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# On The Identification of the Simple Bilinear White Noise Process

**Arimie, C. O.** [1], **Biu, E. O.** [2]

[1, 2] Department of Mathematics and Statistics, University of Port Harcourt

**Abstract:** ***Moments of the squares of simple bilinear process were determined under second order analysis for the purpose of identification and discriminating between bilinear process and linear white noise process. We showed how the variance of the bilinear white noise process can be used to distinguish it from the linear white noise process. The simulation results showed that the squared data of the bilinear white noise series fitted the ARMA(2, 1) model better than ARMA(1, 1) and MA(1)) models respectively.***

**Keywords:** Linear White Noise Process, Bilinear Stochastic Model, Bilinear White Noise Process, Second Order Analysis, Covariance Analysis.

## 1. Introduction

Over the years, much attention has been given to the issue of identification of the simple bilinear white noise process. Many works have been published on ways to discriminate between linear and bilinear time series models (see Granger and Andersen, 1978; Subba Rao, 1981; Pham and Tran, 1981; Hannan, 1982; Akamanam, 1983; Subba Rao and Gabr, 1984; Akamanam et al., 1986; de Gooijer and Heuts, 1987; Iwueze, 1988; Iwueze, 1989; Sesay and Subba Rao, 1991; Subba Rao and da Silva, 1992; Martins, 1999; Goncalves et al., 2005; Omekara, 2008; Iwueze and Johnson, 2011). Granger and Andersen (1978) suggests that one way to distinguish between linear and non linear time series models is to do a second order analysis on the squares of the series. Some authors, including Granger and Andersen (1978), have shown that for a time series $X_t$ that is normally distributed (and therefore linear),

$$\rho_k(X_t^2) = [\rho_k(X_t)]^2 \qquad (1.1)$$

where $\rho_k$ is the lag k autocorrelation.

A white noise process $X_t = \varepsilon_t,\ t \in Z,\ \varepsilon_t \sim N(0, \sigma^2)$ is essentially, a time series. It is an independent and identically distributed (iid) random sequence usually assumed to be Gaussian distributed with zero mean and variance $\sigma^2 < \infty$. According to Granger and Andersen (1978) the bilinear white noise process is a bilinear stochastic model formed by adding a bilinear form to linear ARMA model as shown in (1.2) below:

$$X_t = \sum_{i=1}^{p} \phi_i X_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} + \sum_{i=1}^{r}\sum_{j=1}^{s} \varphi_{ij} X_{t-i}\varepsilon_{t-j} + \varepsilon_t \qquad (1.2)$$

where, $\varepsilon_t$, t ∈ Z = (….., -1, 0, 1, …..) is a sequence of independent and identically distributed random variables with $E(\varepsilon_t) = 0$; $E(\varepsilon_t^2) = \sigma^2 < \infty$; and $\phi_1, \phi_2, \ldots, \phi_p$, $\theta_1, \theta_2, \ldots, \theta_q$, $\varphi_{ij}$,

$1 \le i \le r$, $1 \le j \le s$ are real constants.

In this paper, we considered higher order moments of the bilinear white noise process and how the variance of the powers of the process can be used to distinguish between the linear and simple bilinear white noise process.

## 2. Moments of the Simple Bilinear White Noise Process (SBWNP)

Consider the bilinear white noise process

$$X_t = (\theta_1 X_{t-3} + \theta_2 X_{t-2})\varepsilon_{t-1} + \varepsilon_t, \qquad \varepsilon_t \sim N(0, \sigma^2) \qquad (2.1)$$

where, $\theta_1$ and $\theta_2$ are real constants and $\{\varepsilon_t\}$, t ∈ Z is a sequence of independent and identically distributed random variables with zero mean and variance, $\sigma^2 > 0$.

Assuming normality of $\varepsilon_t$, t ∈ Z, we define the nth central moment of the process as

$$E(\varepsilon^b) = \begin{cases} (2a-1)!!\sigma^{2a}, & b = 2a,\ b,\ even \\ 0, & b,\ odd \end{cases} \qquad (2.2)$$

$$where, (2a-1)!! = \prod_{c=1}^{a}(2c-1) \qquad (Ibrahim,\ 2013)$$

Then, the even moments are $E(\varepsilon_t^2) = \sigma^2$, $E(\varepsilon_t^4) = 3\sigma^4$, $E(\varepsilon_t^6) = 15\sigma^6$, $E(\varepsilon_t^8) = 105\sigma^8$,
$E(\varepsilon_t^{10}) = 945\sigma^{10}$ , etc. For the bilinear white noise model (2.1), with $\varepsilon_t \sim N(0, \sigma^2)$ it can be easily shown that,

$$E(X_t) = E(\theta_1 X_{t-3}\varepsilon_{t-1} + \theta_2 X_{t-2}\varepsilon_{t-1}) + E(\varepsilon_t) = 0 \qquad (2.3)$$

$$Var(X_t) = E(X_t^2) - [E(X_t)]^2 = E(X_t^2) \qquad (2.4)$$

$$X_t^2 = [(\theta_1 X_{t-3} + \theta_2 X_{t-2})\varepsilon_{t-1} + \varepsilon_t]^2$$

$$= (\theta_1 X_{t-3} + \theta_2 X_{t-2})^2 \varepsilon_{t-1}^2 + \varepsilon_t^2 + 2(\theta_1 X_{t-3} + \theta_2 X_{t-2})\varepsilon_{t-1}\varepsilon_t$$

$$= \theta_1^2 X_{t-3}^2 \varepsilon_{t-1}^2 + \theta_2^2 X_{t-2}^2 \varepsilon_{t-1}^2 + 2\theta_1\theta_2 X_{t-3}X_{t-2}\varepsilon_{t-1}^2 + \varepsilon_t^2 + 2\theta_1 X_{t-}$$

$$E\left(X_t^2\right)=\theta_1^2E\left(X_{t-3}^2\right)E\left(\varepsilon_{t-1}^2\right)+\theta_2^2E\left(X_{t-2}^2\right)E\left(\varepsilon_{t-1}^2\right)+E\left(\varepsilon_t^2\right)+2\theta_1\theta_2E\left(X_{t-3}\right)E\left(X_{t-2}\right)E\left(\varepsilon_{t-1}^2\right)$$

$$+2\theta_1E\left(X_{t-3}\right)E\left(\varepsilon_{t-1}\right)E\left(\varepsilon_t\right)+2\theta_2E\left(X_{t-2}\right)E\left(\varepsilon_{t-1}\right)E\left(\varepsilon_t\right)$$

Since $X_t$ and $e_t$ are assumed to be stationary and E( $X_t$ ) = E($e_t$) = 0,

$$E\left(X_t^2\right)=\theta_1^2E\left(X_t^2\right)E\left(\varepsilon_t^2\right)+\theta_2^2E\left(X_t^2\right)E\left(\varepsilon_t^2\right)+E\left(\varepsilon_t^2\right)$$

$$=\sigma^2\theta_1^2E\left(X_t^2\right)+\sigma^2\theta_2^2E\left(X_t^2\right)+\sigma^2$$

$$(1-\sigma^2\theta_1^2-\sigma^2\theta_2^2)E\left(X_t^2\right)=\sigma^2$$

$$\therefore E\left(X_t^2\right)=\frac{\sigma^2}{1-\sigma^2\theta_1^2-\sigma^2\theta_2^2}=\frac{\sigma^2}{1-\left(\theta_1^2+\theta_2^2\right)\sigma^2}$$

$$\Rightarrow Var\left(X_t\right)=\frac{\sigma^2}{1-\sigma^2\sum_{i=1}^{2}\theta_i^2};\qquad \sigma^2\sum_{i=1}^{2}\theta_i^2<1 \quad (2.5)$$

$$R(k)=E\left[\left(X_t-E(X_t)\right)\left(X_{t-k}-E(X_{t-k})\right)\right]=E\left[X_tX_{t-k}\right]$$

$$R(1)=E\left[X_tX_{t-1}\right]$$

$$X_tX_{t-1}=(\theta_1X_{t-3}\varepsilon_{t-1}+\theta_2X_{t-2}\varepsilon_{t-1}+\varepsilon_t)X_{t-1}$$

$$=\theta_1X_{t-3}X_{t-1}\varepsilon_{t-1}+\theta_2X_{t-2}X_{t-1}\varepsilon_{t-1}+X_{t-1}\varepsilon_t$$

$$E\left[X_tX_{t-1}\right]=\theta_1E(X_{t-3}X_{t-1}\varepsilon_{t-1})+\theta_2E(X_{t-2}X_{t-1}\varepsilon_{t-1})+E(X_{t-1}\varepsilon_t)$$

By the assumption of stationarity,

$$E\left[X_tX_{t-1}\right]=\theta_1E(X_{t-2}X_t)E(\varepsilon_t)+\theta_2E(X_{t-1}X_t)E(\varepsilon_t)+E(X_t)E(\varepsilon_t)=0$$

$$\Rightarrow R(k)=\begin{cases}\dfrac{\sigma^2}{1-\sigma^2\sum_{i=1}^{2}\theta_i^2}; & \sigma^2\sum_{i=1}^{2}\theta_i^2<1,\ k=0\\[2ex] 0, & otherwise\end{cases}\qquad (2.6)$$

$$\rho_k=\frac{R(k)}{R(0)}=\begin{cases}1, & k=0\\ 0, & otherwise\end{cases}\qquad (2.7)$$

This is a white noise structure.

### 2.1 Mean and Variance of the Squares of the Simple Bilinear Model

We have shown that the simple bilinear white noise process has the covariance structure of a linear white noise process, what is left is to determine the mean and variance of the squares of the simple bilinear model (1.2). We assumed that $e_t \sim N(0, \sigma^2)$, $E(e_t) = 0$ and $E(e_t^2) = \sigma^2 < \infty$. We also assumed that the process, $\{Y_t\}$ is stationary.

Let,

$$Y_t=X_t^2=\left[(\theta_1X_{t-3}+\theta_2X_{t-2})\varepsilon_{t-1}+\varepsilon_t\right]^2$$

(2.8)

We have shown in section 1 above that

$$E(Y_t)=E(X_t^2)=\frac{\sigma^2}{1-\sigma^2\sum_{i=1}^{2}\theta_i^2}$$

$$(2.9)\ Var\left(Y_t\right)=E\left(Y_t^2\right)-\left[E\left(Y_t\right)\right]^2$$

$$=E\left(Y_t^2\right)-\left(\frac{\sigma^2}{1-\sigma^2\theta_1^2-\sigma^2\theta_2^2}\right)^2$$

$$Y_t^2=X_t^4=\left[X_t^2X_t^2\right]$$

$$=(\theta_1X_{t-3}\varepsilon_{t-1}+\theta_2X_{t-2}\varepsilon_{t-1}+\varepsilon_t)^2(\theta_1X_{t-3}\varepsilon_{t-1}+\theta_2X_{t-2}\varepsilon_{t-1}+\varepsilon$$

$$E\left(Y_t^2\right)=E\left[X_t^2X_t^2\right]$$

Since the process is assumed to be stationary, and $E(X_t^3) = E(X_t) = E(e_t) = 0$ then,

$$E(X^4)=\theta_1^4E(X_{t-3}^4\varepsilon_{t-1}^4)+\theta_2^4E(X_{t-2}^4\varepsilon_{t-1}^4)+6\theta_1^2\theta_2^2E(X_{t-3}^2X_{t-2}^2\varepsilon_{t-1}^4)+$$

$$6\theta_1^2E(X_{t-3}^2\varepsilon_{t-1}^2\varepsilon_t^2)+6\theta_2^2E(X_{t-2}^2\varepsilon_{t-1}^2\varepsilon_t^2)+E(\varepsilon_t^4)$$

$$=3\sigma^4\theta_1^4E(X_{t-2}^4)+3\sigma^4\theta_2^4E(X_{t-1}^4)+18\sigma^4\theta_1^2\theta_2^2E(X_{t-2}^2X_{t-1}^2)+$$

$$6\sigma^4\theta_1^2E(X_{t-2}^2)+6\sigma^4\theta_2^2E(X_{t-1}^2)+3\sigma^4$$

$$=3\sigma^4\left(\theta_1^4+\theta_2^4\right)E\left(Y_t^2\right)+18\sigma^4\theta_1^2\theta_2^2\left[E\left(Y_t\right)\right]^2+6\sigma^4\left(\theta_1^2+\theta_2^2\right)E\left(Y_t\right)+3\sigma^4$$

$$\left(1-3\sigma^4\theta_1^4-3\sigma^4\theta_2^4\right)E\left(Y_t^2\right)=18\sigma^4\theta_1^2\theta_2^2\left(\frac{\sigma^2}{1-\sigma^2\theta_1^2-\sigma^2\theta_2^2}\right)^2$$

$$+6\sigma^4\left(\theta_1^2+\theta_2^2\right)\left(\frac{\sigma^2}{1-\sigma^2\theta_1^2-\sigma^2\theta_2^2}\right)+3\sigma^4$$

$$\Rightarrow E\left(Y_t^2\right)=\left[\frac{18\sigma^8\theta_1^2\theta_2^2+6\sigma^6\left(\theta_1^2+\theta_2^2\right)\left(1-\sigma^2\theta_1^2-\sigma^2\theta_2^2\right)+3\sigma^4\left(1-\sigma^2\theta_1^2-\sigma^2\theta_2^2\right)^2}{\left(1-3\sigma^4\theta_1^4-3\sigma^4\theta_2^4\right)\left(1-\sigma^2\theta_1^2-\sigma^2\theta_2^2\right)^2}\right]$$

$$=\frac{12\sigma^8\theta_1^2\theta_2^2-3\sigma^8\theta_1^4-3\sigma^8\theta_2^4+3\sigma^4}{\left(1-3\sigma^4\theta_1^4-3\sigma^4\theta_2^4\right)\left(1-\sigma^2\theta_1^2-\sigma^2\theta_2^2\right)^2}$$

Hence,

$$Var\left(Y_t\right)=\frac{12\sigma^8\theta_1^2\theta_2^2-3\sigma^8\theta_1^4-3\sigma^8\theta_2^4+3\sigma^4}{\left(1-3\sigma^4\theta_1^4-3\sigma^4\theta_2^4\right)\left(1-\sigma^2\theta_1^2-\sigma^2\theta_2^2\right)^2}-\frac{\sigma^4}{\left(1-\sigma^2\theta_1^2-\sigma^2\theta_2^2\right)^2}$$

$$=\frac{12\sigma^8\theta_1^2\theta_2^2-3\sigma^8\theta_1^4-3\sigma^8\theta_2^4+3\sigma^4-\sigma^4\left(1-3\sigma^4\theta_1^4-3\sigma^4\theta_2^4\right)}{\left(1-3\sigma^4\theta_1^4-3\sigma^4\theta_2^4\right)\left(1-\sigma^2\theta_1^2-\sigma^2\theta_2^2\right)^2}$$

$$\Rightarrow Var\left(Y_t\right)=\frac{12\sigma^8\theta_1^2\theta_2^2+2\sigma^4}{\left(1-3\sigma^4\theta_1^4-3\sigma^4\theta_2^4\right)\left(1-\sigma^2\theta_1^2-\sigma^2\theta_2^2\right)^2} \quad (2.10)$$

## 3. Method of Identification of the Bilinear White Noise Process

Simulation studies were performed to illustrate how the variance of the bilinear white noise can be used to distinguish it from the linear white noise process. Realization of $\{X_t\}$, and $\{Y_t = X_t^2\}$, of length 100 respectively were constructed considering $\{\varepsilon_t\}$ as a sequence of i.i.d. symmetrically distributed random variable with zero mean and, $\varepsilon_t \sim N(0, 1)$. The experiment was repeated 100 times using values of $\theta$ in the interval $-0.7 \le \theta \le 0.7$ where, $\theta$ is assumed to be $(\theta_1 + \theta_2)$. For simplicity, $\theta_1$ was assigned the value zero. The values of $\theta$ were chosen to satisfy the stationary condition for model (2.10) i.e, $(\sigma^4\theta_1^4+\sigma^4\theta_2^4) < 1/\sqrt{3}$ or $\sigma^4\theta^4 < 1/\sqrt{3}$. The Chi-square test for variance (Snedecor and Cochran, 1989) was used to test the hypothesis:

$$H_0: Var(X_t^2)=2(\sigma_0^2)^2 \quad Vs \quad H_1: Var(X_t^2)\neq 2(\sigma_0^2)^2$$

The decision rule was to reject the null hypothesis ($H_0$) at level $\alpha = 5\%$ if $\chi^2_{cal}$ is less than $\alpha/2$ quartile or larger than $1-\alpha/2$ quartile of the Chi-square distribution with $n-1$ degrees of freedom (that is, reject $H_0$ if $\chi^2_{cal} < \chi^2_{\alpha/2,(n-1)}$ or if $\chi^2_{cal} > \chi^2_{1-\alpha/2,(n-1)}$ ).

In section 2, we stated that $E(\varepsilon_t^2)=\sigma^2$ so, if $X_t=\varepsilon_t$ then, it can be easily shown that $Var(X_t^2)=2\sigma^4$.

### 3.1 Fitting MA (1), ARMA (1, 1) and ARMA (2, 1) Models

To fit the above mentioned models, we let $\theta_1 = 0$ thus reducing Model 2.1 to

$$X_t=\theta_2 X_{t-2}\varepsilon_{t-1}+\varepsilon_t, \qquad \varepsilon_t \sim N\left(0,\sigma^2\right) \quad (3.1)$$

Model (3.1) is the simple bilinear white noise process (SBWNP). Iwueze (1988) asserts that the covariance structure of the square of the simple bilinear white noise process is the same as that of linear ARMA (2,1). To confirm this assertion, we simulated the simple bilinear white noise process

$$X_t=0.7X_{t-2}\varepsilon_{t-1}+\varepsilon_t, \qquad \varepsilon_t \sim N\left(0,\sigma^2\right) \quad (3.2)$$

and fitted the simulated data, using various ARMA Models and compared the results to identify the suitable ARMA(p, q) Model as alternative for SBWNP.

## 4. Results

The results of the test to discriminate between the linear and bilinear white noise processes are shown in Table 1.0 below. However, results for only one of the series are given here for illustration for want of space. From Table 1.0, it is shown that the null hypothesis, $H_o$ was not rejected for values of $\theta$ in the interval $-0.3 \le \theta \le 0.5$ thus, implying that the bilinear white noise process is identified as linear white noise for these values of $\theta$ otherwise, the process is bilinear white noise.

### 4.1 Test for the Identification of the SBWNP Parameters

**Table 1:** Result of Test for the identification of the bilinear White Noise Process

| Value of Bilinear coefficient | True value of variance | Estimated values of Variance | | Estimate of test statistic | Decision at 5% L.O.S |
|---|---|---|---|---|---|
| $\theta$ | $\sigma^2$ | $\hat{\sigma}^2 = S^2_{X_t}$ | $\hat{\sigma}^2 = S^2_{X_t^2}$ | $\frac{(n-1)S^2_{X_t^2}}{2\hat{\sigma}_0^2}$ | |
| -0.7 | 1.0000 | 1.4887 | 4.2164 | 42.4931 | Reject |
| -0.6 | 1.0000 | 1.3236 | 2.9623 | 47.7757 | Reject |
| -0.5 | 1.0000 | 1.1984 | 2.2520 | 54.0464 | Reject |
| -0.4 | 1.0000 | 1.1077 | 1.9183 | 63.0715 | Reject |
| -0.3 | 1.0000 | 1.0466 | 1.8014 | 74.3177 | Do not Reject |
| -0.2 | 1.0000 | 1.0106 | 1.7980 | 85.3253 | Do not Reject |
| -0.1 | 1.0000 | 0.9958 | 1.8602 | 93.6432 | Do not Reject |
| 0.1 | 1.0000 | 1.0230 | 2.1574 | 97.5064 | Do not Reject |
| 0.2 | 1.0000 | 1.0668 | 2.4438 | 93.3983 | Do not Reject |
| 0.3 | 1.0000 | 1.1363 | 2.9183 | 86.6489 | Do not Reject |
| 0.4 | 1.0000 | 1.2402 | 3.7498 | 78.4597 | Do not Reject |
| 0.5 | 1.0000 | 1.3928 | 5.2801 | 69.4532 | Do not Reject |
| 0.6 | 1.0000 | 1.6189 | 8.1977 | 59.0768 | Reject |
| 0.7 | 1.0000 | 1.9666 | 13.9319 | 46.1054 | Reject |

### 4.2Numerical Analysis

This section deals with the simulated data analysis of the squared simple bilinear white noise process, compare the results obtained and state the most effective or suitable model. If the time series data, $X_t$, $t = 1, 2, \ldots, n$, admits the SQRT(SBWNP), we achieve stationarity and fit an autoregressive moving average process of order p and q[ $ARMA(p,q)$].

$$Y_t = \mu + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \ldots + \phi_p Y_{t-p} + \theta_1 e_{t-1} + \theta_2 e_{t-2} + \ldots + \theta_q e_{t-q} + e_t \qquad (4.1)$$

*where* $Y_t$ is the transformed series, $\mu$ is a constant and $e_t \sim N(0, \sigma^2)$.

When more than one model is selected from the process enumerated in Equation (4.1), the Akaike's Information Criterion is then used to select the more suitable model amongst them. The Akaike's Information Criterion is most commonly given as;

$$AIC = -n\log\left(\frac{RSS}{n}\right) + 2r \qquad (4.2)$$

Where r is the number of model parameters, N = Effective number of data point used in the estimation procedure and RSS is the estimated residual sum of squares of the model. (Akaike, 1974; Biu and Iwueze, 2011).

The simulated comprises of 100 data points and its series plot is shown as Figure 1.0.

#### 4.2.1 Model Selection Criteria (AIC)

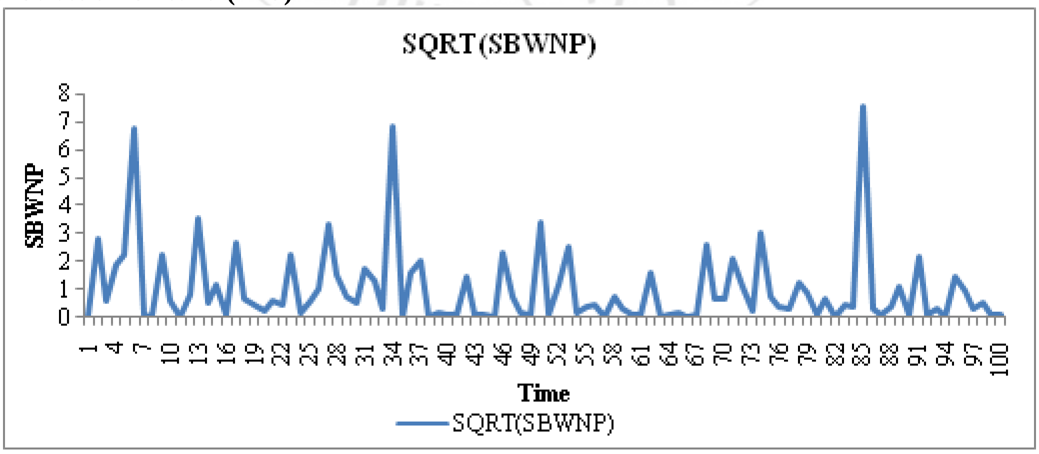


**Figure 1.0:** SQRT (SBWNP) "$Y_t$"

Examining Figure 1.0, we notice that the series is stationary and the variance is constant. We now fit the best ARMA(p, q) model to the transformed series [represented by (3.1)].

**ARMA Modelling of the Series (3.1)**

However, suitable ARMA(p , q) models $(p+q \le 2)$ may also be appropriate.
On the other hand, a test is appropriate, to test if the constant mean " $\mu$ " is involved in the models. In this case, the hypothesis of interest is given as

$$H_0 : \mu = 0 \ against \ H_0 : \mu \neq 0 \qquad (4.3)$$

The test statistics for testing $H_0 : \mu = 0 \ against \ H_0 : \mu \neq 0$ is

$$t = \frac{\overline{Y}_t}{std(Y_t)} \qquad (4.4)$$

The computed t-value using Minitab 16 statistical software is t = 7.04 with p-value = 0.0001 is shown in Appendix A. It can be seen that the p-value is less than the appropriate critical value 0.05; therefore we rejected $H_0$ and concluded that $\mu \neq 0$. That is, $\mu$ is in the models.

Various ARMA(p , q) models were fitted to the series (3.1) with respective residuals as white noise (Appendix B) and is summarized in Table 2.0. The model selection criteria used to select the best model is Akaike's Information Criterion (AIC) [Equation (4.2)]. This is also shown in Table 2.0.

**Table 2.0:** AIC Values for ARMA(p , q) models $(p+q \le 2)$ with constant Computation (3.1)

| **Model** | $k$ | $\sigma^2$ | **N** | **AIC** |
|---|---|---|---|---|
| AR(2) with constant | 3 | 1.986 | 100 | 176.20 |
| MA(2) with constant | 3 | 1.988 | 100 | 176.16 |
| ARMA(1 , 1) with constant | 3 | 1.989 | 100 | 176.14 |
| ARMA(1 , 2) with constant | 4 | 2.008 | 100 | 177.72 |
| **ARMA(2 , 1)** | **3** | **2.014** | **100** | **175.59** |
| ARMA(2 , 1) with constant | 4 | 2.013 | 100 | 177.62 |
| ARMA(2 , 2) | 4 | 2.045 | 100 | 176.16 |
| ARMA(2, 2) with constant | 5 | 1.885 | 100 | 182.47 |

The model identified using Akaike's Information Criterion in Table (2.0) is

**ARMA(2, 1) with constant**

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \theta_1 \varepsilon_{t-1} + \varepsilon_t \qquad (4.5)$$

**Estimation Parameters of the Identified ARMA(2, 1) Model with Constant for (3.1)**
Estimates were obtained using Minitab 16.0 software and the results are tabulated in Table (3.0).

**Table 3.0:** Parameter Estimates of AR(2, 1) Models for (3.1)

| **AR(2, 1) Model** |
|---|
| $\phi_1 = 0.9025 \pm 0.1012$ |
| $\phi_2 = 0.0975 \pm 0.1010$ |
| $\theta_1 = 0.9801 \pm 0.0166$ |
| $RSS = \sigma^2 = 2.014$ |

**Footnote:** values after ( $\pm$ ) are their standard errors

By Substitution; Equation (4.5) becomes

$$Y_t = 0.90Y_{t-1} + 0.10Y_{t-2} + 0.98\varepsilon_{t-1} + \varepsilon_t \quad (4.6)$$

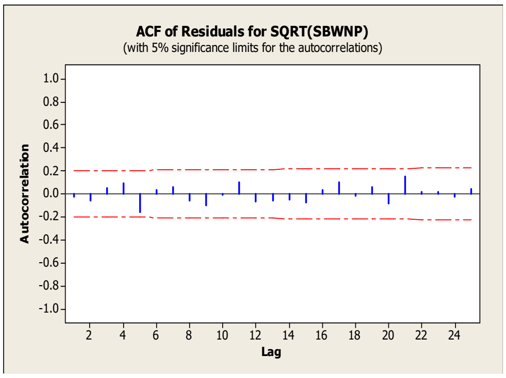


**Figure 2.0:** Residual ACF correlogram of AR(2, 1) with constant

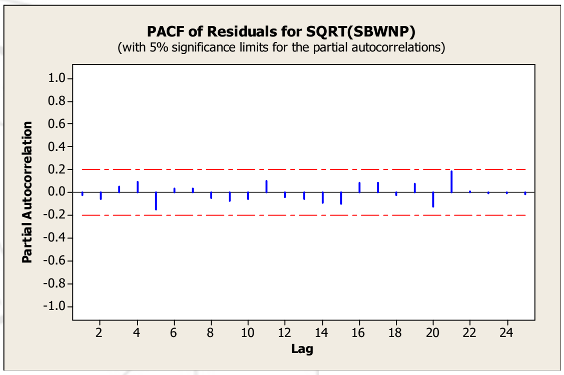


**Figure 3.0:** Residual PACF correlogram of AR(2,1) with constant

The residuals ACF and PACF in Figures (2.0) and (3.0) reveal that the models are adequate for (3.1). The adequacies of the models were also checked by the use of Ljung-Box (1978) Chi-square statistics and the results are summarized in Table 4.0.

**Table 4.0:** (Ljung-Box) Chi-square Statistic for Adequacy of (4.6)

| $k$ | $df$ | $Q(k)$ for AR(2,1) | Chi-square Table $\chi^2_{(h)}$ |
|---|---|---|---|
| 12 | 9 | 8.2 | 14.7 |
| 24 | 21 | 15.7 | 30.1 |
| 36 | 33 | 23.4 | 44.8 |
| 48 | 45 | 28.4 | 65.7 |

From Table (4.0), comparing $Q(k)$ with $X^2_{(df)}$, [i.e. $Q(k) < X^2_{(df)}$, $k = 12, 24, 36, 48$], it is obvious that the models are adequate and they can be used for forecasting the square simple bilinear white noise process.

## 5. Conclusion

This results of the analysis show that the ARMA(2, 1) model fitted the squared data of the simple bilinear white noise process better than ARMA(1, 1) and MA(1)) models respectively thus, agreeing with theory. We therefore, conclude that the suggested model in this research work [ARMA(2, 1)] is a better alternative for modelling and forecasting a simple bilinear white noise series (SBWNP).

The results of the test to discriminate between the linear and bilinear white noise processes showed that the values θ should lie in the interval $-0.3 \leq \theta \leq 0.5$ for identification.

## References

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## Appendix A

### One-Sample T: SQRT(SBWNP)

Test of mu = 0 vs not = 0

| Variable | N | Mean | StDev | SE Mean | 95% CI | T | P |
|---|---|---|---|---|---|---|---|
| SQRT(SBWNP) | 100 | 0.989994 | 1.405621 | 0.140562 | (0.711088, 1.268899) | 7.04 | 0.00001 |

## Appendix B

**Table 5.0:** Analysis of **SQRT(SBWNP)** [Fitting AR(p, q) models without constant]

| ARMA(p, q) $p+q\leq 2$ | Estimates $\hat{\phi}_1$ | $\hat{\phi}_2$ | $\hat{\theta}_1$ | $\hat{\theta}_2$ | $Q(k)$ $k=12$ | $k=24$ | $k=36$ | $k=48$ | $\hat{\sigma}^2$ | AIC |
|---|---|---|---|---|---|---|---|---|---|---|
| AR(1) | 0.2634 (0.0970) | | | | 20.8 (11) | 32.3 (23) | 42.9 (35) | 48.0 (47) | 2.760 | Correlated Residuals |
| MA(1) | | | -0.1886 (0.0987) | | 17.0 (11) | 28.4 (23) | 38.0 (35) | 42.6 (47) | 2.823 | Correlated Residuals |
| AR(2) | 0.1961 (0.0976) | 0.2564 (0.0976) | | | 25.6 (10) | 32.8 (22) | 45.0 (34) | 53.9 (46) | 2.605 | Correlated Residuals |
| MA(2) | | | -0.1647 (0.1000) | -0.1381 (0.1000) | 20.5 (10) | 28.4 (22) | 39.4 (34) | 46.5 (46) | 2.770 | Correlated Residuals |
| ARMA(1,1) | 1.0007 (0.0034) | | 0.9854 (0.0024) | | 9.7 (10) | 17.9 (22) | 26.2 (34) | 31.1 (46) | 2.025 | Correlated Residuals |
| ARMA(1,2) | -0.9712 (0.0321) | | -1.1301 (0.0001) | -0.1195 (0.0093) | 13.8 (9) | 25.2 (21) | 33.8 (33) | 38.7 (45) | 2.794 | Correlated Residuals |
| ARMA(2,1) | 0.9025 (0.1012) | 0.0975 (0.1010) | 0.9801 (0.0166) | | 8.2 (9) | 15.7 (21) | 23.4 (33) | 28.4 (45) | 2.014 | 175.59 |
| ARMA(2,2) | 0.1581 (0.3633) | 0.8413 (0.3749) | 0.1821 (0.3826) | 0.7763 (0.4291) | 9.9 (8) | 17.2 (20) | 26.3 (32) | 31.6 (44) | 2.045 | 176.16 |

**Table 6:** Analysis of **SQRT(SBWNP)** [Fitting AR(p, q) models with constant]

| ARMA(p, q) $p+q\leq 2$ | Estimates $\hat{\mu}$ | $\hat{\phi}_1$ | $\hat{\phi}_2$ | $\hat{\theta}_1$ | $\hat{\theta}_2$ | $Q(k)$ $k=12$ | $k=24$ | $k=36$ | $k=48$ | $\hat{\sigma}^2$ | AIC |
|---|---|---|---|---|---|---|---|---|---|---|---|
| AR(1) | 0.9918 (0.1264) | -0.1107 (0.1006) | | | | 7.7 (10) | 15.4 (22) | 22.7 (34) | 27.6 (46) | 1.972 | Correlated Residuals |
| MA(1) | 0.9920 (0.1235) | | | 0.1198 (0.1006) | | 7.3 (10) | 15.1 (22) | 22.2 (34) | 26.9 (46) | 1.969 | Correlated Residuals |
| AR(2) | 0.9922 (0.1200) | -0.1167 (0.1016) | -0.0574 (0.1018) | | | 6.6 (9) | 14.9 (21) | 21.5 (33) | 25.7 (45) | 1.986 | 176.20 |
| MA(2) | 0.9922 (0.1215) | | | 0.1129 (0.1018) | 0.0258 (0.1021) | 7.0 (9) | 15.0 (21) | 21.9 (33) | 26.3 (45) | 1.988 | 176.16 |
| ARMA(1,1) | 0.9921 (0.1228) | 0.0952 (0.8497) | | 0.2125 (0.8348) | | 7.2 (9) | 15.0 (21) | 22.0 (33) | 26.6 (45) | 1.989 | 176.14 |
| ARMA(1,2) | 0.9921 (0.1221) | -0.2171 (3.1377) | | -0.1039 (3.1341) | 0.0556 (0.3577) | 6.9 (8) | 15.0 (20) | 21.8 (32) | 26.1 (44) | 2.008 | 177.72 |
| ARMA(2,1) | 0.9918 (0.1319) | 0.7748 (0.5176) | 0.0952 (0.1279) | 0.8857 (0.5046) | | 7.7 (8) | 15.4 (20) | 22.6 (32) | 27.5 (44) | 2.013 | 177.62 |
| ARMA(2,2) | 0.9801 (0.1301) | 0.0497 (0.0618) | -0.9227 (0.0556) | 0.1736 (0.0560) | -0.9699 (0.0457) | 4.1 (7) | 14.4 (19) | 18.8 (31) | 21.2 (43) | 1.885 | 182.47 |



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