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Stress Analysis of Laminated Composite Beam by Using MATLAB Software

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Abstract: ***Composite materials have fascinating properties like high strength to weight ratio, easy fabrication, sensible electrical and thermal properties compared to metals. A laminated material includes of many layers of a composite mixture consisting of matrix and fibers. during this paper, the normal stress and shear stress inside the laminated composite beam are analyzed by using MATLAB software, the stress and strain developed in plies of each sub-laminates due to an axial loading (N_X) at the centroid of the beam. the main goal of this paper is to illustration the benefits and ease use of MATLAB software to analysis the laminated composite beam and to find the stress and strain on each layer of laminated composite beam. The reinforcement fiber was used a carbon fiber as material. The engineering properties values of multilayered beam and several others of the various results of the stress-strain distributions in the layers of the analyzed laminated beam are given. The numerical results in this work were calculated by using MATLAB software that is ready to analyzing of multilayered beam consist of material. Based on the results the maximum axial stress (σ_x) occurs on 0^0 ply, the maximum vertical stress (σ_y) occurs on 90^0 ply and the maximum shear stress (τ_{max}) occurs on 45^0 ply. They are presented in tabular and graphical forms.***

Keywords: Laminated composite beam, MATLAB software, stress, strain

1. Introduction

Composite materials are materials constituted for two or more components, which remain independent at the macroscopic level when they become part of a structure. The main advantages for the use composite materials are high strength and high stiffness to weight ratio. Those advantages are why composite materials are used in many fields of industry. It is more common to see composite material products in many forms and applications. Even though composite materials in their form of fiber reinforced composites offer many design possibilities, its complexity in design increases. The design engineer must be aware of the results and analyze many cases and possible designs; this is possible nowadays with the aid of a computer and with some computer software. The effect of transverse shear deformation is more pronounced in thick beams made of fibrous composite material which has a high extensional modulus to shear modulus ratio [1].

Many authors analyzed the laminated beam structures. **Later, Rojas and Chan** [2] in a study integrated an analysis of laminates including calculation of structural section properties, failure prediction, and analysis of composite laminated beams. **Rios** [3] presented a unified analysis of stiffener reinforced composite beams. The study presented a general analytical method to study the structural response of composite laminated beams. **Avinash Ramsaroop and Krishnan Kanny** [4] present the generation of MATLAB script files that assists the user in the design of a composite laminate to operate within safe conditions. **Sayyad** [5] presented a refined shear deformation theory for the static flexure and free vibration analysis of thick isotropic beams considering parabolic, trigonometric, hyperbolic and exponential functions in terms of thickness co-ordinate associated with transverse shear deformation effect. This theory is further extended by **Sayyad et al.** [6] for the flexural analysis of single layered composite beams.

2. Laminated Composite Structures

A laminate is constructed by stacking a number of laminas in the thickness (z) direction. Each layer is thin and may have different fiber orientation. The fiber orientation, stacking arrangements and material properties influence the response from the laminate. The theory of lamination is same whether the composite structure beams. Fig 1. shows a laminated beam considered in most of the analysis[7].

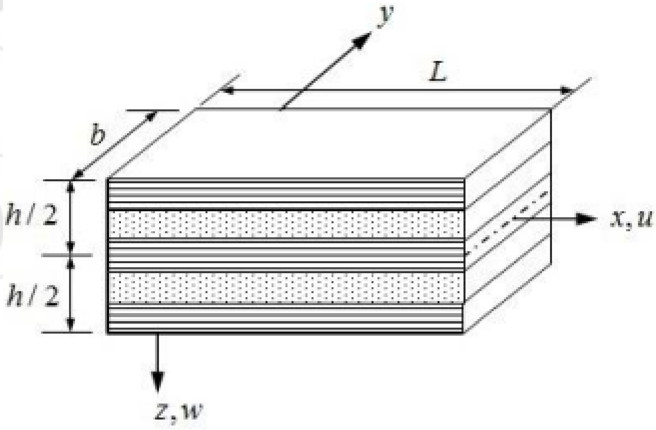

Figure 1: Geometry and Coordinate System of Laminated Composite Beam.

3. Materials and Methods

3.1 Problem Description

The normal stress (σ) and strain (ε) in the laminated composite beam are analyzed by MATLAB software, The stresses and strains developed in plies of each sub-laminates due to an axial loading (N_x = 1.0 N/m) at the centroid of the beam cross-section as fig 2, we write a MATLAB program that assists the user to find out the stiffness matrix of a laminate composite. The dimensions of laminated composite beam are illustrated in table 1.

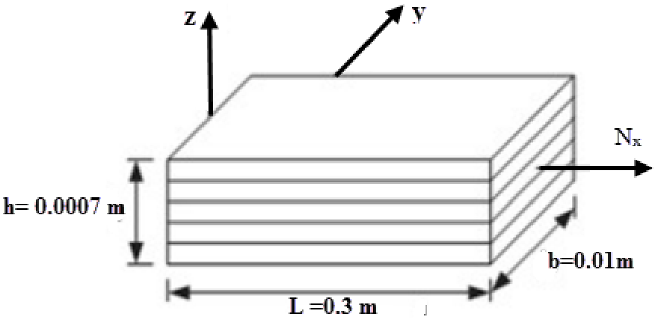

Figure 2: Geometry of laminated composite beam

Table 1: The dimension of laminated composite beam

Length	*Width*	*Thickness*
0.3 m	0.01 m	0.0007 m

The Stacking of plies with different angles of laminated composite beam are illustrated in fig 3.

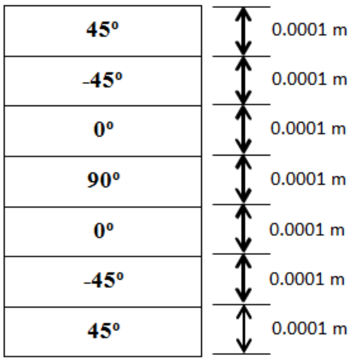

Figure 3: The Stacking of plies with different angles of laminated composite beam

4. Mathematical Formations

4.1 Assumptions in Classical Lamination Theory

The following assumptions are made in formulations:

1) Each lamina of the laminated composite is nonhomogeneous and isotropic.
2) The middle plane of the beam is taken as the reference plane.
3) The laminated beam consists of arbitrary number of homogeneous, linearly elastic orthotropic layers perfectly bonded to each other.
4) The analysis follows linear constitutive relations i.e. obeys generalized Hooke's law for the material.
5) The lateral displacements are small compared to beam thickness.
6) The laminate is thin and edge dimensions of composite beam are much larger than its thickness. The loadings are applied in the laminate's plane and the laminate (except for their edges) are in a state of plane stress ($\sigma_z = \tau_{yz} = 0$).
7) In plane displacements in the x and y, directions are linear functions of z.
8) Transverse shear strains γxz and γyz are negligible.
9) Stress-strain and strain-displacement relations are linear.
10) The transverse normal strain εz is negligible.

4.2 Derivation of the Formula for Normal Stress and Shear Stress

The use of mathematics is one of the many approaches to solving real-world problems. Others include experimentation either with scaled physical models or with the real world directly. Mathematical modeling is the process by which a problem as it appears in the real world is interpreted and represented in terms of abstract symbols. This makes mathematical modeling challenging and at the same time demanding since the use of mathematics and computers for solving real world problems is very widespread and has an impact in all branches of learning [8]

The coordinate system for a laminated beam used in the present paper is shown in fgure.4. The xyz coordinate system is assumed to have its origin at the corner of the middle plane of the beam. The surfaces of the beam are at z=+h/2 and h is the thickness of the beam .ɵ is the angle of fiber orientation with the respect to the axis.1-,2- and 3-directions are principal axes in the longitudinal, transverse and normal directions ,respectively, it is assumed that the transverse deflection is small, so that the out-of-plane components of the in plane results , and are negligible [8].

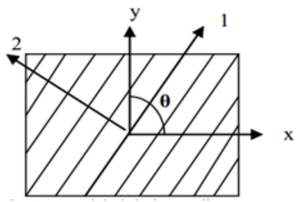

Figure 4: Local / global coordinate systems

The analyzed angle-ply symmetric layered composite beams have been made from three orthotropic laminas with equal thickness that had bonded together normal to their principal plane (1, 2). In a symmetric laminate all layers above the mid-plane of the laminate (a plane of symmetry) have the different angle as the ply in the equivalent position below the mid-plane. By using of the generalized Hooke's law in the principal material coordinates of an orthotropic lamina. The stress-strain relationships for a composite material in linear elastic area can be written in shorthand matrix form as [9, 10]:

$$\{\sigma\} = [C]\{\varepsilon\} \qquad \text{Or in inverted form: } \{\varepsilon\} = [S]\{\sigma\} \tag{1}$$

Where $[C]$ is the stiffness matrix and $[S]$ is the compliance matrix of a lamina.

4.3.1 Stress-Strain Relationship for 00 Lamina [11]

Since the lamina is thin, the state of stress can be considered in the plane stress condition. That means,

$$\sigma_3 = 0 \quad ,\tau_{23} = 0 \quad ,\tau_{13} = 0 \tag{2}$$

Hence, the stress-strain relationship for thin lamina in the matrix form along the principal axis can be written as,

$$[\varepsilon]_{1-2} = \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} \quad (3)$$

The elements in compliance matrix [S] are the functions of elastic constant of the composite lamina and can be expressed as,

$$S_{11} = \frac{1}{E_1}, S_{22} = \frac{1}{E_2}, S_{12} = -\frac{\vartheta_{12}}{E_1} = -\frac{\vartheta_{21}}{E_2}, S_{66} = \frac{1}{G_{12}} \quad (4)$$

The engineering properties of an unidirectional lamina in the principal material coordinate axes (1, *2*), Figure. 2., are: E_1 – the longitudinal modulus of elasticity in the 1 direction, E_2 – the transverse modulus of elasticity in the 2 direction, G_{12} – the shear modulus in (1, 2)-plane, and ν_{12} – the major Poisson's ratio. Inverting Equation (3) we have,

$$[\sigma]_{1-2} = \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{bmatrix} \quad (5)$$

The elements in stiffness matrix [Q] can be expressed as,

$$Q_{11} = \frac{E_1}{1 - \vartheta_{12}\vartheta_{21}}$$

$$Q_{22} = \frac{E_2}{1 - \vartheta_{12}\vartheta_{21}}$$

$$Q_{12} = \frac{\vartheta_{21}E_1}{1 - \vartheta_{12}\vartheta_{21}} = \frac{\vartheta_{12}E_2}{1 - \vartheta_{12}\vartheta_{21}}$$

$$Q_{66} = G_{12} \quad (6)$$

4.3.2 Stress-Strain Transformation Matrices [11]

Generally, the lamina reference axes (x, y) do not coincide with the lamina principle axes (1, 2). Therefore, the relation between the stress and strain components in principal axes making an angle with respect to reference axes can be expressed using transformation matrices as,

$$[\sigma]_{1-2} = [T_\sigma][\sigma]_{x-y}$$

And

$$[\epsilon]_{1-2} = [T_\varepsilon][\epsilon]_{x-y} \quad (7)$$

Where $[T_\sigma]$ and $[T_\varepsilon]$ are the transformation matrices for stress and strain, respectively. They are:

$$[T_\sigma] = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2 - n^2 \end{bmatrix}$$

$$[T_\varepsilon] = \begin{bmatrix} m^2 & n^2 & mn \\ n^2 & m^2 & -mn \\ -2mn & 2mn & m^2 - n^2 \end{bmatrix} \quad (8)$$

Where $m = Cos\theta$ and $n = Sin\theta$

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \overline{Q_{11}} & \overline{Q_{12}} & \overline{Q_{16}} \\ \overline{Q_{21}} & \overline{Q_{22}} & \overline{Q_{26}} \\ \overline{Q_{61}} & \overline{Q_{62}} & \overline{Q_{66}} \end{bmatrix} \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix} \quad (9)$$

Where the elements in the stiffness matrix, $\overline{Q}$ matrix are

$$\overline{Q_{11}} = m^4Q_{11} + n^4Q_{22} + 2m^2n^2Q_{12} + 4m^2n^2Q_{66}$$

$$\overline{Q_{22}} = n^4Q_{11} + m^4Q_{22} + 2m^2n^2Q_{12} + 4m^2n^2Q_{66}$$

$$\overline{Q_{12}} = \overline{Q_{21}} = m^2n^2(Q_{11} + Q_{22} - 4Q_{66}) + (m^4 + n^4)Q_{12}$$

$$\overline{Q_{16}} = \overline{Q_{61}} = m^3n(Q_{11} - Q_{22} - 2Q_{66}) + mn^3(Q_{12} - Q_{22} + 2Q_{66})$$

$$\overline{Q_{26}} = \overline{Q_{62}} = mn^3(Q_{11} - Q_{22} - 2Q_{66}) + m^3n(Q_{12} - Q_{22} + 2Q_{66})$$

$$\overline{Q_{66}} = m^3n^3(Q_{11} + Q_{22} - 2Q_{12}) + (m^2 - n^2)^2Q_{66} \quad (10)$$

$$[\overline{Q}] = [T]^{-1}[Q][T] \quad (11)$$

The analysis of layered structures is based on the classical lamination theory [1, 2] and the orthotropic beam is analyzed with the coordinate system (x, y, z) on the middle surface of the beam (z = 0), Figure.5. The strains and curvatures on the middle surface of beam are in this case.

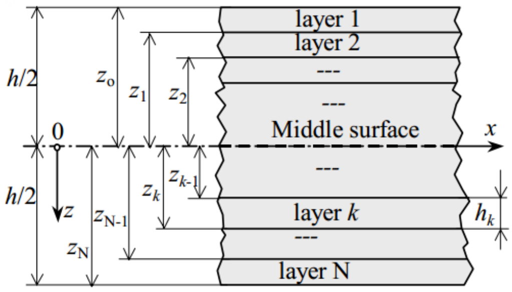

Figure 5: Laminated beam geometry and layer numbering System [11].

Resultant forces and moments:

$$N_x = \int_{-h/2}^{h/2} \sigma_x\, dz \qquad M_x = \int_{-h/2}^{h/2} \sigma_x\, z\, dz \quad (12)$$

$$N_y = \int_{-h/2}^{h/2} \sigma_y\, dz \qquad M_y = \int_{-h/2}^{h/2} \sigma_y\, z\, dz \quad (13)$$

$$N_{xy} = \int_{-h/2}^{h/2} \tau_{xy}\, dz \qquad M_{xy} = \int_{-h/2}^{h/2} \tau_{xy}\, z\, dz \quad (14)$$

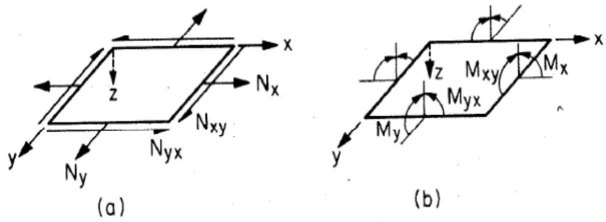

Figure 6: Positive Sense of Resultant Forces and moments. [11]

The total constitutive equation or load-deformation relation for the laminate is as follows:

$$\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{bmatrix} \epsilon^0 \\ k \end{bmatrix} \quad (15)$$

Where:

$$[A] = \sum_{k=1}^{n} [\overline{Q}]^k (h_k - h_{k-1})$$

$$[B] = \frac{1}{2} \sum_{k=1}^{n} [\overline{Q}]^k (h_k^2 - h_{k-1}^2)$$

$$[D] = \frac{1}{3} \sum_{k=1}^{n} [\overline{Q}]^k (h_k^3 - h_{k-1}^3) \quad (13)$$

5. Numerical Results

The stresses and strains developed in plies of each sub-laminates due to an axial loading at the centroid of the cross-section by using numerical method and MATLAB code for this study. The stresses from analytical expression for each sub laminated are confirmed from the MATLAB results in table: 1.

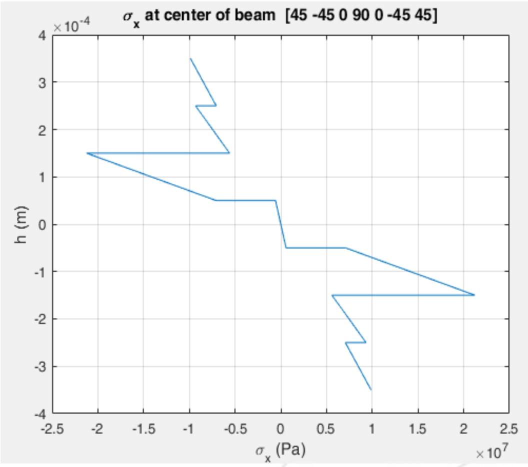

Figure 7: The axial stress (σ_x) in Laminated Composite beam

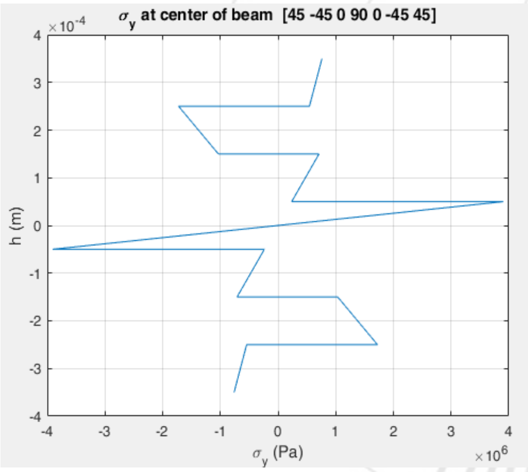

Figure 8: The vertical stress (σ_y) in Laminated Composite beam

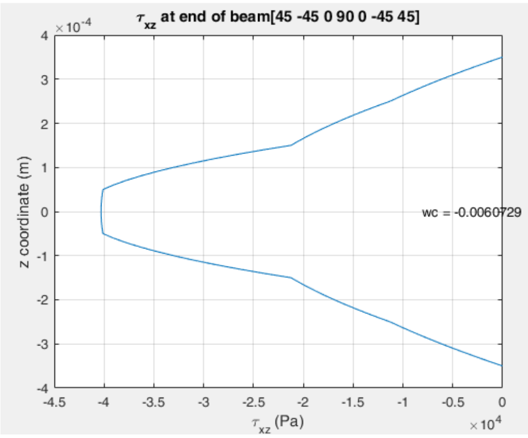

Figure 9: The shear stress (τ_{xy}) in Laminated Composite Beam

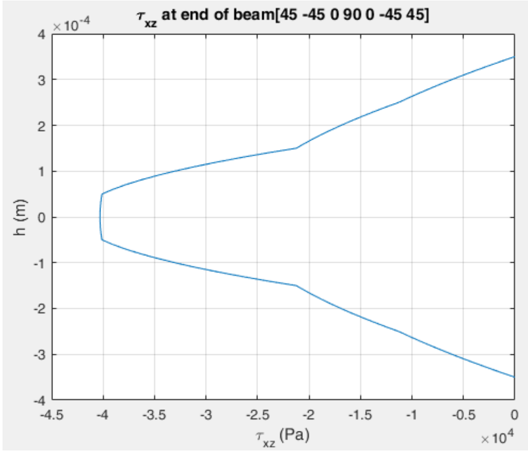

Figure 10: The shear stress (τ_{xz}) at end of Laminated composite beam

Table 2: The Stresses in Laminated Composite Beam

Ply No:	*Angle*	*Stress* (σ_x)	*Stress* (σ_y)	*Shear stress* (τ_{xy})
Ply # 1	45	9.8784e+06	7.6176e+05	-3.1147e+06
Ply # 2	-45	9.3240e+06	-1.7239e+06	4.8557e+06
Ply # 3	0	2.1234e+07	7.1065e+05	2.0337e+05
Ply # 4	90	5.7675e+05	3.9060e+06	6.7793e+04
Ply # 5	0	-2.1234e+07	-7.1065e+05	-2.0337e+05
Ply # 6	-45	-9.3240e+06	1.7239e+06	-4.8557e+06
Ply # 7	45	-9.8784e+06	-7.6176e+05	3.1147e+06

The results show the maximum axial stress occurs in 0^0 ply in both laminates. The 0^0 ply in sub-laminates provide the pure axial stiffness to the laminated composite beam and the maximum shear stress occurs in 45^0 ply in both laminates.

6. Acknowledgements

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7. Conclusion

In this paper the mechanics of the laminated composite beam mistreatment MATLAB software have been considered and an analytical method was developed for stress analysis of composite beam. Finally, the stress and strain in every ply of laminates is calculated by using MATLAB software to get the stiffness of every ply and the stress of every ply under an axial loading at the centroid of the beam.

From this paper, following conclusion can be made.

- Analytical expression to analyze stress shows excellent resulted when using MATLAB software.
- There is a symmetrical distribution of stress and shear stress around the neutral axis of the beam.
- The stress and strain in each ply of beam subjected to axial load at the centroid had a difference results.
- The axial stress (σ_x) in 0^0 ply, due to axial load at the centroid, was at its maximum for the laminated composite beams.
- The stress (σ_y) in 90^0 ply, due to axial load at the centroid, was at its maximum for the laminated composite beams.

- The shear stress (τ_{xy}) in 45^0 ply, due to axial load at the centroid, was at its maximum for the laminated composite beams.
- Based on these results a designer can choose the right ply orientations to control the maximum stress and shear stress of laminated composite beams.

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