50,50)	0.00290	0.00014	0.00058	0.00015	Sh1
(50,100)	0.04322	0.00011	0.00250	0.00012	Sh1
(100,10)	0.08266	0.00028	0.05753	0.00092	Sh1
(100,30)	0.05176	0.00013	0.01656	0.00019	Sh1
(100,50)	0.02505	0.00009	0.00496	0.00011	Sh1
(100,100)	0.00149	0.000080	0.00030	0.000082	Sh1

Table 5: $R_{(s,k)}$ and $\hat{R}_{(s,k)}$ when s=2 with k=4 , α_1=2, α_2=4 and θ=3

(n,m)	$R_{(s,k)}$	$\hat{R}_{(s,k)mle}$	$\hat{R}_{(s,k)sh\,1}$	$\hat{R}_{(s,k)sh\,2}$	$\hat{R}_{(s,k)th}$
(10,10)	0.40000	0.40183	0.39978	0.39983	0.39978
(10,30)	0.40000	0.70534	0.40108	0.46815	0.40306
(10,50)	0.40000	0.80247	0.39970	0.47944	0.40204
(10,100)	0.40000	0.89351	0.39947	0.49094	0.40211
(30,10)	0.40000	0.13701	0.39981	0.26564	0.39484
(30,30)	0.40000	0.40452	0.40048	0.40153	0.40050
(30,50)	0.40000	0.54808	0.39960	0.43695	0.40073
(30,100)	0.40000	0.72845	0.40099	0.47167	0.40308
(50,10)	0.40000	0.07005	0.40114	0.19291	0.39084
(50,30)	0.40000	0.25837	0.39934	0.34581	0.39770
(50,50)	0.40000	0.39769	0.39946	0.39878	0.39945
(50,100)	0.40000	0.60137	0.40027	0.44873	0.40174
(100,10)	0.40000	0.02341	0.39977	0.10133	0.37609
(100,30)	0.40000	0.11852	0.40054	0.25308	0.39406
(100,50)	0.40000	0.21517	0.39997	0.32420	0.39724
(100,100)	0.40000	0.40185	0.40027	0.40067	0.40028

Table 6: MSE for $\hat{R}_{(s,k)}$ when s=2 with k=4 , α_1=2, α_2=4, θ=3, and $R_{(s,k)}$= 0.40000

(n,m)	$\hat{R}_{(s,k)mle}$	$\hat{R}_{(s,k)sh\,1}$	$\hat{R}_{(s,k)sh\,2}$	$\hat{R}_{(s,k)th}$	Best
(10,10)	0.01636	0.00088	0.00359	0.00091	Sh1
(10,30)	0.09966	0.00063	0.00671	0.00065	Sh1
(10,50)	0.16530	0.00063	0.00830	0.00065	Sh1
(10,100)	0.24459	0.00059	0.01026	0.00061	Sh1
(30,10)	0.07274	0.00051	0.02120	0.00061	Sh1
(30,30)	0.00553	0.00029	0.00114	0.00030	Sh1
(30,50)	0.02614	0.00024	0.00219	0.00025	Sh1
(30,100)	0.10990	0.00022	0.00586	0.00024	Sh1
(50,10)	0.11009	0.00044	0.04551	0.00064	Sh1
(50,30)	0.02330	0.00021	0.00399	0.00023	Sh1
(50,50)	0.00327	0.00016	0.00065	0.00017	Sh1
(50,100)	0.04267	0.00014	0.00284	0.00014	Sh1
(100,10)	0.14201	0.00039	0.09067	0.00125	Sh1
(100,30)	0.08019	0.00015	0.02260	0.00021	Sh1
(100,50)	0.03576	0.00012	0.00641	0.00014	Sh1
(100,100)	0.00168	0.00008	0.00034	0.00009	Sh1

Table 7: $R_{(s,k)}$ and $\hat{R}_{(s,k)}$ when s=1 with k=3 , α_1=4, α_2=2, θ=3

(n,m)	$R_{(s,k)}$	$\hat{R}_{(s,k)mle}$	$\hat{R}_{(s,k)sh\,1}$	$\hat{R}_{(s,k)sh\,2}$	$\hat{R}_{(s,k)th}$
(10,10)	*0.85714*	0.84852	0.85684	0.85568	0.85682
(10,30)	*0.85714*	0.94664	0.85707	0.88211	0.85789
(10,50)	*0.85714*	0.96695	0.85678	0.88635	0.85775
(10,100)	*0.85714*	0.98357	0.85700	0.89065	0.85809
(30,10)	*0.85714*	0.65383	0.85688	0.78327	0.85475
(30,30)	*0.85714*	0.85505	0.85717	0.85697	0.85717
(30,50)	*0.85714*	0.90697	0.85692	0.87139	0.85739
(30,100)	*0.85714*	0.95235	0.85714	0.88366	0.85800
(50,10)	*0.85714*	0.53193	0.85694	0.72179	0.85240
(50,30)	*0.85714*	0.78053	0.85746	0.83310	0.85678
(50,50)	*0.85714*	0.85616	0.85731	0.85720	0.85731
(50,100)	*0.85714*	0.92228	0.85690	0.87555	0.85750
(100,10)	*0.85714*	0.36804	0.85723	0.60810	0.84689
(100,30)	*0.85714*	0.63802	0.85715	0.77701	0.85440
(100,50)	*0.85714*	0.74633	0.85700	0.82069	0.85583
(100,100)	*0.85714*	0.85593	0.85707	0.85685	0.85706

Table 8: MSE for $\hat{R}_{(s,k)}$ when s=1 with k=3 , α_1=4, α_2=2, θ=3, and $R_{(s,k)}$=0.85714

(n,m)	$\hat{R}_{(s,k)mle}$	$\hat{R}_{(s,k)sh\,1}$	$\hat{R}_{(s,k)sh\,2}$	$\hat{R}_{(s,k)th}$	Best
(10,10)	0.00334	0.00013	0.00058	0.00014	Sh1
(10,30)	0.00837	0.00008	0.00086	0.00009	Sh1
(10,50)	0.01218	0.00007	0.00105	0.00008	Sh1
(10,100)	0.01601	0.00006	0.00130	0.00007	Sh1
(30,10)	0.04832	0.00011	0.00712	0.00014	Sh1
(30,30)	0.00108	0.000052	0.00020	0.000054	Sh1
(30,50)	0.00284	0.000038	0.00031	0.000039	Sh1
(30,100)	0.00915	0.000028	0.00077	0.000029	Sh1
(50,10)	0.11318	0.00011	0.02100	0.00016	Sh1
(50,30)	0.00745	0.00004	0.00085	0.00005	Sh1
(50,50)	0.00056	0.000027	0.00010	0.000028	Sh1
(50,100)	0.00438	0.00001	0.00039	0.00002	Sh1
(100,10)	0.24491	0.00010	0.06649	0.00028	Sh1
(100,30)	0.05032	0.00003	0.00696	0.00005	Sh1
(100,50)	0.01342	0.00002	0.00153	0.00003	Sh1
(100,100)	0.00030	0.000016	0.00006	0.000017	Sh1

Table 9: $R_{(s,k)}$ and $\hat{R}_{(s,k)}$ when s=2 with k=3 , α_1=4, α_2=2, θ=3

(n,m)	$R_{(s,k)}$	$\hat{R}_{(s,k)mle}$	$\hat{R}_{(s,k)sh\,1}$	$\hat{R}_{(s,k)sh\,2}$	$\hat{R}_{(s,k)th}$
(10,10)	0.68571	0.67812	0.68632	0.68548	0.68632
(10,30)	0.68571	0.87271	0.68543	0.73456	0.68699
(10,50)	0.68571	0.92033	0.68465	0.74352	0.68652
(10,100)	0.68571	0.96048	0.68624	0.75425	0.68836
(30,10)	0.68571	0.37516	0.68602	0.55769	0.68202
(30,30)	0.68571	0.68049	0.68506	0.68427	0.68505
(30,50)	0.68571	0.78714	0.68536	0.71392	0.68625
(30,100)	0.68571	0.88523	0.68568	0.73764	0.68732
(50,10)	0.68571	0.24354	0.68680	0.46664	0.67839
(50,30)	0.68571	0.54967	0.68570	0.68570	0.68438
(50,50)	0.68571	0.68232	0.68548	0.68485	0.68547
(50,100)	0.68571	0.81871	0.68528	0.72161	0.68644
(100,10)	0.68571	0.10759	0.68548	0.31241	0.66574
(100,30)	0.68571	0.35007	0.68564	0.54517	0.68045
(100,50)	0.68571	0.49901	0.68590	0.61972	0.68370
(100,100)	0.68571	0.68313	0.68570	0.68512	0.68569

Table 10: MSE for $\hat{R}_{(s,k)}$ when s=2 with k=3 , α_1=4, α_2=2, θ=3, and $R_{(s,k)}$=*0.68571*

(n,m)	$\hat{R}_{(s,k)mle}$	$\hat{R}_{(s,k)sh\,1}$	$\hat{R}_{(s,k)sh\,2}$	$\hat{R}_{(s,k)th}$	Best
(10,10)	0.01181	0.00059	0.00245	0.00061	Sh1
(10,30)	0.03668	0.00033	0.00333	0.00034	Sh1
(10,50)	0.05566	0.00025	0.00411	0.00026	Sh1
(10,100)	0.07566	0.00023	0.00546	0.00024	Sh1
(30,10)	0.10669	0.00041	0.02042	0.00049	Sh1
(30,30)	0.00368	0.00019	0.00074	0.00020	Sh1
(30,50)	0.01195	0.00013	0.00122	0.00014	Sh1
(30,100)	0.68732	0.00010	0.00300	0.00011	Sh1
(50,10)	0.20218	0.00040	0.05341	0.00057	Sh1
(50,30)	0.02256	0.00015	0.00296	0.00017	Sh1
(50,50)	0.00215	0.00010	0.00042	0.00011	Sh1
(50,100)	0.01837	0.00007	0.00150	0.000074	Sh1
(100,10)	0.33641	0.00037	0.14485	0.00102	Sh1
(100,30)	0.11614	0.00014	0.02116	0.00019	Sh1
(100,50)	0.03747	0.00010	0.00499	0.00011	Sh1
(100,100)	0.00117	0.00005	0.00023	0.00006	Sh1

Table 11: $R_{(s,k)}$ and $\hat{R}_{(s,k)}$ when s=2 with k=4 , α_1=4, α_2=2, θ=3

(n,m)	$R_{(s,k)}$	$\hat{R}_{(s,k)mle}$	$\hat{R}_{(s,k)sh\,1}$	$\hat{R}_{(s,k)sh\,2}$	$\hat{R}_{(s,k)th}$
(10,10)	*0.76190*	0.74531	0.76118	0.75818	0.76114
(10,30)	*0.76190*	0.90643	0.76075	0.80022	0.76203
(10,50)	*0.76190*	0.94227	0.76074	0.80760	0.76227
(10,100)	*0.76190*	0.971453	0.76159	0.81545	0.76331
(30,10)	*0.76190*	0.47741	0.76219	0.65196	0.75889
(30,30)	*0.76190*	0.75690	0.76162	0.76077	0.76161
(30,50)	*0.76190*	0.84188	0.76136	0.78445	0.76209
(30,100)	*0.76190*	0.91724	0.76166	0.80365	0.76301
(50,10)	*0.76190*	0.32815	0.76174	0.56298	0.75464
(50,30)	*0.76190*	0.64460	0.76175	0.72352	0.76066
(50,50)	*0.76190*	0.75968	0.76178	0.76151	0.76177
(50,100)	*0.76190*	0.86809	0.76203	0.79189	0.76299
(100,10)	*0.76190*	0.16735	0.76241	0.41425	0.74622
(100,30)	*0.76190*	0.44941	0.76173	0.63954	0.75739
(100,50)	*0.76190*	0.59739	0.76198	0.70627	0.76017
(100,100)	*0.76190*	0.76052	0.76170	0.76151	0.76170

Table 12: MSE for $\hat{R}_{(s,k)}$ when s=2 with k=4 , α_1=4, α_2=2, θ=3, and $R_{(s,k)}$=0.76190

(n,m)	$\hat{R}_{(s,k)mle}$	$\hat{R}_{(s,k)sh\,1}$	$\hat{R}_{(s,k)sh\,2}$	$\hat{R}_{(s,k)th}$	Best
(10,10)	0.00951	0.00042	0.00178	0.00044	Sh1
(10,30)	0.02189	0.00021	0.00208	0.00022	Sh1
(10,50)	0.03288	0.00018	0.00260	0.00019	Sh1
(10,100)	0.04399	0.00016	0.00334	0.00017	Sh1
(30,10)	0.09225	0.00030	0.01557	0.00036	Sh1
(30,30)	0.00251	0.00012	0.00048	0.00013	Sh1
(30,50)	0.00745	0.000101	0.00081	0.000104	Sh1
(30,100)	0.02438	0.00006	0.00193	0.00007	Sh1
(50,10)	0.19695	0.00027	0.04465	0.00041	Sh1
(50,30)	0.01702	0.00010	0.00210	0.00011	Sh1
(50,50)	0.00158	0.000082	0.00031	0.000084	Sh1
(50,100)	0.01173	0.00005	0.00106	0.00006	Sh1
(100,10)	0.35721	0.00025	0.12712	0.00068	Sh1
(100,30)	0.10139	0.00009	0.01615	0.00013	Sh1
(100,50)	0.02931	0.00006	0.00355	0.00007	Sh1
(100,100)	0.00068	0.000036	0.00013	0.000037	Sh1

4. Discussion Numerical Simulation Results

For all *n*=(10,30,50,100) and for all *m*=(10,30,50,100), in this work, the minimum mean square error (MSE) for reliability estimation of $R_{(s,k)}$ model for the exponentiated Weibull distribution is held using the shrinkage weight factor as a function of sizes *n* and *m* ($\hat{R}_{sh1}$). This implies that, the shrinkage for reliability estimation ($\hat{R}_{sh1}$) is the best and follows by using Thompson type shrinkage estimator $\hat{R}_{th}$. For any *n*, some of the proposed estimator (sh_1,th) is decreasing with *m* and the other methods are vibration. Finally, when *n*=*m*=100, the third order best estimator is shrinkage estimation method using constant shrinkage weight function ($\hat{R}_{sh2}$) after $\hat{R}_{sh1}$and $\hat{R}_{th}$.

5. Conclusion

From the numerical results, one can find the proposal using the shrinkage weight factor as a function of sizes *n* and *m* ($\hat{R}_{sh1}$) which depends on unbiased estimator and prior estimate (moment method) as a linear combination, performance good behavior and it is the best estimator than the others in the sense of MSE.

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