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# Theory of Quasi C-Reducible Finsler Manifold

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**Abstract:** ***Purpose of this paper is to study the theory of quasi C-reducible Finsler manifold. In this paper, we have obtained some important theorems on quasi C-reducible Finsler manifold.***

**Keywords:** (h)hv-torsion tensor, angular metric tensor, C-2 like, C-reducible, quasi C-reducible Finsler manifold.

Let $F^n$ be an n-dimensional Finsler manifold with the metric tensor $g_{ij}$, the angular metric tensor $h_{ij}$ and (h)hv-torsion tensor $C_{ijk}$. We have the following definitions:

**Definition 1.1:**
A Finsler manifold $F^n$ is said to be C-2 like Finsler manifold, if the (h)hv-torsion tensor $C_{ijk}$ satisfies the following condition
(1.1) $C_{ijk} = (1/C^2)C_iC_jC_k$.
Wherein
(1.2) $g^{jk} C_{ijk} = C_i$
is the contracted torsion tensor.

**Definition 1.2:**
A Finsler manifold $F^n$ is said to be C-reducible Finsler manifold, if the (h)hv-torsion tensor $C_{ijk}$ satisfies the following condition
(1.3) $C_{ijk} = \{1/(n+1)\}(h_{ij}C_k + h_{jk}C_i + h_{ki}C_j)$.
Wherein
(1.4) $h_{ij} = g_{ij} - l_il_j$
is angular metric tensor.

**Definition 1.3:**
A Finsler manifold $F^n$ is said to be quasi C-reducible Finsler manifold, if the (h)hv-torsion tensor $C_{ijk}$ satisfies the following condition
(1.5) $C_{ijk} = A_{ij}C_k + A_{jk}C_i + A_{ki}C_j$.
Wherein $A_{ij}$ is a symmetric indicatric tensor and satisfies the condition
(1.6) $A_{ij} g^{ij} = A$.
In this regard, we have the following theorems:

**Theorem 1.1:**
In the quasi C-reducible Finsler manifold, if the indicatric tensor is symmetric then (h)hv-torsion tensor is also symmetric with respect to first two indices.

**Proof:**
Interchanging the indices i and j in equation (1.5), we get
(1.7) $C_{jik} = A_{ji}C_k + A_{ik}C_j + A_{kj}C_i$
If the indicatric tensor $A_{ij}$ is symmetric then the equation (1.7) becomes
(1.8) $C_{jik} = A_{ij}C_k + A_{ki}C_j + A_{jk}C_i$
From equations (1.5) and (1.8), we obtain
(1.9) $C_{ijk} = C_{jik}$
Hence, the (h)hv-torsion tensor is symmetric with respect to first two indices in the quasi C-reducible Finsler manifold.

**Theorem 1.2:**
In the quasi C-reducible Finsler manifold, if the indicatric tensor is symmetric then (h)hv-torsion tensor is also symmetric with respect to last two indices.

**Proof:**
Interchanging the indices j and k in equation (1.5), we get
(1.10) $C_{ikj} = A_{ik}C_j + A_{kj}C_i + A_{ji}C_k$
If the indicatric tensor $A_{ij}$ is symmetric then the equation (1.10) becomes
(1.11) $C_{ikj} = A_{ki}C_j + A_{jk}C_i + A_{ij}C_k$
From equations (1.5) and (1.11), we obtain
(1.12) $C_{ijk} = C_{ikj}$
Hence, the (h)hv-torsion tensor is symmetric with respect to last two indices in the quasi C-reducible Finsler manifold.

## References

[1] C.K. Mishra and Gautam Lodhi : On decomposability of the curvature tensor in recurrent conformal Finsler spaces, Differential Geometry, Dynamical Systems, Vol. 10, p.(235-242), (2008).
[2] K. Yano and S. Sasaki : Riemannian manifolds admitting a conformal transformation group, J. Diff. Geom., p.(161-184), 2, (1968).
[3] M. Matsumoto : On three dimensional Finsler spaces satisfying the T and $B^p$-conditions, Tensor, N.S., p. (13-20), 29, (1975).
[4] M. Matsumoto : On Finsler spaces with Randers metric and special forms of important tensors, J. Math. Kyoto Univ., 14, p.(477-498), (1977).
[5] M. Matsumoto and C. Shibata : On semi C-reducibility, T-tensor = 0 and S4-likeness of Finsler spaces, J. Math. Kyoto Univ., p.(301-314), 19, (1979).
[6] T.S. Chauhan, I.S. Chauhan and Hari Shanker : Some problems on quasi C-reducible Finsler space, Acta Ciencia Indica, Vol. XXXII M, No. 2, p.(625-628), (2006).

## Author Profile

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