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Theory of Quasi C-Reducible Finsler Manifold

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Abstract: ***Purpose of this paper is to study the theory of quasi C-reducible Finsler manifold. In this paper, we have obtained some important theorems on quasi C-reducible Finsler manifold.***

Keywords: (h)hv-torsion tensor, angular metric tensor, C-2 like, C-reducible, quasi C-reducible Finsler manifold.

Let F^n be an n-dimensional Finsler manifold with the metric tensor g_{ij}, the angular metric tensor h_{ij} and (h)hv-torsion tensor C_{ijk}. We have the following definitions:

Definition 1.1:
A Finsler manifold F^n is said to be C-2 like Finsler manifold, if the (h)hv-torsion tensor C_{ijk} satisfies the following condition

(1.1) $C_{ijk} = (1/C^2)C_iC_jC_k$.

Wherein

(1.2) $g^{jk} C_{ijk} = C_i$

is the contracted torsion tensor.

Definition 1.2:
A Finsler manifold F^n is said to be C-reducible Finsler manifold, if the (h)hv-torsion tensor C_{ijk} satisfies the following condition

(1.3) $C_{ijk} = \{1/(n+1)\}(h_{ij}C_k + h_{jk}C_i + h_{ki}C_j)$.

Wherein

(1.4) $h_{ij} = g_{ij} - l_il_j$

is angular metric tensor.

Definition 1.3:
A Finsler manifold F^n is said to be quasi C-reducible Finsler manifold, if the (h)hv-torsion tensor C_{ijk} satisfies the following condition

(1.5) $C_{ijk} = A_{ij}C_k + A_{jk}C_i + A_{ki}C_j$.

Wherein A_{ij} is a symmetric indicatric tensor and satisfies the condition

(1.6) $A_{ij} g^{ij} = A$.

In this regard, we have the following theorems:

Theorem 1.1:
In the quasi C-reducible Finsler manifold, if the indicatric tensor is symmetric then (h)hv-torsion tensor is also symmetric with respect to first two indices.

Proof:
Interchanging the indices i and j in equation (1.5), we get

(1.7) $C_{jik} = A_{ji}C_k + A_{ik}C_j + A_{kj}C_i$

If the indicatric tensor A_{ij} is symmetric then the equation (1.7) becomes

(1.8) $C_{jik} = A_{ij}C_k + A_{ki}C_j + A_{jk}C_i$

From equations (1.5) and (1.8), we obtain

(1.9) $C_{ijk} = C_{jik}$

Hence, the (h)hv-torsion tensor is symmetric with respect to first two indices in the quasi C-reducible Finsler manifold.

Theorem 1.2:
In the quasi C-reducible Finsler manifold, if the indicatric tensor is symmetric then (h)hv-torsion tensor is also symmetric with respect to last two indices.

Proof:
Interchanging the indices j and k in equation (1.5), we get

(1.10) $C_{ikj} = A_{ik}C_j + A_{kj}C_i + A_{ji}C_k$

If the indicatric tensor A_{ij} is symmetric then the equation (1.10) becomes

(1.11) $C_{ikj} = A_{ki}C_j + A_{jk}C_i + A_{ij}C_k$

From equations (1.5) and (1.11), we obtain

(1.12) $C_{ijk} = C_{ikj}$

Hence, the (h)hv-torsion tensor is symmetric with respect to last two indices in the quasi C-reducible Finsler manifold.

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