International Journal of Science and Research (IJSR)
ISSN (Online): 2319-7064
Index Copernicus Value (2015): 78.96 | Impact Factor (2015): 6.391

Analysis of Transient Heat Conduction in Different Geometries

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Abstract: ***Present work deals with the analytical solution of unsteady state one-dimensional heat conduction problems. An improved lumped parameter model has been adopted to predict the variation of temperature field in a long slab and cylinder. Polynomial approximation method is used to solve the transient conduction equations for both the slab and tube geometry. A variety of models including boundary heat flux for both slabs and tube and, heat generation in both slab and tube has been analyzed. Furthermore, for both slab and cylindrical geometry, a number of guess temperature profiles have been assumed to obtain a generalized solution. Based on the analysis, a modified Biot number has been proposed that predicts the temperature variation irrespective the geometry of the problem. In all the cases, a closed form solution is obtained between temperature, Biot number, heat source parameter and time. The result of the present analysis has been compared with earlier numerical and analytical results. A good agreement has been obtained between the present prediction and the available results.***

Keywords: lumped model, polynomial approximation method, transient, conduction, modified Biot number.

1. Introduction

1.1 General Background

Heat transfer is the study of thermal energy transport within a medium or among neighboring media by molecular interaction, fluid motion, and electro-magnetic waves, resulting from a spatial variation in temperature. This variation in measured temperature is governed by the principle of energy conservation, which when applied to a control volume or a control mass, states that the sum of the flow of energy and heat across the system, the work done on the system, and the energy stored and converted within the system, is zero. Heat transfer finds application in many important areas, namely design of thermal and nuclear power plants including heat engines, steam generators, condensers and other heat exchange equipments, catalytic convertors, heat shields for space vehicles, furnaces, electronic equipments etc, internal combustion engines, refrigeration and air conditioning units, design of cooling systems for electric motors generators and transformers, heating and cooling of fluids etc. in chemical operations, construction of dams and structures, minimization of building heat losses using improved insulation techniques, thermal control of space vehicles, heat treatment of metals, dispersion of atmospheric pollutants. A thermal system contains matter or substance and this substance may change by transformation or by exchange of mass with the surroundings. To perform a thermal analysis of a system, we need to use thermodynamics, which allows for quantitative description of the substance. This is done by defining the boundaries of the system, applying the conservation principles, and examining how the system participates in thermal energy exchange and conversion.

1.2 Modes of Heat Transfer

Heat transfer generally takes place by three modes such as conduction, convection and radiation. Heat transmission, in majority of real situations, occurs as a result of combinations of these modes of heat transfer. Conduction is the transfer of thermal energy between neighboring molecules in a substance due to a temperature gradient. It always takes place from a region of higher temperature to a region of lower temperature, and acts to equalize temperature differences. Conduction needs matter and does not require any bulk motion of matter. Conduction takes place in all forms of matter such as solids, liquids, gases and plasmas. In solids, it is due to the combination of vibrations of the molecules in a lattice and the energy transport by free electrons. In gases and liquids, conduction is due to the collisions and diffusion of the molecules during their random motion. Convection occurs when a system becomes unstable and begins to mix by the movement of mass. A common observation of convection is of thermal convection in a pot of boiling water, in which the hot and less-dense water on the bottom layer moves upwards in plumes, and the cool and denser water near the top of the pot likewise sinks. Convection more likely occurs with a greater variation in density between the two fluids, a larger acceleration due to gravity that drives the convection through the convecting medium. Radiation describes any process in which energy emitted by one body travels through a medium or through space absorbed by another body. Radiation occurs in nuclear weapons, nuclear reactors, radioactive radio waves, infrared light, visible light, ultraviolet light, and X-rays substances.

1.3 Heat Conduction Problems

The solution of the heat conduction problems involves the functional dependence of temperature on various parameters such as space and time. Obtaining a solution means determining a temperature distribution which is consistent with conditions on the boundaries.

One Dimensional analysis

In general, the flow of heat takes place in different spatial coordinates. In some cases the analysis is done by considering the variation of temperature in one-dimension. In a slab one dimension is considered when face dimensions in each direction along the surface are very large compared to the region thickness, with uniform boundary condition is applied to each surface. Cylindrical geometries of one-dimension have axial length very large compared to the maximum conduction region radius. At a spherical geometry to have one-dimensional analysis a uniform condition is applied to each concentric surface which bounds the region.

Steady and unsteady analysis

Steady state analysis

A steady-state thermal analysis predicts the effects of steady thermal loads on a system. A system is said to attain steady state when variation of various parameters namely, temperature, pressure and density vary with time. A steady-state analysis also can be considered the last step of a transient thermal analysis. We can use steady-state thermal analysis to determine temperatures, thermal gradients, heat flow rates, and heat fluxes in an object which do not vary over time. A steady-state thermal analysis may be either linear, by assuming constant material properties or can be nonlinear case, with material properties varying with temperature. The thermal properties of most material do vary with temperature, so the analysis becomes nonlinear. Furthermore, by considering radiation effects system also become nonlinear.

Unsteady state analysis

Before a steady state condition is reached, certain amount of time is elapsed after the heat transfer process is initiated to allow the transient conditions to disappear. For instance while determining the rate of heat flow through wall, we do not consider the period during which the furnace starts up and the temperature of the interior, as well as those of the walls, gradually increase. We usually assume that this period of transition has passed and that steady-state condition has been established.

1.4 Description of Analytical Method and Numerical Method

In general, we employ either an analytical method or numerical method to solve steady or transient conduction equation valid for various dimensions (1D/2D). Numerical technique generally used is finite difference, finite element, relaxation method etc. The most of the practical two dimensional heat problems involving irregular geometries is solved by numerical techniques. The main advantage of numerical methods is it can be applied to any two-dimensional shape irrespective of its complexity or boundary condition. The numerical analysis, due to widespread use of digital computers these days, is the primary method of solving complex heat transfer problems.

The heat conduction problems depending upon the various parameters can be obtained through analytical solution. An analytical method uses Laplace equation for solving the heat conduction problems. Heat balance integral method, hermite-type approximation method, polynomial approximation method, wiener–Hopf Technique are few examples of analytical method

1.5 Objective of Present Work

1) An effort will be made to predict the temperature field in solid by employing a polynomial approximation method.
2) Effort will be made analyze more practical case such as heat generation in solid and specified heat flux at the solid surface is investigated.
3) Effort will be made to obtain new functional parameters that affect the transient heat transfer process.
4) It is tried to consider various geometries for the analysis.

2. Analytical Solutions

Keshavarz and Taheri [1] have analyzed the transient one-dimensional heat conduction of slab/rod by employing polynomial approximation method. In their paper, an improved lumped model is being implemented for a typical long slab, long cylinder and sphere. It has been shown that in comparison to a finite difference solution, the improved model is able to calculate average temperature as a function of time for higher value of Biot numbers. The comparison also presents model in better accuracy when compared with others recently developed models. The simplified relations obtained in this study can be used for engineering calculations in many conditions. He had obtained the temperature distribution as:

$$\theta = \exp\left(-\frac{B(m+1)(m+3)}{m+B+3}\tau\right)$$

Jian Su [2] have analyzed unsteady cooling of a long slab by asymmetric heat convection within the framework of lumped parameter model. They have used improved lumped model where the heat conduction may be analyzed with larger values of Biot number. The proposed lumped models are obtained through two point Hermite approximations method.

Closed form analytical solutions are obtained from the lumped models. Higher order lumped models, ($H_{1.1}/H_{0,0}$ approximation) is compared with a finite difference solution and predicts a significance improvement of average temperature prediction over the classical lumped model. The expression was written as-

$$\theta = \exp\left(-\frac{3(B1+B2+2B1B2)}{2(3+2B1+2B2+B1B2)}\tau\right)$$

Su and Cotta [3] have modeled the transient heat transfer in nuclear fuel rod by an improved lumped parameter approach. Average fuel and cladding temperature is derived using hermite approximation method. Thermal hydraulic behavior of a pressurized water reactor (PWR) during partial loss of coolant flow is simulated by using a simplified mathematical model. Transient response of fuel, cladding and coolant is analyzed.

3. Conclusions

An improved lumped parameter model is applied to the transient heat conduction in a long slab and long cylinder. Polynomial approximation method is used to predict the transient distribution temperature of the slab and tube geometry. Four different cases namely, boundary heat flux for both slab and tube and, heat generation in both slab and tube has been analyzed. Additionally different temperature profiles have been used to obtain solutions for a slab. A unique number, known as modified Biot number is, obtained from the analysis. It is seen that the modified Biot number, which is a function of Biot number, plays important role in the transfer of heat in the solid. Based on the analysis the following conclusions have been obtained.

1- Initially a slab subjected to heat flux on one side and convective heat transfer on the other side is considered for the analysis. Based on the analysis, a closed form solution has been obtained.

$$\theta = \left(\frac{e^{-U\tau} + V}{U} \right)$$

Where $U = \frac{B}{1+B/3}$, $V = \frac{Q}{1+B/3}$

2- A long cylinder subjected to heat flux on one side and convective heat transfer on the other side is considered for the analysis. Based on the analysis, a solution has been obtained.

$$\theta = \left(\frac{e^{-U\tau} + V}{U} \right)$$

Where $U = \frac{B}{(4+B)/8}$, $V = \frac{Q}{(4+B)/8}$

3- A slab subjected to heat generation at one side and convective heat transfer on the other side is considered for the analysis. Based on the analysis, a closed form solution has been obtained.

$$\theta = \frac{e^{-\tau U} + V}{U}$$

Where $U = \frac{B}{(4+B)/8}$, $V = \frac{G}{(1+\frac{B}{3})}$

4- A long cylinder subjected to heat generation at one side and convective heat transfer on the other side is considered for the analysis. Based on the analysis a closed form solution has been obtained.

$$\theta = \frac{e^{-\tau U} + V}{U}$$

Where $U = \left(\frac{2B}{1+B/4} \right)$, $V = \frac{G}{(1+B/4)}$

5. Based on the analysis a unique parameter known as modified Boit number obtained from the analysis. With higher value of heat source parameter, the temperature inside the tube does not vary with time. However at lower values of heat source parameters, the temperature decreases with increase of time. With lower value of Biot numbers, the temperature inside the tube does not vary with time. For higher value of Biot numbers, the temperature decreases with the increase of time.

4. Scope for Further Work

1) Polynomial approximation method can be used to obtain solution of more complex problem involving variable properties and variable heat transfer coefficients, radiation at the surface of the slab.
2) Other approximation method, such as Heat Balance Integral method, Biots variation method can be used to obtain the solution for various complex heat transfer problems.
3) Efforts can be made to analyze two dimensional unsteady problems by employing various approximate methods.

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