

Does Repetition with Variation Improve Students' Mathematics Conceptual Understanding and Retention?

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Abstract: *Western education culture viewed repetitive learning as opposite of deep learning and understanding, while Asian mathematics education considered repetition as an important route to understanding. With these conflicting views, this study was undertaken to determine the influence of repetition with variation in students' achievement scores, conceptual understanding and retention. Pretest-posttest control group research design was employed. 31-item teacher-made multiple choice test with open-ended questionnaire was the main instrument of the study. Results of the analysis revealed that students exposed to repetition with variation approach had significantly higher achievement, conceptual understanding and improved retention.*

Keywords: repetition with variation, conceptual understanding, retention

1. Introduction

Western and Eastern education have conflicting views on repetition in learning and instruction. Western education culture often viewed repetitive learning as an opposite of deep learning and understanding, while East Asian mathematics education idea of repetition with variation was often seen as an important route to understanding. Western educators oppose the concept of repetition and emphasize the need for students to construct a conceptual understanding of mathematical symbols and rules before they practice the rules (Li, 2006). Similarly, many Western educators hold the view that students should be encouraged to understand rather than to memorize what they are learning as they believe that understanding is more likely to lead to high quality outcomes than memorizing (Dahlin & Watkins, 2000).

On the other hand, Marton, Wen and Wong (2005) pointed out that the likelihood of being able to recall something is higher if the learners hear or see something several times than if they do not. Furthermore, they commented that, unlike when you read the same presentation of something several times in the same way and thus repeat the same thing again and again, or read the same presentation in different ways, something is repeated and something is varied. They also reported that Chinese learners recognise the mechanism of repetition as an important part of the process of memorization and that understanding can be developed through memorisation. Dahlin and Watkins (2000) asserted that the traditional Asian practice of repetition can create a deep impression on the mind and enhance memorization, but they also argue that repetition can be used to deepen and develop understanding.

The Western idea of rote drilling is not the same as the East Asian idea of repetition with variation. The idea of repetition with variation is often seen in East Asian mathematics education. With a set of practicing exercises that vary

systematically, repeated practice may become an important route to understanding (Leung, 2006).

With these aforementioned views, this paper sought to explore the theory of variation and repetition in its place in the development of mathematical understanding. This study also aimed to investigate the influence of repetition with variation on students' understanding and retention.

2. Review of Related Literature

2.1 Repetition with variation on conceptual understanding

This study was to investigate the influence of repetition with variation on students' achievement, conceptual understanding and repetition is anchored on variation theory of learning which emerged from the phenomenographic research tradition described by Marton and Booth (1997). There are two fundamentals in the variation theory. The first one is that learning always has an object; the second one is that the object of learning is experienced and conceptualized by learners in different ways.

This study is also based on the teaching with variation developed by Gu (2004). Gu independently based his theory on the result of the longitudinal mathematics teaching experiments in China and the influence of cognitive science and constructivism. According to this theory, meaningful learning enables learners to establish a substantial and non-arbitrary connection between new knowledge and their previous knowledge. Classroom activities are developed to help students establish this kind of connection by experiencing certain dimensions of variation. This theory suggests that two types of variation are helpful for meaningful learning. One is called conceptual variation which provides students with multiple experiences from different perspectives. The other is called procedural variation which is concerned with the process of forming a

concept, logically or chronologically in arriving at solutions to problems.

Marton & Morris(2002) and Marton & Tsui (2004) described variation theory as learning concepts for learning through the experience of discernment, simultaneity and variation. For every concept, situation or phenomena, it has particular aspects, and if an aspect is varied and another remained invariant, the varied aspect will be discerned. In addition, understanding of the concept in a certain way requires the simultaneous discernment of the critical aspects of the concept of learning. This theory on discernment, simultaneity and variation is related to learning and is believed to be critical for learning to happen which can also be used as an analytical tool for analyzing teaching. As a result, learning and teaching are brought closer together. In this study on the use of repetition with variation, the class would be given different mathematics problems of the same concept with varied questions. Here, students would apply the concept learned to differentiate one problem from the other, and plan another strategy to solve it; hence, the conceptual understanding as well as retention would be improved.

Watson & Mason (2005) further claimed that teaching with variation helps students to actively try things out, and then to construct mathematical concepts that meet specified constraints with related components richly interconnected. Building on this idea, teaching with variation matches the central idea of constructivism that is, seeing learners as constructors of meaning. Hence, using repetition with variation as an approach can help the students develop their ability to explain, interpret and apply certain concepts in mathematics.

Bruner (1961) stated that it is only through the exercise of problem solving and the effort of discovery that one learns the working heuristics of discovery. The more one has practice, the more likely is one to generalize what one has learned into a style of problem solving or inquiry that serves for any kind of task or almost any kind of task. Bruner also believed that it was by translating redundancy into a manipulable model that the child is able to go beyond the information before him. The importance of repetition to Bruner's concept of learning was particularly clear in his description of the spiral curriculum which, he said, as it develops basic ideas repeatedly, building upon them until the student has grasped the full formal apparatus that goes with them. In repetition with variation, the students were given mathematics problems such that they would be able to practice solving and thus, helping them to conceptualize such principles. Retention is being developed as well, for the given problems are of the same concept but the question is stated in varied ways.

Also, according to Piaget (1963), development is the result of repeated patterns of exercise of the reflex, the circular reaction, the reuse of known schemes of assimilation employed in novel situations, the gradual accommodation to external reality through repeated use, and in short, the tendency toward repetition of behavior patterns and toward the utilization of external objects in the framework of such repetition. Brooks & Brooks (1993) affirmed this by stating

that the role of repetition in constructive learning theory is the similarities found when relating new experience to previous experience. They further stressed that deep understanding occurs when the presence of new information prompts the emergence or enhancement of cognitive structures that enable a person to rethink his prior ideas. Likewise, according to Vygotsky (1978), through repeated experiences children learn covertly to plan their activities. Such repeated experience, he said, proceeds not in a circle but in a spiral, passing through the same point at each new revolution while advancing to a higher level.

Tong (2012) affirmed with these theories and stated that there is no one way of understanding or experiencing a particular phenomenon depending on the context and prior experiences. Applied to learning, this means that individual students will understand new concepts in varying ways depending on their existing framework of knowledge. With this present study varying problems were given to students for them to consolidate concepts by extending the original problem by varying the conditions, changing the results and making generalization. It provides students with an opportunity to experience a way of mathematical thinking, investigating the cases from special to general, from which students can see and construct mathematical concept.

Lai (2015), in his study entitled teaching with procedural variation: a Chinese way of promoting deep understanding of mathematics, also found out that by creating this form of procedural variation, students are able to comprehend different components of a concept and hence upgrade their structure of knowledge, while, a non-arbitrary relationship between different components of teaching with procedural variation the concept can be built. In other words, this form of procedural variation offers a structured and structural approach to exposing underlying mathematical forms and therefore, can enhance students' conceptual understanding of a series of related concepts.

On similar aspect, Noche & Yu (2015) found out from her study on supplemental self-paced instruction that focuses on the mastery of either concepts or procedures through repetition with variation, helps young adults improve their performance in tasks designed and proportional reasoning understanding and skills. Hence, according to Olteanu & Olteanu (2011), in classroom situations, it is very important that the teacher is able to bring critical features of the object of learning into students' focal awareness. The learning theory of variation serves as a useful theoretical framework to help teachers plan and structure their lessons. It guides teachers to decide what aspects to focus on, what aspects to vary simultaneously, and what aspects remain invariant or constant. Furthermore, it guides teachers to consciously design patterns of variation to bring about the desired learning outcomes. Student's lived object of learning can be compared against categories of description as a means of assessing the level of learning achieved or against the enacted level of learning to determine whether the enacted concept of learning is being transferred to the lived concept of learning as expected.

2.2 Repetition with variation on retention

Retention is the ability of the students to retain things in mind, preserve information about the concept discussed as aftereffects of learning experiences that makes recall or recognition possible after a period of time. However, attaining and gaining retention requires an intensive process and effort. Ritter & Schooler (2010) suggested that there should be extended practice to take place to have a potentially strong retention. In other words, learning by practice helps the students understand the concept well and their long term memory will somehow help them retrieve whatever concepts or ideas that may be necessary in the future. In this present study, students were exposed to varied repetitive exercises to encourage constant practice through to promote retention. Also, the students in this study were given a retention test two weeks after it was given the first time to find out if they preserved concepts or ideas and could retrieve it in answering the test.

Furthermore, Chanson, Kurumeh and Obida (2010) stated that consistent elaboration and explaining of a topic would surely bring deep retention of a concept. Their findings stressed out that students were able to retain concepts of a specific topic for longer period of time when they were asked to explain and reasons during discussion. This is related to the present study because in the open ended-questions the learners were told to explain and justify their solution after they arrived at the answer.

Furthermore, various studies shows that variations in classroom activities promotes retention, such as the study of Haltiwanger and Simpson (2013) who emphasized that allowing students to construct ideas through writing in mathematics can promote thinking for writing in mathematics can develop students' skills to illustrate an awareness of mathematical connections, communicate their thoughts and share their ideas comfortably with pairs. If students were able to show all these manifestations, this means that they acquire conception which promotes retention of learning. Ubalde (2015), on her study on the effect of bridging the knowing-doing gap through the zone of generativity on pupils' achievement, retention and anxiety towards mathematics, found out that knowing-doing gap through the zone of generativity had the best effect on the students' retention. This study is related to the present study because it also dealt with retention. Another study is that of Herrera (2007), which focused on problem-based and activity-based instructions and their influence on the students' achievement and retentions scores in probability and statistics and attitude towards mathematics, found out that the activity method in teaching mathematics can promote better retention. This is related to the present study because the researcher used similar method in teaching by giving an activity to be answered after the discussion. Tan (2015) also studied retention based on the influence of problem posing and sense making. Her study revealed that problem posing and sense making in mathematics class is effective in improving students' retention in conceptual understanding. This is related to the present study because one of the variables in the study was retention and the students were allowed to

explain the given problem based from their ability to use the knowledge and experience about the given lessons.

3. Methodology

The study employed a pretest-posttest control group design. 453 Grade-10 students of Bulua National High School were randomly distributed to 10 sections and one intact section composed of 55 students was randomly assigned as control group and another 55 students as one section for experimental group. A teacher-made test was used in the study, the 31-item multiple choice which assessed students' achievement and with open-ended questions which required students to interpret, to write the step-by-step process of the solutions, and to provide justification and explanation on how and why to apply such concept in solving the problem on circles and geometry. These tests were prepared in accordance with a table of specification and validated with coefficient of reliability index of 0.95.

The researcher handled the two classes to minimize the possible effect of the teacher factor that might affect the outcome of the study. To ensure that the two approaches were implemented appropriately and distinctively in the control and experimental groups, the researcher invited the mathematics department coordinator and one mathematics teacher to observe the two classes under study. This was done to avoid bias. There were three observations done for each class.

The discussion in both groups started with a lecture of the basic terms and steps in solving each problem of identifying what was asked in the problem, listing down the given facts, sketching the diagram, indicating the part that needed to be solved. However, in the control group, the teacher illustrated concepts by solving sample problems. Then the teacher gave repetitively similar problems for students to solve. This repetitive way of asking mathematics problems could make the learners master a certain topic because students focused only on the same type of problem. The class was told first to answer the given example in their seat, and discuss their solution with the group. After which a student was asked to present the solution. Then the class was given an activity to be written in their activity notebook. The students were instructed to submit their activity notebook with their solutions to be checked by the teacher. Their outputs were returned right after checking with the correct solution written on it. While in the experimental group, students were given varied problems as an example. Then, the class answered it first in their seat, once they have their answer, they were asked to discuss it with the group. After the group activity, volunteers were asked to write the answer on the board and to explain it before the class. Immediately after the discussion, an activity was given where they were given varied mathematics problems of the same concept. After the responses were written, the students were instructed to submit their activity notebook and have it checked by the teacher with the correct solutions written on it. Their outputs were returned right after checking, then they were instructed to rewrite their answers correctly and their rewritten work were collected again by the researchers to be reviewed for

accuracy. Upon completion of the series of lessons, the participants were required to complete the post-test.

Answers on the open-ended questions were evaluated using rubric scale adapted from the study of Lomibao (2016) where students were required to explain, interpret and apply. There were three mathematics teachers, including the researcher, who rated the answers of the participants.

The analysis of covariance (ANCOVA) was used to determine the effects of the treatment because the samples were intact. The performance in terms of achievement, conceptual understanding and retention of the students of both groups were described using the mean and standard deviation. In testing the hypotheses, alpha is set at 0.05 level of significance.

4. Results and Discussion

Table 1: Mean and Standard Deviation of Students' Achievement Scores

	Experimental Group N=35		Control Group N=35	
	Pretest	Posttest	Pretest	Posttest
Mean	11.43	21.31	9.23	14.66
SD	2.78	4.90	2.76	3.43

*Perfect Score: 31

Table 2: One-way ANCOVA Summary for students' Achievement Scores

Source	Adj. SS	DF	Mean Square	F	p-value
Treatment within	405.599	1	405.599	27.556	0.0001
Error	986.165	67	14.719		
Total	1391.764	68			

*Significant at .05 level

Table 1 shows the mean and standard deviation of achievement scores. It can be notice that there is a little difference of the achievement which means that both groups had a little knowledge about the lessons. On other hand, in the post test, the experimental group gets a higher mean score compare to the control group, which means that there is an increase of their achievement scores. Pretest standard deviation in the achievement test showed that both groups were of the same ability before the treatment was implemented since standard deviation difference is negligible at 0.02. As regards to posttest, the experimental group scores were widely spread compared to the control group, this indicates that students have heterogeneous ability after the treatment.

To determine if there was significant difference between the retention of the students on procedural knowledge test, the analysis of covariance ANCOVA was employed. Table 2 shows the analysis of covariance of pre-test and post-test scores of students' achievement scores. The analysis yielded a computed probability value of .001 which is less than 0.05 level of significance. This led the researcher to reject the null hypothesis. This implies that there is a significant difference in the students' mathematics performance in favor of the experimental group. This means that the experimental group exposed to repetition with variation approach performed

better than those exposed to repetition with no variation. It manifests further that, individual student would understand new concepts in varying ways depending on their existing framework of knowledge. Varying the unknown of mathematics problems could enhance deep understanding, proving that students performed better if they were exposed to repetition with variation. The result concurred with the finding of Tong (2012), Noche & Yu (2015), Olteanu, & Olteanu, (2011) that repetition with variation improved students' achievement.

Table 3: Mean and Standard Deviation of Students' Conceptual Understanding

	Experimental Group N=35		Control Group N=35	
	Pretest	Posttest	Pretest	Posttest
Mean	13.82	94.56	15.70	69.31
SD	5.98	23.88	5.10	18.17

Table 4: One-way ANCOVA Summary for students' Conceptual Understanding

Source	Adj. SS	df	Mean Square	F	p-value
Treatment within	10610.749	1	405.599	27.556	0.0001
Error	30545.444	67	14.719		
Total	41156.193				

*Significant at .05 level

Table 3 shows that the mean score of the experimental group's conceptual understanding. Posttest scores reveal that the students in the experimental group got higher mean scores compare to the control group, indicating that both groups have increase their scores in conceptual understanding. This means that they had acquired knowledge on the lessons after a series of discussions made by the teacher. However, a greater increase can be observed from the students in the experimental group compared to the control group. With regards to pretest of students' standard deviation in conceptual understanding, both experimental and control groups got 5.98 and 5.10, respectively, which means that students had comparable initial knowledge of the subject. As regards to posttest, the experimental group with 23.88, which means that the scores were widely spread compared to the control group with 18.17. This is an indication that some of the students' scores were low, while those of the others were high.

Table 4 shows the analysis of covariance of pre-test and post-test scores of students' conceptual understanding. The analysis yielded a computed probability value of .0001 which is less than 0.05 level of significance. This led to non-acceptance of the null hypothesis. This means that there is a significant difference in the students' conceptual understanding between the experimental and control groups. This implies that the conceptual understanding of students exposed to repetition with variation approach is significantly higher than those exposed to repetition without variation. This happened because students were engaged in critical thinking that facilitated learning of important mathematics concepts and mathematical processes. In this case, conceptual understanding was acquired because students were required to explain and interpret what they were doing in mathematical operation, why it worked, and where and

when it could be applied. In creating this form of variation, the students were able to comprehend different components of a concept and upgrade their structure of knowledge. This form of variation offered a structured and structural approach to exposing underlying mathematical forms and, therefore, could enhance students' conceptual understanding of a series of related concepts. This finding confirmed the claims of Ketterlin-Geller (2007), and Lai, (2015), on enhancing conceptual understanding.

Table 5: Mean and Standard Deviation of Students' Achievement Scores on the Retention Test on Circles and Plane Coordinate Geometry

	Experimental Group N=35		Control Group N=35	
	Posttest	Retention	Posttest	Retention Test
Mean	21.31	21.60	14.66	16.46
SD	4.90	4.97	3.43	6.66

Table 6: One-way ANCOVA Summary for students' Retention on Achievement Scores Test

Source	Adj. SS	df	Mean Square	F	p-value
Treatment within	242.839	1	242.839	6.95	0.010
Error	2341.188	67	34.943		
Total	2584.027	68			

*Significant at .05 level

Table 5 shows the mean and standard deviation of students' achievement on the retention test on Circles and Plane Coordinate Geometry. It can be observed that the retention mean score is higher than the posttest mean scores for both experimental and control groups, indicating that students remembered more concepts after two weeks. In addition to that, Table 5 shows that the retention mean score increase of the control group is a little bit higher compared to the experimental group since the increase of the control group from posttest to retention test is 1.8 while that of the experimental group, it is only 0.29, a difference of 1.51 in favor of the control group. The standard deviation of students' retention test reveals that the control group scores is more spread than that of the experimental group. This implies that some of the students' scores were low while those of the others were high.

Table 6 shows the summary of the analysis of covariance of the posttest and retention of the experimental and control groups in the achievement test. The analysis yielded a computed F-ratio of 6.95 and a probability-value of 0.010 which is lesser than the 0.05 level of significance. This led to the rejection of the null hypothesis. This means that there is enough evidence to conclude that the retention score of the experimental group in the test is significantly higher than those exposed to the conventional method which is repetition without variation. This further implies that their experience in the previous tests helped the students understand the concept well and the repetition with variation have helped develop their long term memory which allow to retrieve whatever concepts or ideas they needed. Despite the difference, the experimental group still showed better retention because their mean score is higher compared to the control group. This denotes that consistent elaboration or explanation of a topic would surely bring deep retention of the concept; strong

retention took place as a result of extended practice. In other words, learning by practice helped the students understand the concept well and their long term memory would help them retrieve whatever concepts or ideas they need for future use. In addition, writing could help enhance students' performance and improve their communication ability and problem solving competence. Moreover, writing developed a more positive attitude towards mathematics, allowed students to construct ideas in mathematics that promoted thinking and illustrated an awareness of mathematical connections. This finding confirmed the claims of Chanson, Kurumeh and obida (2010), Ritter (2010), Haltiwanger and Simpson (2013), Ubalde (2015), Herrera (2007) and Tan (2115).

Table 7: Mean, Standard Deviation of Students' Conceptual Understanding on the Retention Test on Circles and Plane Coordinate Geometry.

	Experimental Group N=35		Control Group N=35	
	Posttest	Retention Test	Posttest	Retention Test
Mean	94.56	95.01	69.31	69.70
SD	23.88	17.98	18.17	15.93

Table 8: One-way ANCOVA Summary for students' Retention on Conceptual Understanding Test

Source	Adj. SS	df	Mean Square	F	p-value
Treatment within	1272.575	1	1272.575	10.255	0.002
Error	314.487	67	124.097		
Total	9587.062	68			

*Significant at .05 level

Table 7 shows the mean and standard deviation of students' conceptual understanding on the retention test on Circles and Plane Coordinate Geometry. It can be observed that there is no noticeable difference between the posttest and retention test means for both groups, an indication that the students had retained conceptual understanding, however, a greater improvement can be observed with the students in the experimental group. The standard deviation of the experimental group is higher compared to the control group. This indicates that the scores of the experimental group in the retention test were more dispersed than that of the control group. This explains why participants in the experimental group got a very high score while others got a very low score.

Table 8 shows the summary of the analysis of posttest and retention of the experimental and control groups in the conceptual understanding test. The analysis yielded a computed F-ratio of 10.255 and a probability-value of .002 which is lesser than the .05 level of significance. This led to the rejection of the null hypothesis. This means that there is enough evidence to conclude that the retention score of the experimental group in conceptual understanding test is significantly higher than those exposed to the conventional method which is repetition without variation. This implies that the said approach in mathematics class is an effective teaching method to improve students' conceptual understanding and retention. In this study on the use of repetition with variation, it showed that this method has helped students' improved their learning of certain concepts because they have higher retention when given mathematics

problems they would be able to solve it using the concept and principles appropriately. The more one has practiced, the more likely is one to generalize what one has learned using the style of problem solving or inquiry that is appropriate for any kind of task. Hence, using repetition with variation as an approach could help the students develop their ability to explain, interpret and apply certain concepts in mathematics. This finding supports the theory of variation described by Marton and Booth (1997), Gu (2004), Marton and Morris (2002), Watson & Mason (2005), Bruner (1961), Piaget (1963), and Vygotsky (1978) for students' retention on conceptual understanding.

5. Conclusion and Recommendations

Based on the analysis and findings of the study the researchers concluded that repetition with variation is effective in teaching mathematics to improve students' achievement and conceptual understanding and enhanced students' retention. Hence, they recommended that teachers could use repetition with variation as an approach in teaching word problems in Mathematics that involve the four fundamental operations to enhance the K-12 lesson guides. Teachers and researchers could use this method as a basis for future studies for more insights on instruction that use repetition with variation and similar studies may be conducted to wider scope using different population in different institutions for better generalizability of the method.

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