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An Application of Finite Affine Plane of Order *n*, in an Experiment Planning

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Abstract: ***In this paper we present an application possibility of the affine plane of order n, in the planning experiment, taking samples as his point. In this case are needed n^2 samples. The usefulness of the support of experimental planning in a finite affine plane consists in avoiding the partial repetition combinations within a proof. Reviewed when planning cannot directly drawn over an affine plane. In this case indicated how the problem can be completed, and when completed can he, with intent to drawn on an affine plane.***

Keywords: incidence structure, affine plane, experiment planning, equilibrium, combinations.

MSC2010: 51-XX, 51Axx, 51A45, 51Exx, 51E15.

1. Affine Plane, General Considerations

Let be $\mathcal{P},\mathcal{D}$ non empty set, and sub-set $\mathcal{I}$ to cartesian product $\mathcal{P}\times\mathcal{D}$.

Definition 1.1: [1], [2] *The incidence Structure called the triple* $\mathcal{S}=(\mathcal{P},\mathcal{D},\mathcal{I})$ *where* $\mathcal{P}\cap\mathcal{D}=\varnothing$ *and* $\mathcal{I}\subseteq\mathcal{P}\times\mathcal{D}$.

Definition 1.2: [1],[2],[3],[5],[10] *Affine plane called the incidence structure* $\mathcal{A}=(\mathcal{P},\mathcal{L},\mathcal{I})$ *that satisfies the following axioms:*

A1: *For every two different points **P** and **Q*** $\in\mathcal{P}$, *there exists exactly one line* $\ell\in\mathcal{L}$ *incident with that points.*

The line ℓ, determined from the point ***P*** and ***Q*** will denoted ***PQ***.

A2: *For a point* $P\in\mathcal{P}$, *and an line* $\ell\in\mathcal{L}$ *such that* $(P,\ell)\notin\mathcal{I}$, *there exists one and only one line* $r\in\mathcal{L}$, *incident with point **P** and such that* $\ell\cap r=\varnothing$.

A3: *In* $\mathcal{A}$ *there are three non-incident points with a line..*

From axioms **A1** implicates that tow different lines of $\mathcal{L}$ many have a common point, in other words *tow different lines of* $\mathcal{L}$ *or no have common point or have only one common point.*

Definition 1.3: [2], [7] *Two lines* $\ell,m\in\mathcal{L}$ *that matching or do not have common point of called parallel and in this case write* $\ell\parallel m$, *and when they have only one common point say that they expected.*

PROPOSITION 1.1: [1],[2],[4],[7] ***Parallelism relation on*** $\mathcal{L}$ ***is an equivalence relation in*** $\mathcal{L}$.

Definition 1.4: [2], [3] *Three different points* $P,Q,R\in\mathcal{P}$ *we called collinearly, if there are incidents with the same line.*

Definition 1.5: [1],[3],[10] *An affine plane* $\mathcal{A}=(\mathcal{P},\mathcal{L},\mathcal{I})$, *that there are a natural number of points will be called finite affine plane.*

PROPOSITION 1.2: [1],[2] ***In an finite affine plane*** $\mathcal{A}=(\mathcal{P},\mathcal{L},\mathcal{I})$ ***every line contains the same number of points and each point passes through the same number of line. Furthermore, exists the natural number*** $n\in\mathbb{N}$ $(n\geq 2)$ ***of which are true following propositions:***

1) In every line $\ell\in\mathcal{L}$, ***the number of incidents points with him is*** n.

2) For every point $P\in\mathcal{P}$, ***there are*** $n+1$ ***lines incidents with to.***

3) In the affine plane $\mathcal{A}$ ***there are*** n^2 ***points.***

4) In the affine plane $\mathcal{A}$ ***there are*** $n\cdot(n+1)$ ***lines.***

Definition 1.6: [2] *The number* n *in the Propositions 1.2 called order of the affine plane* $\mathcal{A}=(\mathcal{P},\mathcal{L},\mathcal{I})$.

PROPOSITION 1.3: [2] ***In an affine plane*** $\mathcal{A}=(\mathcal{P},\mathcal{L},\mathcal{I})$ ***with order n, there are*** $n+1$ ***the equivalence classes by parallelism of lines and each of which has*** n ***lines.***

Example 1.1: The minimal model of the affine plane there are order **2**. It contains **4** points $\mathcal{P}=\{A,B,C,D\}$ and **6** lines (Fig.1)

$$\mathcal{L}=\begin{Bmatrix}(A,B),(A,C),(A,D),\\(B,C),(B,D),(C,D)\end{Bmatrix}.$$

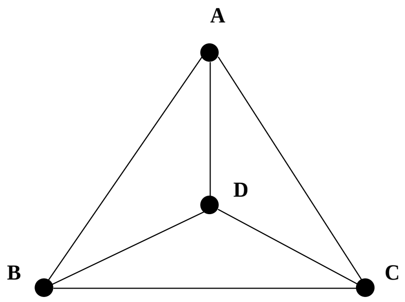

Fig. 1 *The affine plane of order 2.*

In this model there are three equivalence classes of parallel lines

$$\{(A,B),(C,D)\};\{(A,D),(B,C)\};$$
$$\{(A,C),(B,D)\}.$$

Example 1.2. The affine plane of order **3** are **9** points and **12** lines. In every line have **3** points (Fig.2): $\mathcal{P}=\{1,2,3,4,5,6,7,8,9\}$ and

$$\mathcal{L}=\begin{Bmatrix}\ell_1,\ell_2,\ell_3,\ell_4,\ell_5,\ell_6,\\ \ell_7,\ell_8,\ell_9,\ell_{10},\ell_{11},\ell_{12}\end{Bmatrix}$$

where

$$\ell_1=\{1,2,3\};\ell_2=\{4,5,6\};\ell_3=\{7,8,9\};$$
$$\ell_4=\{1,4,7\};\ell_5=\{2,5,8\};\ell_6=\{3,6,9\};$$
$$\ell_7=\{1,5,9\};\ell_8=\{2,6,7\};\ell_9=\{3,4,8\};$$
$$\ell_{10}=\{1,6,8\};\ell_{11}=\{2,4,9\};\ell_{12}=\{3,5,7\}.$$

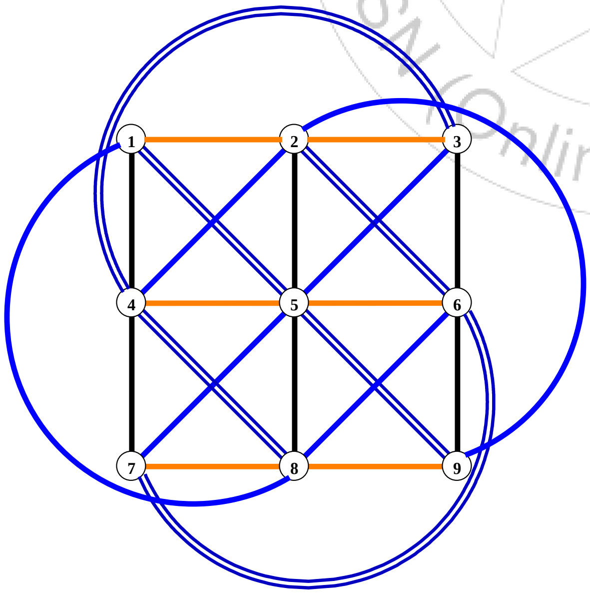

Fig. 2. *The affine plane of order 3.*

In this example we have four equivalence classes of parallel lines (Fig.3):

$$K_1=\{\ell_1,\ell_2,\ell_3\};K_2=\{\ell_4,\ell_5,\ell_6\};$$
$$K_3=\{\ell_7,\ell_8,\ell_9\};K_4=\{\ell_{10},\ell_{11},\ell_{12}\};$$

It is clear that these four classes are partition of the above figure:

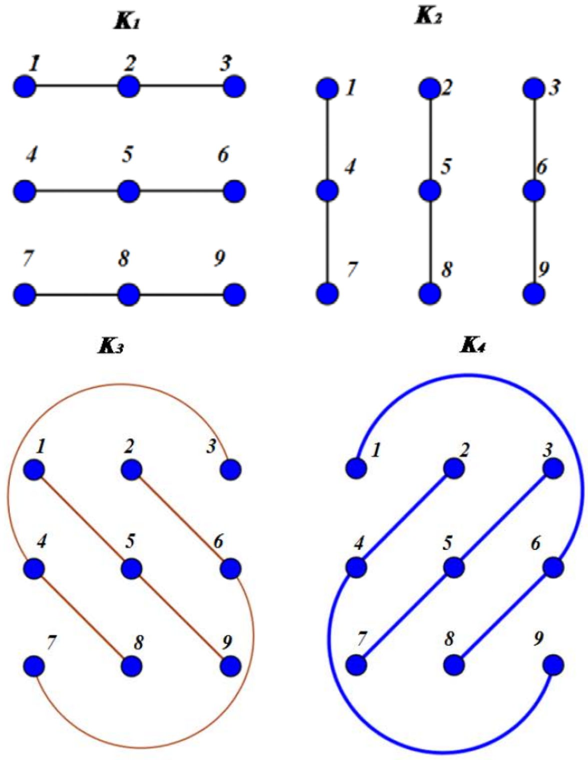

Fig.3. *Equivalence classes of parallel lines*

2. An Application of finite affine plane of order *n*, in an experiment planning

Examples of construction of finite affine planes brought in the preceding paragraph find interesting use in planning experiment.

PROBLEM 1

Suppose that an experiment must be carried out in $'n'$ 'levels. In a test we suppose that have n affecting feature. Each of the n affecting features we divide in n levels. If we perform a direct test we would need n^{n+1} the sample, because such as are possible combinations. By taking samples as points to an affine plane of order n, will take us were needed n^2 the sample. The usefulness of the experimental planning in support of an affine plane consists in avoiding the repetition of partial (at least twice for two different levels of **A1** axioms of affine plane) combinations within a test. Through this planning method we would have a lower cost of the experiment.

From proposition 1.2, in an affine plane we have

1. n^2 Point. Points on us will be samples.

2. Every line has $n-$points. Tests and affecting traits are divided into $n-$levels.

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3. In an affine plane we have $n(n+1)$ lines. In our planning these will be the test of all levels together with levels of affecting traits.

From propositions 1.3, we have $n+1$ equivalence classes by parallelism of parallel line. One class will be the proof and $n-$classes other of its are affecting traits. And each of class has by $n-$levels (line) her.

Example 2.1: Suppose an medical research firm wants to prove the benefits of an new medicament. In this experiment, the medicament is to be administered in three dosage levels;

Level I, Level II and **Level III.**

With three affecting traits divided into three levels each Affecting trait **1**, Affecting trait **2** and Affecting trait **3**, with three levels each. These data are presented in the table below:

Affecting trait / Level	Affecting trait **T.N.1.**	Affecting trait **T.N.2.**	Affecting trait **T.N.3.**
I	***(a)***	***(1)***	***(A)***
II	***(b)***	***(2)***	***(B)***
III	***(c)***	***(3)***	***(C)***

One way for being make sure that we have balanced each of the three groups of selected samples in each of the above extensions (Affecting Trait **1**, Affecting Trait **2**, Affecting Trait **3**), and reducing the effects of different combinations of to these factors, we have to prove the following combinations of traits and training at various levels of **81** samples:

I- (a)-1-A; I-(a)-1-B; I-(a)-1-C;
I-(a)-2-A; I-(a) -2-B; I-(a)-2-C;
I-(a)-3-A; I-(a) -3-B; I-(a)-3-C;

etc … to the ultimate combination

III-(c)-3-A; III-(c)-3-B; III-(c)-3-C;

But it takes 81 the sample (or more if we try to test some samples with every possible combination traits affecting in each of the three levels of treatment) in fact may be very difficult to locate the sample with all combinations possible traits. One way of "light" which can provide the satisfying equilibrium is to choose only nine individuals with the following combinations traits and treatments which are outline in Fig.4:

I-(a)-1-B.
(Level **I** tested in the sample **1,** with **T.N.1(a), T.N.2(1)** amd **T.N.3.(B)**)
I-(b)-3-C.
(Level **I** tested in the sample **2,** with **T.N.1(b), T.N.2(3)** and **T.N.3.(C)**)
I-(c) -2-A.
(Level **I** tested in the sample **3**, with **T.N.1(c), T.N.2(2)** and **T.N.3.(A)**)
II-(a)-3-A.
(Level **II** tested in the sample **4,** with **T.N.1(a), T.N.2(3)** and **T.N.3.(A)**)
II-(b)-2-B.
(Level **II** tested in the sample **5,** with **T.N.1(b), T.N.2(2)** and **T.N.3.(B)**)
II-(c)-1-C.
(Level **II** tested in the sample **6**, with **T.N.1(c), T.N.2(1)** and **T.N.3.(C)**)
III-(a)-2-C.
(Level **III** tested in the sample **7**, with **T.N.1(a), T.N.2(2)** and **T.N.3.(C)**)
III-(b)-1-A.
(Level **III** tested in the sample **8,** with **T.N.1(b), T.N.2(1)** and **T.N.3.(A)**)
III-(c)-3-B.
(Level **III** tested in the sample **9,** with **T.N.1(c), T.N.2(3)** and **T.N.3.(B)**)

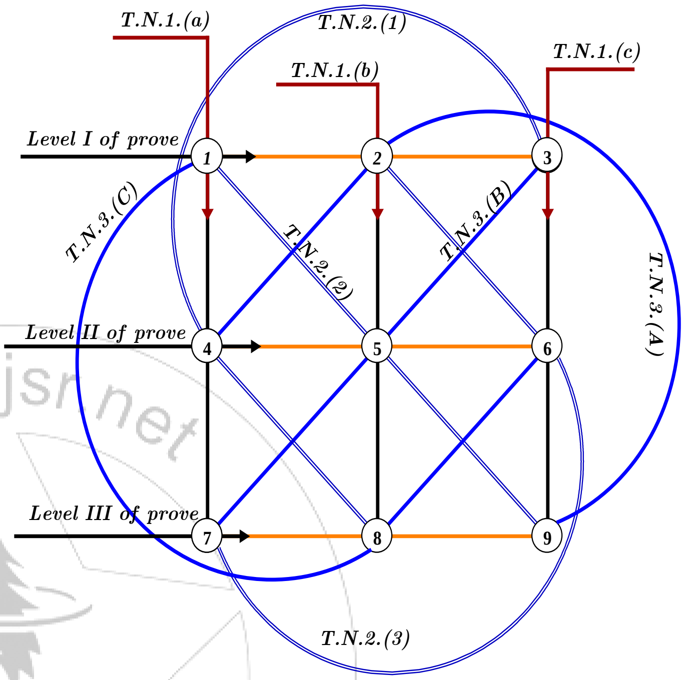

Fig. 4. Modeling the experiment on the third-order affine plane.

Here we see the affine plane of order **3**, in which nine points were nine people, and representing the twelve line treatment levels or extensions of an particular trait.

Parallel lines representing the different levels of the same quality (or different levels of the proof).

If we test nine samples with combinations shown traits over a one-week, then an ANOVA the careful (analysis of variance) will give an table with the results of how effective it would be medicament, and will tell the usefulness varies as individuals with different traits.

The designs of finite affine geometries figures of the different sizes can be used in planning the experiment, in order to obtain an bigger amount of information for an small cost as of possible.

PROBLEM 2.

Suppose that the experiment must be carried out in $'k'-$levels. In an proof suppose that we have $'m'$ Affecting Trait. Each of the $'m'$ affecting traits we have divided respectively $q_1, q_2, \dots, q_m$ levels.

Before starting the experiment, we recommend become a complement to bring the problem in the form of **Problem 1.**

This supplementation is recommended be done if the total number of combinations of proof is higher than the square of the largest number of indicators between levels of the test, the number traits impactful, and the number of levels of impactful traits, so if

$$\left[\max\left\{k, m, q_1, q_2, \dots, q_m\right\}\right]^2 < k \cdot m \cdot \prod_{i=1}^{m} q_i.$$

only thus this method it would be efficient.

Suppose we are the conditions when this method would be appropriate. We will act with the help of this supplementation algorithm:

Algorithm 1:

Step 1 Write down $n = \max\left\{k, m, q_1, q_2, \dots, q_m\right\}$

Step 2 Supplemented with the proof and influencing traits by $n-$levels (added in the levels thinking the fictitious levels).
Step 3 We are planning experiment over an affine finite plane of order n. Community distinguish all of lines of this plan, the line will be, respectively, the levels of evidence and the levels of influencing traits.
Step 4 From planning the experiment over the affine finite plane of order n, taken, combinations for to experimented.

Remember that the points of the affine finite plane are that we need to take samples for experimentation.

Example 2.2: We need to conduct a proof on her **5**-levels. For conducting this proof have **3** traits influencing: T_1, T_2 and T_3, with by **2, 3** and **4** levels each respectively. How many the sample are needed us to carry out this proof?
Either directly to obtain all the combinations we would need:

$$5 \cdot 3 \cdot 2 \cdot 3 \cdot 4 = 360$$

the sample. If we follow the recommendation in **Problem 2** and following the Algorithm 1**.**
We would need to we add levels in the influencing traits T_1, T_2 and T_3, even we have to add another two other influencing features T_4 and T_5, with **5** levels each (supplements we have said that are **fictitious**, but can also be the traits with very little influence, to addition the appropriations in an influential feature we definitely receive more information).
Now, following the design of the plan order afin **5**th, we need only **25** samples, for conduct the experiment.

3. Conclusions

In this paper, present a highly efficient method of how the affine plane can be applied to an experiment planning. We showed that the main advantage of this method was **the small number** of samples selected to perform a particular experiment.

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