Axi-Symmetric Thermal Analysis of Regenerative Cooled Cryogenic Engine Nozzle Using Fem

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Abstract: *An axi-symmetric thermal analysis on the cooling channels of a regenerative cooled cryogenic engine nozzle is carried out. Taylor Galerkin Finite Element Method is used as a means for spatial & time discretization of the energy equation in cylindrical coordinate system. Numerically simulated results for temperature distribution in the nozzle wall is obtained for a given set of mass flow rate of coolant and hot gas side heat transfer coefficients, set as boundary conditions. The results obtained are used to determine the optimum mass flow rate of coolant required for the proper functioning of the engine.*

Keywords: cryogenic engine, cooling channels, finite element method, nozzle wall, mass flow rate, Bartz equation, regenerative cooling

1. Introduction

Modern cryogenic rocket engine operates under severe pressure and temperature conditions leading to generate very high heat flux ($0.5 – 160$ MW/m^2) in rocket thrust chamber[1],[2].The combustion products from cryogenic propellants such as liquid hydrogen and liquid oxygen can reach the temperature as high as 3500K, which is much higher than the melting point of the material generally used in thrust chambers. In-order to achieve a failsafe operation of the engine, certain specific design considerations is to be taken into account. One such consideration is to maintain the material strength of the thrust chamber even at such a hostile environment of operation, as stated above. The engine nozzle expands and accelerates the combustion gases for generating thrust. Being next to thrust chamber and of reduced cross section, it experiences much higher heat flux compared to thrust chamber. Hence, these kinds of rocket engines need re-generative cooling system to prevent the thrust chamber and nozzle from becoming excessive hot by maintaining a balance between the heat energy released by the combusted gas and the heat energy absorbed by the coolant [3].

Given this background, it is proposed to develop a comprehensive thermal model for re-generatively cooled rocket engine nozzle accounting for heat flux from hot gases, conduction within the nozzle wall and convection to the coolant in cooling channels. However, determination of co-efficient of gas side heat transfer poses a complex problem. The boundary layer that controls the heat transfer rate to the wall is greatly affected by the turbulent combustion process, local gas compositions and temperature, thereby governing the magnitude of local heat transfer co-efficient[4].

Galerkin based FEM[5]solver in conjugation with the Taylor explicit scheme of time integration is developed and validated. The problem is modeled as an axi-symmetric problem where linear triangular elements are used in cylindrical coordinate system for space discretization. The considered geometry is meshed using mesh generating software of GAMBIT [6] to generate nodes and elements.

The heat transfer co-efficient for hot gas side convection is estimated in terms of gas temperature using Bartz correlation [7], based on thermal properties of combusted gases and isentropic gas equations and coolant side heat transfer by Nusselt-type Gnielinski co-relation [8] within valid range of Reynolds and Prandtl numbers.

Numerically simulated results for temperature distribution in nozzle wall is found out, from which appropriate coolant mass flow rate through the coolant channel is evaluated for safe operation of the engine.

2. Numerical Modeling

The thrust chamber wall geometry is modeled using the heat conduction equation in cylindrical coordinates. The problem is thus projected as an axi-symmetric problem. The solution of the resulting differential equation, with prescribed boundary conditions, gives the temperature distribution in the nozzle.

2.1 Governing Equations and boundary conditions

Thermal analysis on the effect of regenerative cooling to the nozzle wall temperature distribution is described by applying the energy conservation law to a differential control volume [9]. The solution of the resulting differential equation, with prescribed boundary conditions, gives the temperature distribution in the computational domain. Considering isotropic (constant thermal conductivity) condition, the heat conduction equation is written as

$$k\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_{actual}}{\partial r}\right)+\frac{1}{r^2}\left(\frac{\partial^2 T_{actual}}{\partial \theta^2}\right)+\frac{\partial^2 T_{actual}}{\partial z^2}\right)+Q=\rho C_p \frac{\partial T_{actual}}{\partial t} \quad \text{--- (1)}$$
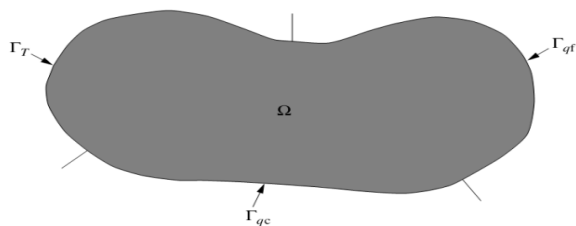
Figure 1: Boundary conditions for an arbitrary control volume in space

The Equations (2) and (3),mentioned below describe the boundary conditions to be implemented for the analysis of the system. Here, T_0 is the prescribed temperature; Γ the boundary surface; n is the outward direction normal to the surface and C is the constant flux given. The insulated, or adiabatic, condition can be obtained by substituting C =0.The set boundary condition can be applied in equation (4).The boundary condition of convective heat transfer also falls into the Neumann category and can be expressed as

$$-k\frac{\partial T}{\partial n} = h(T_w - T_a) \text{ on } \Gamma_{qc}$$

--- (2)

Since time appears as a first-order term, only one initial value (i.e., at some instant of time all temperatures must be known) needs to be specified for the entire body, that is,

$$T = T_0 \text{ all over the domain } \Omega \text{ at } t = t_0$$

--- (3)

where t_0 is a reference time. The constant or variable temperature conditions are generally easy to implement as temperature is a scalar. Hence, equation (2) and(3)can be expressed with the direction cosines of the outward normal as

$$k_x\frac{\partial T}{\partial x}l + k_y\frac{\partial T}{\partial y}m + k_z\frac{\partial T}{\partial z}n = C \text{ on } \Gamma_{qc}$$

---(4)

$$k_x\frac{\partial T}{\partial x}l + k_y\frac{\partial T}{\partial y}m + k_z\frac{\partial T}{\partial z}n = h(T - T_a) \text{ on } \Gamma_{qf}$$

--(5)

Where l,m and n are the direction cosines of the appropriate outward surface normal's.

2.2 Taylor-Galerkin Discretization of Heat Conduction Equation

The isotropic heat conduction equation given below is discretized using Taylor-Galerkin discretization method[10].

$$k\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_{actual}}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T_{actual}}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}\right) + Q = \rho C_p\frac{\partial T_{actual}}{\partial t}$$

---(6)

The Boundary Condition is,

$$k\left(\frac{\partial T^*}{\partial r}n_1 + \frac{\partial T^*}{\partial z}n_2\right) + q + h(T^* - T_a) = 0$$

---(7)

This on further Re-arranging gives,

$$[C]T^{n+1} = [C]T^n + \left(\{f\} - [K]\{T\}\right)\Delta t$$

-- (8)

This Taylor-Galerkin Solution of the Heat Conduction equation is valid as long as the Von-Neumann stability criterion is satisfied.

Von Neumann Stability Criterion [11]:

$$\Delta t \le 0.5\frac{\Delta x^2}{\alpha}$$

— (9)

The above equation (8) is solved by the Taylor Galerkin explicit scheme to obtain the temperature distribution throughout the computational domain .

3. Computation Model Description and Assumptions

Nozzle description: OFHC Copper/Stainless Steel
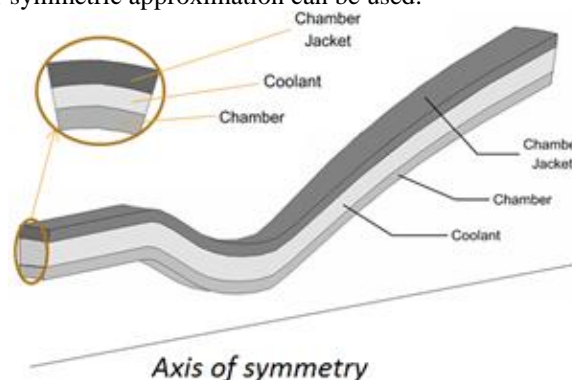
Material properties:
Copper:
Melting pt.: 1358K, Th. Cond.: 401W/mK, Density: 8933Kg/m^3, Cp: 385J/Kg K

Stainless Steel:
Melting pt.: 1670K, Th. Cond.: 20W/m K, Density: 7800Kg/m^3, Cp: 450J/Kg K

An element of cooling channel as shown in Fig.2is considered. The Coolant Channel is taken as annular so that axi-symmetric approximation can be used.

Figure 2: A cross-sectional view of the coolant channel

3.1 Computational Domain

The Nozzle Geometry was obtained from a CAD software and meshed using GAMBIT[6] to generate **105666 nodes** and 184620**elements**.
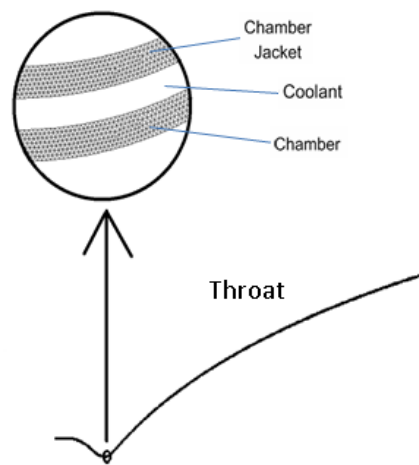
Figure 3: Nozzle Geometry meshed with three node triangular elements

GAMBIT was used to specify the boundary conditions, the hot gas side and coolant side are forced convection, and the outside is free convection since the analysis is for ground test. The outer Free Convection was specified on the Coolant Jacket with heat convection coefficient; h= 5W/m²K and Ta= 300K.

3.2 Gas Side Heat Transfer

Bartz Correlation [7] is used for estimation of rocket nozzle convective heat transfer co-efficient based on thermal properties of products of combustion and isentropic gas equations.

$$h = \left(\frac{0.026}{D^{*0.2}}\right)\left(\frac{\mu_0^{0.2}C_p}{Pr^{0.6}}\right)\left(\frac{P_0}{C^*}\right)^{0.8}\left(\frac{D^*}{r_c}\right)^{0.1}\left(\frac{A^*}{A}\right)^{0.9}\sigma \quad \text{-- (10)}$$

$$where, \sigma = 1$$

$$\mu_0 = 11.848 \times 10^{-8} \times \sqrt{M_w} \times T_0^{0.6}$$

$$C^* = \frac{\sqrt{\gamma R T_0}}{\gamma \sqrt{\left(\frac{2}{\gamma+1}\right)^{\left(\frac{\gamma+1}{\gamma-1}\right)}}} \quad \text{--- (11)}$$

To=3418K
Po=58bar
Cp=5161.105 J/Kg.K
Molecular Weight of Combustion gases: M_w=13.142
Pr=0.47
Throat Area: A*=1591.5mm² and Throat Diameter: D*=45.00mm
Radius of Curvature, r_c= 90mm

3.3 Coolant Side Heat Transfer

The Coolant is taken as Gaseous Hydrogen throughout the channel contour. The Heat Transfer between the coolant and the thrust chamber wall is by forced convection. The correlationsusedforcoolantsideheattransferareprincipallybase dontheconventionalNusselt-type correlations for turbulent, thermally fully developed flow for fluids with constant property values[12],[13].

The Coolant Properties and conditions are:
Coolant Temperature: Ta=250K.
Pressure: Po=105bar
Molecular Weight: M_w=2 (GH₂)
Pr=0.707
Cp=14060 J/Kg.K
K=0.157W/m.K
Coefficient of Viscosity: μ=7.89 x 10⁻⁶Ns/m²
Density: ρ= 10.10344 kg/m³

The Reynolds Number of flow is estimated to the order of10⁵,thus heat transfer correlation is estimated using Nusselt-type correlation developed by Gnielinski; since Reynolds number and Prandtl number of the coolant flow lie in the permissible limits for using Gnielinski correlation [9].

$$Nu = \frac{(\frac{f}{8})(\text{Re}-1000)\,Pr}{1+12.78\left(\sqrt{\frac{f}{8}}\right)\left(Pr^{\frac{2}{3}}-1\right)} \quad \text{--- (12)}$$

The Gnielinski Correlation is valid for

$$0.5 \leq Pr \leq 2000$$

$$3000 \leq \text{Re}_D \leq 5 \times 10^6$$

Where f is the Darcy friction factor that can be obtained from the correlation developed by Petukhov[14].

$$f = (0.782 \ln \text{Re} - 1.64)^{-2} \quad \text{--- (13)}$$

The value of coefficient of heat transfer is then calculated from the above equations and used accordingly in order to set the appropriate boundary conditions for the solver.

4. Results and Discussions

The Analysis was done for Steady state Condition. Steady state time was obtained by computing the change in the temperature between two time steps. Steady state was assumed to be reached when this change in Temperature was less than the magnitude of 10⁻⁵ K.
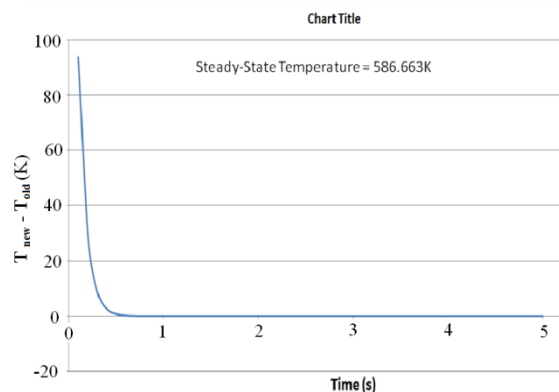
Figure 4: Change in temperature with increasing time step.

The total run time is taken as 5 seconds since steady state condition is reached before 5s.
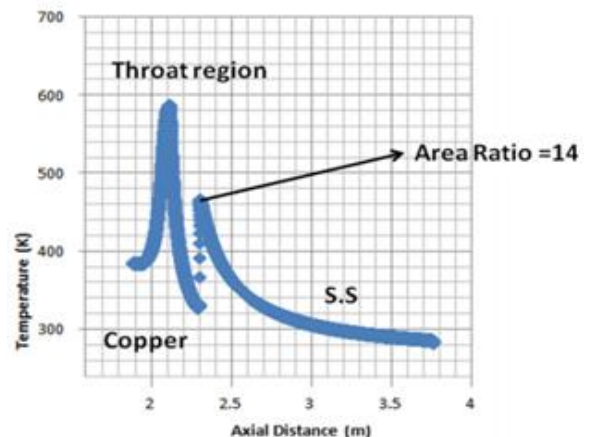
Figure 5: Steady State Gas Side Wall Temperature vs Axial distance 'z'

Fig 5. shows the steady state heat transfer condition. The maximum temperature was observed at the throat region which is depicted by the first thermal spike. The second thermal spike is because of the change in thermal conductivities from Copper to Stainless Steel at area ratio of 14. Multi material configuration is easily handled by the code and the results provide the capacity of the code in effective prediction of temperature with different boundary conditions.

Analysis on GH_2 Cooled Nozzle

The Analysis was carried out for 4 cases, findings of which are described below:

Case 1: Normal Coolant Mass flow Rate of 2.4Kg/s and Normal Hot Side Heat Convection obtained from Bartz equation.
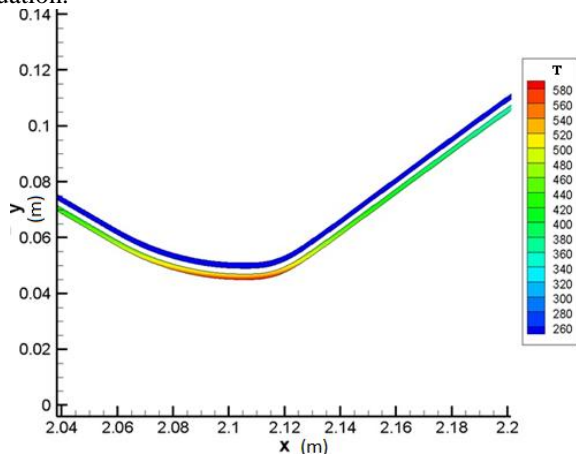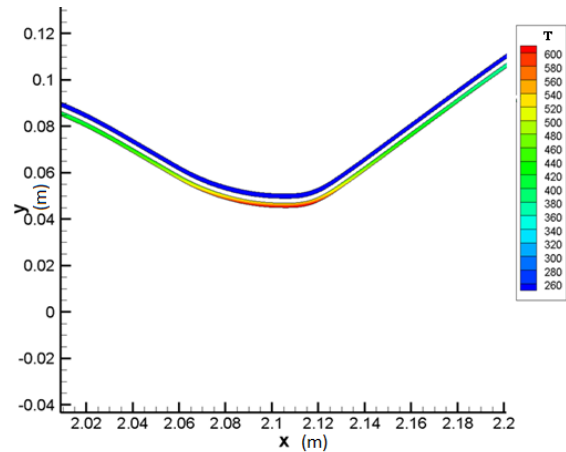

Figure 6: Steady State Temperature versus Axial distance near the throat region for Case 1.

T_{max}= 578.832K
T_{min}=260 K

The maximum temperature of 578.832K is predicted at the throat where the material is copper and it is much lesser than melting point of copper (1358K) at area ratio of 14 the temperature is 475K which is much less than the melting point of stainless steel (1670K). The temperature contour shows that highest temperature is at the throat region of the chamber wall. The temperature in the chamber jacket is nearly the same as the coolant temperature, the slight increase is due to the boundary conditions of outer free convection.

Case 2: Reduced Coolant Mass flow Rate of 2Kg/s and Normal Hot Side Heat Convection obtained from Bartz equation.
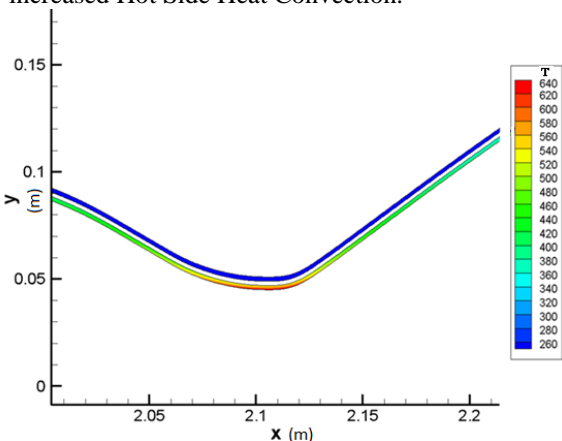

Figure 7: Steady State Temperature versus Axial distance near the throat region for Case 2.

T_{max}=605.832K
T_{min}=260 K

In the present case, 20% decrease in coolant mass flow rate (from 2.4 kg/s to 2 kg/s) caused 3.3% increase in temperature at the throat region. The temperature increased from 586.663K to605.832K.Therefore this substantiates that the coolant mass flow rate has a direct effect on the temperature distribution throughout the nozzle geometry.

Case 3: Normal Coolant Mass flow Rate of 2.4Kg/s and 20% increased Hot Side Heat Convection.
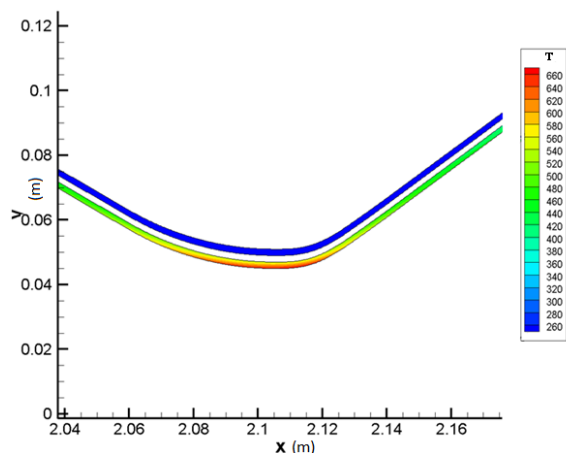

Figure 8: Steady State Temperature versus Axial distance near the throat region for Case 3.

T_{max}=645.664K
T_{min}=260 K

In the case presented above, 20% increased hot side heat convection is imposed for temperature calculations. This change has caused a maximum temperature of 645.664 K in the throat region. Temperature increased by 10 %.

Case 4: Reduced Coolant Mass flow Rate of 2Kg/s and 20% increased Hot Side Heat Convection.

Figure 9: Steady State Temperature versus Axial distance near the throat region for Case 4.

T_{max}=667.704K
T_{min}=260 K

An adverse situation occurred in this case with rising of temperature as high as 667.704K.at the throat region. This clearly indicates the importance of both the mass flow rate of the coolant &the convection heat transfer coefficient as obtained from the Bartz correlation.
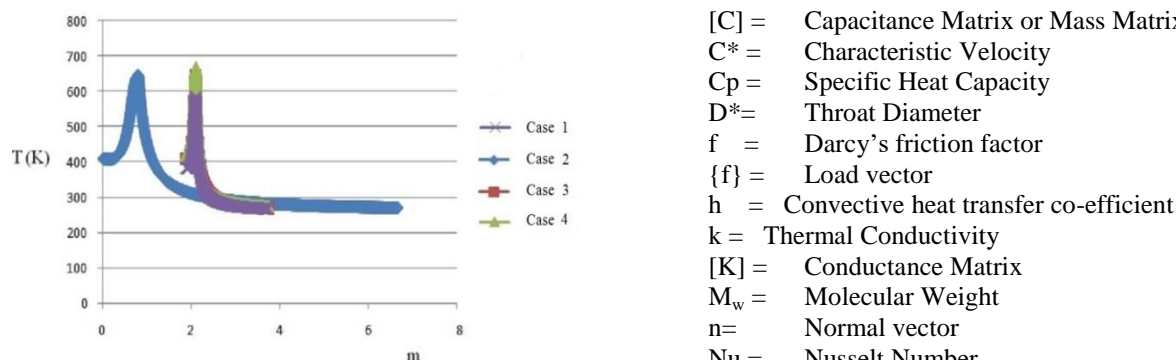
Figure 10: Comparison of temperature distributions throughout the entire nozzle geometry for all the cases.

Table 1: Variation of temperature with respect to mass flow rate and convection heat transfer coefficient.

Case	Mass flow rate (Kg/s)	Convection heat transfer coefficient	Max Temp(K)	Min Temp(K)
1	2.4	Normal	578.832	260
2	2	Normal	605.832	260
3	2.4	20% increased	645.664	260
4	2	20% increased	667.704	260

The above table summarizes the entire set of observations made.

5. Conclusions

Validated Heat Conduction Solver was used for the analysis of Regenerative cooled Cryogenic Engine involving different boundary conditions. Temperature prediction for annular channel is done using the Axi-symmetric approximation. The cryogenic engine is modeled to operate on a liquid hydrogen/ liquid oxygen mixture at a chamber pressure of around 58 bar and chamber temperature of 3418K.

Multi-Material configuration is easily handled by the Code and the results proved the capability of the code in effective temperature prediction with different boundary conditions. Validation of the model is tested through four different case studies. The important findings are as follows:
1) The highest temperature occurred at the throat region where the material used was copper. A variation of heat flux in hot gas side, affects regenerative cooling more severely than a corresponding variation in the coolant mass flow rate.
2) Temperature at the throat region with reduced coolant mass flow rate of 2Kg/s and 20% increased hot side heat convection was observed to be 667.704 K.
3) Since the highest observed Temperature was found to be lesser than the melting point of the materials used, it can be said that the regenerative cooling is efficient even in cases of reduced coolant mass flow rate and increased hot gas side heat flux.
4) Coolant mass flow rate of 2.4kg/s is adequate for Regenerative Cooling of the Cryogenic Engine Nozzle considered.

Nomenclature
A   =   Arbitrary channel Area
A* =   Throat Area
C =   Constant heat flux
[C] =   Capacitance Matrix or Mass Matrix
C* =   Characteristic Velocity
Cp =   Specific Heat Capacity
D* =   Throat Diameter
f   =   Darcy's friction factor
{f} =   Load vector
h   =   Convective heat transfer co-efficient
k =   Thermal Conductivity
[K] =   Conductance Matrix
M_w =   Molecular Weight
n=   Normal vector
Nu =   Nusselt Number
Po =   Prescribed Pressure
Pr =   Prandtl Number
q   =   Heat flux
Q =   Heat generation
r =   Radial co-ordinate
r_c=   Radius of Curvature
R =   Universal Gas Constant
Re =   Reynolds Number
t =   time
T =   Temperature
T_0 =   Prescribed temperature
T_a =   Ambient temperature
z=   z Coordinate
α=   Thermal Diffusivity
Δ =   Change
Δx =   Minimum Element Edge Length
γ   =   Ratio of Specific heats
Γ =   Boundary surface
Ω =   Domain
ρ =   Density
μ =   Co-eff.of viscosity
θ =   Angular Co-ordinate

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Subscripts

max = Maximum
min = Minimum
qc = Boundary qc
qf = Boundary qf
c = Curvature
T = Total

6. Acknowledgement

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