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Bayesian Estimation of the Parameter of Inverse – Maxwell Distribution under SELF and Precautionary Loss Functions

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Abstract: ***In this paper, we have discussed Bayesian estimation of the parameter of an Inverse Maxwell Distribution. Bayes estimators of the scale parameter 'θ' of the Inverse Maxwell Distribution using quasi-prior under squared error, precautionary, entropy and another two loss functions have been obtained and the corresponding risk functions of these estimators relative to squared error loss function have also been calculated for the sake of comparison. The relevant graphs have been plotted.***

Keywords: Inverse Maxwell Distribution, SELF, Precautionary Loss, Entropy Loss, Loss function -I, Loss function-II, Risk function etc.

1. Introduction

There has been increased interest shown in the application of Bayesian methodology in the last four decades and a number of methods have been proposed for Bayesian inference. More generally, Bayesian methods are data analysis tools that are derived from the principles of Bayesian inference. In addition to their formal interpretation as a means of induction, Bayesian methods provide (Hoff, Peter D (2009)):

- Parameter estimates with good statistical properties;
- Parsimonious descriptions of observed data;
- Predictions for missing data and forecasts of future data;
- A computational framework for model estimation, selection and validation.

Let $f(y \mid \theta); \theta \in \Theta$ be the probability density function of lifetime distribution of a component or an animate, where the parameter space Θ is known but the true value of θ is unknown. Let $g(\theta)$ is the prior density function of the random variable θ. Let $\underline{y} = (y_1, \ldots, y_n)$ be an n independent observations from $f(y; \theta)$. Then using Bayes' theorem (1763) the posterior distribution $f\left(\theta \mid \underline{y}\right)$ of θ is given by

$$f\left(\theta \mid \underline{y}\right) = \frac{f\left(\underline{y} \mid \theta\right) g(\theta)}{\int_{\Theta} f\left(\underline{y} \mid \theta\right) g(\theta) d\theta}. \quad (1.1)$$

where $f\left(\underline{y} \mid \theta\right)$ is the joint probability density function of $\underline{y} = (y_1, \ldots, y_n)$. For a given sample $\underline{y}$, the posterior p.d.f. $f\left(\theta \mid \underline{y}\right)$ Bayes theorem. In order to define Bayes estimators we must specify a loss function

$$L(\hat{\theta}, \theta) \geq 0, \text{ for all } \hat{\theta} \text{ and } \theta;$$

The corresponding Bayes risk is defined as the expected value of the risk $R(\hat{\theta}, \theta)$ with respect to the prior distribution $g(\theta)$ on Θ and is given as,

$$R(\hat{\theta}, \theta) = E[r(\hat{\theta}, \theta)] = \int_{\Theta} r(\hat{\theta}, \theta)\, g(\theta) dy$$

where the risks function $r(\hat{\theta}, \theta)$ is defined as

$$r(\hat{\theta}, \theta) = \int_{\chi} L(\hat{\theta}, \theta)\, f\left(\underline{y} \mid \theta\right) d\underline{y}$$

where χ stands for the sample space of $\underline{y}$. The fundamental problems in Bayesian analysis is that of the choice of prior distribution $g(\theta)$ and loss function $L(\hat{\theta}, \theta)$ which may be appropriate for the situation at hand.

Singh, Kusum Lata & Srivastava, R.S., (2014 a,b) have defined an Inverse Maxwell distribution, obtained its pdf and established it as a survival model also discussed the estimation of the parameter of size-biased Inverse Maxwell distribution. The Inverse Maxwell distribution is chosen to be used in the study of the propagation time of Dark Matter, Geoffry Blewitt(2016) university of Nevada, Reno, USA. Let Y is a random variable having pdf

$$f(y; \theta) = \frac{4}{\sqrt{\pi}\theta^{\frac{3}{2}}} \frac{1}{y^4} . e^{-\frac{1}{\theta y^2}} .\ y>0,\ \theta>0 \quad (1.2)$$

where θ is a scale parameter, the raw moment of IMD is given by

$$E(X^r) = \mu_r' = \int_0^\infty y^r f(y) dy$$
$$= \frac{2}{\sqrt{\pi}\theta^{\frac{r}{2}}} \Gamma\left(\frac{-r+3}{2}\right) \quad (1.3)$$

The mean, variance and harmonic mean are obtained as :-
Mean $(\mu_1') = \frac{2}{\sqrt{\pi\theta}}$, **Variance**$(\mu_2) = \frac{2(\pi-2)}{\theta\pi}$, and **Harmonic Mean** $(H) = \frac{1}{2}\sqrt{\frac{\pi}{\theta}}$. (Singh, Kusum Lata & Srivastava, R.S., (2014 a))

2. Prior Distribution

The prior distribution should be specific to the situation. We often have prior information on the sign of parameters, on the relative or approximate magnitude of parameters, and even on (sometimes complex) functions of parameters.

In Bayesian analysis the fundamental problem are that of the choice of prior distribution $g(\theta)$ and a loss function L (.;.). Let us consider a suitable prior (e.g. quasi – prior) for θ to obtain the bayes estimator's in this case assuming independent among the parameters is:

$$g(\theta) = \frac{1}{\theta^d};\ \theta > 0,\ d > 0 \quad (2.1)$$

3. Various Loss function

Let θ be an unknown parameter of some distribution $f(x \mid \theta)$ and suppose we estimate θ by some statistic $\hat{\theta}$. Let $L(\hat{\theta}, \theta)$ represent the loss incurred when the true value of the parameter is θ and we are estimating θ by the statistic $\hat{\theta}$.

(a) Squared error loss function (SELF)

$$L(\hat{\theta}, \theta) = (\hat{\theta} - \theta)^2 \qquad (3.1)$$

The Bays estimator under the above loss function, say is $\hat{\theta}$ the posterior mean, i.e.

$$\hat{\theta}_B = E_\pi(\theta) \qquad (3.2)$$

The risk function is given by:

$$R_B(\hat{\theta}) = E_\theta(\hat{\theta})^2 - 2\theta E_\theta(\hat{\theta}) + \theta^2 . \qquad (3.3)$$

(b) Precautionary Loss Function :

Norstrom (1996) introduced an alternative asymmetric precautionary loss function and also presented a general class of precautionary loss functions with quadratic loss function as a special case (Srivastava, R.S.,et al. (2004)). A very useful and simple asymmetric precautionary loss function is given as

$$L(\hat{\theta}, \theta) = \frac{(\hat{\theta} - \theta)^2}{\hat{\theta}} . \qquad (3.4)$$

The posterior expectation of loss function in (6) is

$$E_\pi[L(\hat{\theta} - \theta)] = E_\pi\left(\frac{\theta^2}{\hat{\theta}}\right) + E_\pi(\hat{\theta}) - E_\pi(\theta) \qquad (3.5)$$

The value of $\hat{\theta}$ that minimize (7), denoted by $\hat{\theta}_p$ is obtained by solving the following equation

$$\frac{d}{d\theta} E_\pi[L(\hat{\theta} - \theta))] = 0$$

$$\hat{\theta}_P = [E_\pi(\theta^2)]^{\frac{1}{2}} . \qquad (3.6)$$

(c) Entropy Loss Function :

In many practical situations, it appears to be more realistic to express the loss iSn terms of the ratio $\frac{\hat{\theta}}{\theta}$. In this case, Calabria and Pulcini (1994) points out that a useful asymmetric loss function is the entropy loss given by

$$L(\delta) = [\delta^p - p\log_e(\delta) - 1] \qquad (3.7)$$

where

$$\delta = \frac{\hat{\theta}}{\theta} ,$$

The posterior expectation of loss function in (9) is

$$E_\pi[L(\delta)] = b\left[E_\pi\left(\frac{\hat{\theta}}{\theta}\right) - E_\pi\left(\log_e\left(\frac{\hat{\theta}}{\theta}\right)\right) - 1\right] \qquad (3.8)$$

The value of $\hat{\theta}$ that minimum (10), denoted by $\hat{\theta}_e$ is obtained by solving the following equation

$$\frac{d}{d\theta} E_\pi [L(\Delta)] = 0$$

$$\hat{\theta}_e = \left[E_\pi\left(\frac{1}{\theta}\right)\right]^{-1} \qquad (3.9)$$

(d) Loss function-L_1 :

Consider the loss function given by

$$L_1(\hat{\theta}, \theta) = \left(\frac{\hat{\theta}}{\theta} - 1\right)^2 \qquad (3.10)$$

The Bayes estimator under loss function- $\mathbf{L_1}$, say $\hat{\theta}_1$ using the value of $f(\theta|\underline{y})$,

$$\hat{\theta}_1 = \frac{E_\pi\left(\frac{1}{\theta}\right)}{E_\pi\left(\frac{1}{\theta^2}\right)} \qquad (3.11)$$

(e) Loss function-L_2 :

Consider the loss function given by

$$L_2(\hat{\theta}, \theta) = \left(\frac{\theta}{\hat{\theta}} - 1\right)^2 \qquad (3.12)$$

The Bayes estimator under loss function- $\mathbf{L_2}$, say $\hat{\theta}_2$ using the value of $f(\theta|\underline{y})$,

$$\hat{\theta}_2 = \frac{E_\pi(\theta^2)}{E_\pi(\theta)} \qquad (3.13)$$

In this paper, we have discussed Bayesian estimation of the parameter of an Inverse Maxwell Distribution. Bayes estimators of the scale parameter 'θ' of the Inverse Maxwell distribution (IMD) using quasi-prior under squared error, precautionary, entropy and another two loss functions have been obtained and the corresponding risk functions of these estimators relative to squared error loss function have been obtained for the sake of comparison. The relevant graphs have also been plotted.

4. Bayesian Estimation under g(θ)

Under **g**(θ), the posterior distribution is defined by

$$f(\theta \mid \underline{y}) = \frac{f(\underline{y}|\theta)g(\theta)}{\int_\Theta f(\underline{y}|\theta)g(\theta)d\theta} \qquad (4.1)$$

substituting the value of g(θ) and $f(\underline{y} \mid \theta)$ in (4.1)

In order to carry out the estimation procedure, let us suppose that very small information is available about the parameter (the suitable prior for this case (Singh, Kusum Lata & Srivastava, R.S., (2014 c)). Assuming independence among the parameters, consider a quasi prior (2.1),

$$g(\theta) = \frac{1}{\theta^d} .$$

The joint density function of IMD is given by

$$f(\underline{y}|\theta) = \left(\frac{4}{\theta^{\frac{3}{2}}\sqrt{\pi}}\right)^n \prod_{i=1}^n \frac{1}{y_i^4} e^{\left(\sum \frac{1}{y_i^2}\right)\left(\frac{1}{\theta}\right)} \qquad (4.2)$$

where

$$T_n = \left(\sum \frac{1}{y_i^2}\right).$$

Now using the Bayes theorem, the joint density function (4.2) along with the prior (2.1), we obtain the following joint posterior density function of IMD is

$$f(\theta \mid \underline{y}) = \frac{f(y|\theta)g(\theta)}{\int_0^\infty f(y|\theta)g(\theta)d\theta} , \qquad (4.3)$$

which on substituting the value of g(θ) and f(y|θ) gives

$$f(\theta \mid \underline{y}) = \frac{\left(\frac{1}{\theta}\right)^{\frac{3n}{2}+d} [T_n]^{\frac{3n}{2}+d-1} e^{-\left(\frac{T_n}{\theta}\right)}}{\Gamma\left(\frac{3n}{2}+d-1\right)} . \qquad (4.4)$$

(a) **Squared error loss function** : the Bayes estimator under squared error loss function is the posterior mean given by

$$\hat{\theta}_S = \int_0^\infty \theta \, f(\theta|\underline{y}) \, d\theta \qquad (4.5)$$

Substituting the value of $f(\theta \mid \underline{y})$ from equation (4.4) in equation (4.5) and solving it, we get

$$\hat{\theta}_S = \int_0^\infty \theta \frac{\left(\frac{1}{\theta}\right)^{\frac{3n}{2}+d} [T_n]^{\frac{3n}{2}+d-1} e^{-\left(\frac{T_n}{\theta}\right)}}{\Gamma\left(\frac{3n}{2}+d-1\right)} . \qquad (4.6)$$

Solving equation (4.6), we get

$$\hat{\theta}_S = \frac{T_n}{\frac{3n}{2}+d-2} . \qquad (4.7)$$

(b) **Precautionary loss function**: The Bayes estimator under precautionary loss function

$$\hat{\theta}_P = [E_\pi(\theta^2)]^{\frac{1}{2}} = \left[\int_0^\infty \theta^2 \ f(\theta \mid \underline{y}) \ d\theta\right]^{\frac{1}{2}} \qquad (4.8)$$

which on simplification leads to

$$\hat{\theta}_P = \frac{T_n}{\left[\left(\frac{3n}{2}+d-2\right)\left(\frac{3n}{2}+d-3\right)\right]^{\frac{1}{2}}}. \qquad (4.9)$$

(c) **Entropy loss function**: The Bayes estimator under entropy loss function

$$\hat{\theta}_e = \left[E_\pi\left(\frac{1}{\theta}\right)\right]^{-1} = \left[\int_0^\infty \frac{1}{\theta} \ f(\theta \mid \underline{y}) \ d\theta\right]^{-1} \qquad (4.10)$$

Which on simplification comes out to be

$$\hat{\theta}_e = \frac{S}{\left(\frac{3n}{2}+d-1\right)}. \qquad (4.11)$$

(d) **Other loss function-L_1 :**

Consider the loss function given by

$$L_1(\hat{\theta}, \theta) = \left(\frac{\hat{\theta}}{\theta} - 1\right)^2$$

The Bayes estimator under loss function- $\mathbf{L_1}$ say $\hat{\theta}_1$ using the expression of f $(\theta|\underline{y})$ in equation (4.4) is the solution of equation given

$$\hat{\theta}_1 = \frac{E_\pi\left(\frac{1}{\theta}\right)}{E_\pi\left(\frac{1}{\theta^2}\right)} = \frac{\int_0^\infty \frac{1}{\theta} f(\theta \mid \underline{y}) \ d\theta}{\int_0^\infty \frac{1}{\theta^2} f(\theta \mid \underline{y}) \ d\theta} \qquad (4.12)$$

$$\text{Or } \hat{\theta}_1 = \frac{T_n}{\left(\frac{3n}{2}+d\right)} \qquad (4.13)$$

(e) **Other loss function- L_2 :**

Consider the loss function given by

$$L(\hat{\theta}, \theta) = \left(\frac{\theta}{\hat{\theta}} - 1\right)^2.$$

The Bayes estimator under loss function-$\mathbf{L_2}$, say, $\hat{\theta}_2$ using the value of $f(\theta|\underline{y})$ from equation (4.4) is the solution of equation given by

$$\hat{\theta}_2 = \frac{E_\pi(\theta^2)}{E_\pi(\theta)} = \frac{\int_0^\infty \theta^2 \ f(\theta \mid \underline{y}) \ d\theta}{\int_0^\infty \theta \ f(\theta \mid \underline{y}) \ d\theta}, \qquad (4.14)$$

$$\text{or } \hat{\theta}_2 = \frac{T_n}{\left(\frac{3n}{2}+d-3\right)}. \qquad (4.15)$$

4.1 The Risk Functions Under The Squared Error Loss Function

(i) The risk function of $\hat{\theta}_S$, relative to squared error loss function is denoted by $R_S(\hat{\theta}_S)$ and accordance with (5), is given by

$$R_S(\hat{\theta}_S) = E_\theta\left(\hat{\theta}_S^{\ 2}\right) - 2\theta E_\theta(\hat{\theta}_S) + \theta^2. \qquad (5.1)$$

Substituting the value of $\hat{\theta}_S$ from (4.7) and evaluating various expectations in (5.1), we get

$$R_S(\hat{\theta}_S) = \theta^2\left[\frac{\left(\frac{3n}{2}-1\right)\left(\frac{3n}{2}+1\right)}{\left(\frac{3n}{2}+d-2\right)^2} - \frac{2\left(\frac{3n}{2}-1\right)}{\left(\frac{3n}{2}+d-2\right)} + 1\right]. \qquad (5.2)$$

(ii) The risk function of $\hat{\theta}_p$, relative to squared error loss function is denoted by $R_S(\hat{\theta}_p)$ is given by

$$R_S(\hat{\theta}_p) = E_\theta\left(\hat{\theta}_p^{\ 2}\right) - 2\theta E_\theta(\hat{\theta}_p) + \theta^2. \qquad (5.3)$$

Substituting the value of $\hat{\theta}_p$ from (4.9) and evaluating various expectations in (5.3), we get

$$R_S(\hat{\theta}_p) = \theta^2\left[\frac{\left(\frac{3n}{2}-1\right)\left(\frac{3n}{2}+1\right)}{\left[\left(\frac{3n}{2}+d-2\right)\left(\frac{3n}{2}+d-3\right)\right]} - \frac{2\left(\frac{3n}{2}-1\right)}{\left[\left(\frac{3n}{2}+d-2\right)\left(\frac{3n}{2}+d-3\right)\right]^{\frac{1}{2}}} + 1\right]. \qquad (5.4)$$

(iii) The risk function of $\hat{\theta}_e$, relative to squared error loss function is denoted by $R_S(\hat{\theta}_e)$ is given by

$$R_S(\hat{\theta}_e) = E_\theta\left(\hat{\theta}_e^{\ 2}\right) - 2\theta E_\theta(\hat{\theta}_e) + \theta^2. \qquad (5.5)$$

Substituting the value of $\hat{\theta}_e$ from (4.11) and evaluating various expectations in (5.5), we get

$$R_S(\hat{\theta}_e) = \theta^2\left[\frac{\left(\frac{3n}{2}-1\right)\left(\frac{3n}{2}+1\right)}{\left(\frac{3n}{2}+d-1\right)^2} - \frac{2\left(\frac{3n}{2}-1\right)}{\left(\frac{3n}{2}+d-1\right)} + 1\right]. \qquad (5.6)$$

(iv) The risk function of $\hat{\theta}_1$, relative to squared error loss function is denoted by $R_S(\hat{\theta}_S)$, is given by

$$R_S(\hat{\theta}_1) = E_\theta\left(\hat{\theta}_1^{\ 2}\right) - 2\theta E_\theta(\hat{\theta}_1) + \theta^2. \qquad (5.7)$$

Substituting the value of $\hat{\theta}_1$ from (4.13) and evaluating various expectations in (5.7) we get

$$R_S(\hat{\theta}_1) = \theta^2\left[\frac{\left(\frac{3n}{2}-1\right)\left(\frac{3n}{2}+1\right)}{\left(\frac{3n}{2}+d\right)^2} - \frac{2\left(\frac{3n}{2}-1\right)}{\left(\frac{3n}{2}+d\right)} + 1\right]. \qquad (5.8)$$

(v) The risk function of $\hat{\theta}_2$, relative to squared error loss function is denoted by $R_S(\hat{\theta}_2)$, is given by

$$R_S(\hat{\theta}_2) = E_\theta\left(\hat{\theta}_2^{\ 2}\right) - 2\theta E_\theta(\hat{\theta}_2) + \theta^2. \qquad (5.9)$$

Substituting the value of $\hat{\theta}_2$ from (4.15) and evaluating various expectations in (5.10), we get

$$R_S(\hat{\theta}_2) = \theta^2\left[\frac{\left(\frac{3n}{2}-1\right)\left(\frac{3n}{2}+1\right)}{\left(\frac{3n}{2}+d-3\right)^2} - \frac{2\left(\frac{3n}{2}-1\right)}{\left(\frac{3n}{2}+d-3\right)} + 1\right]. \qquad (5.10)$$

5. The Risk Functions Under the Precautionary Loss Function

(i) The risk function of $\hat{\theta}_S$, relative to squared error loss function is denoted by $R_S(\hat{\theta}_S)$ and accordance with (5), is given by

$$R_P(\hat{\theta}_S) = \theta^2 E_\theta\left(\frac{1}{\hat{\theta}_S}\right) - E_\theta(\hat{\theta}_S) - 2\theta. \qquad (6.1)$$

Substituting the value of $\hat{\theta}_S$ from (4.7) and evaluating various expectations in (6.1), we get

$$R_P(\hat{\theta}_S) = \theta\left[\frac{\left(\frac{3n}{2}+d-2\right)}{\left(\frac{3n}{2}-1\right)} - \frac{\left(\frac{3n}{2}\right)}{\left(\frac{3n}{2}+d-2\right)} - 2\right]. \qquad (6.2)$$

(ii) The risk function of $\hat{\theta}_p$, relative to squared error loss function is denoted by $R_S(\hat{\theta}_p)$ is given by

$$R_P(\hat{\theta}_p) = \theta^2 E_\theta\left(\frac{1}{\hat{\theta}_p}\right) - E_\theta(\hat{\theta}_p) - 2\theta. \qquad (6.3)$$

Substituting the value of $\hat{\theta}_p$ from (4.9) and evaluating various expectations in (6.3), we get

$$R_P(\hat{\theta}_p) = \theta\left[\frac{\left[\left(\frac{3n}{2}+d-2\right)\left(\frac{3n}{2}+d-3\right)\right]^{\frac{1}{2}}}{\left(\frac{3n}{2}-1\right)} - \right.$$

(3n2)[(3n2+d−2)(3n2+d−3)]12−2 . (6.4)

(iii) The risk function of $\hat{\theta}_e$, relative to squared error loss function is denoted by $R_S(\hat{\theta}_e)$ is given by

$$R_P(\hat{\theta}_e) = \theta^2 E_\theta\left(\frac{1}{\hat{\theta}_e}\right) - E_\theta(\hat{\theta}_e) - 2\theta. \qquad (6.5)$$

Substituting the value of $\hat{\theta}_e$ from (4.11) and evaluating various expectations in (6.5), we get

$$R_P(\hat{\theta}_e) = \theta\left[\frac{\left(\frac{3n}{2}+d-1\right)}{\left(\frac{3n}{2}-1\right)} - \frac{\left(\frac{3n}{2}\right)}{\left(\frac{3n}{2}+d-1\right)} - 2\right]. \qquad (6.6)$$

(iv) The risk function of $\hat{\theta}_1$, relative to squared error loss function is denoted by $R_S(\hat{\theta}_S)$, is given by

$$R_P(\hat{\theta}_1) = \theta^2 E_\theta\left(\frac{1}{\hat{\theta}_1}\right) - E_\theta(\hat{\theta}_1) - 2\theta. \qquad (6.7)$$

Substituting the value of $\hat{\theta}_1$ from (4.13) and evaluating various expectations in (6.7) we get

$$R_P(\hat{\theta}_1) = \theta\left[\frac{(\frac{3n}{2}+d)}{(\frac{3n}{2}-1)} - \frac{(\frac{3n}{2})}{(\frac{3n}{2}+d)} - 2\right] \quad (6.8)$$

(v) The risk function of $\hat{\theta}_2$, relative to squared error loss function is denoted by $R_S(\hat{\theta}_2)$, is given by

$$R_S(\hat{\theta}_2) = \theta^2 E_\theta\left(\frac{1}{\hat{\theta}_2}\right) - E_\theta(\hat{\theta}_2) - 2\theta \quad (6.9)$$

Substituting the value of $\hat{\theta}_2$ from (4.15) and evaluating various expectations in (6.9), we get

$$R_S(\hat{\theta}_2) = \theta\left[\frac{(\frac{3n}{2}+d-3)}{(\frac{3n}{2}-1)} - \frac{(\frac{3n}{2})}{(\frac{3n}{2}+d-3)} - 2\right] \quad (6.10)$$

6. Conclusion

In this paper, we have discussed Bayesian estimation of one parameter of Inverse Maxwell Distribution. It is evident from the equations (4.7), (4.9), (4.11), (4.13) and (4.15) that the MLE of $\hat{\theta}$, under squared, precautionary, entropy, and other two loss function L_1 & L_2 using quasi-prior. The Bayes estimators depend upon the parameter of the prior distribution.

In figure 1(a), 1(b), 2(a) and 2(b), we have plotted the ratio of risk functions of the Bayes estimators $\widehat{\theta_s}, \widehat{\theta_p}, \widehat{\theta_e}, \widehat{\theta_1}$ & $\widehat{\theta_2}$, as given in the equation (5.2), (5.4), (5.6), (5.8), (5.10), (6.2),(6.4), (6.6), (6.8) and (6.10) for d=0.0(0.5)5.0, θ =1 and n= 5 and 10. It is clear that neither of the estimators uniformly dominates the any other.

Table 1(a): Risk function under squared error (θ = 1, n= 5)

d	Rs	Rp	Re	R1	R2
0	0.46281	0.619225	0.307692	0.248889	0.839506
0.5	0.368056	0.468202	0.270408	0.238281	0.61
1	0.307692	0.371225	0.248889	0.235294	0.46281
1.5	0.270408	0.309533	0.238281	0.237654	0.368056
2	0.248889	0.271435	0.235294	0.243767	0.307692
2.5	0.238281	0.249409	0.237654	0.2525	0.270408
3	0.235294	0.238484	0.243767	0.263039	0.248889
3.5	0.237654	0.235296	0.2525	0.274793	0.238281
4	0.243767	0.237532	0.263039	0.287335	0.235294
4.5	0.2525	0.243569	0.274793	0.300347	0.237654
5	0.263039	0.252257	0.287335	0.3136	0.243767

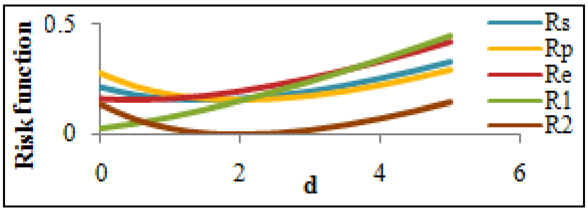


Table 1(b): Risk function (θ = 1, n= 10)

d	Rs	Rp	Re	R1	R2
0	0.171598	0.194103	0.142857	0.128889	0.222222
0.5	0.155007	0.171966	0.134364	0.125911	0.1936
1	0.142857	0.155271	0.128889	0.125	0.171598
1.5	0.134364	0.14304	0.125911	0.125803	0.155007
2	0.128889	0.134483	0.125	0.128028	0.142857
2.5	0.125911	0.128958	0.125803	0.131429	0.134364
3	0.125	0.125941	0.128028	0.135802	0.128889
3.5	0.125803	0.125	0.131429	0.140979	0.125911
4	0.128028	0.12578	0.135802	0.146814	0.125
4.5	0.131429	0.127986	0.140979	0.153189	0.125803
5	0.135802	0.131373	0.146814	0.16	0.128028

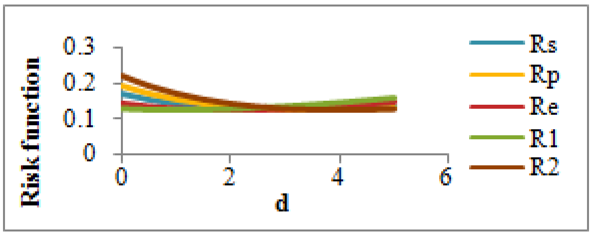


Table 2(a): Risk function under precautionary (θ = 1, n= 5)

D	Rs	Rp	Re	R1	R2
0	0.20979	0.272932	0.153846	0.020513	0.136752
0.5	0.173077	0.211956	0.148352	0.043269	0.069231
1	0.153846	0.174229	0.153846	0.072398	0.027972
1.5	0.148352	0.154312	0.168269	0.106838	0.00641
2	0.153846	0.148345	0.190045	0.145749	0
2.5	0.168269	0.153509	0.217949	0.188462	0.005495
3	0.190045	0.167699	0.251012	0.234432	0.020513
3.5	0.217949	0.189311	0.288462	0.283217	0.043269
4	0.251012	0.217099	0.32967	0.334448	0.072398
4.5	0.288462	0.250082	0.374126	0.387821	0.106838
5	0.32967	0.287477	0.421405	0.443077	0.145749

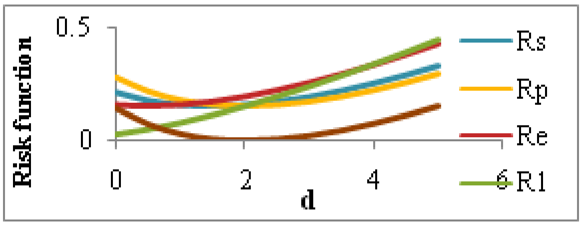
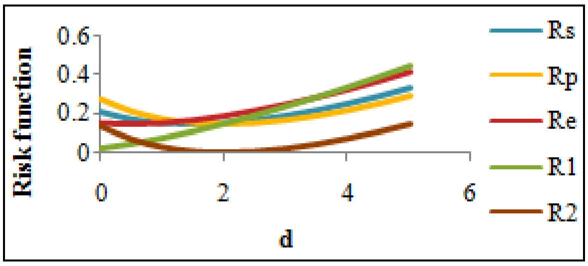


Table 2(b): Risk function (θ = 1, n= 10)

d	Rs	Rp	Re	R1	R2
0	0.082418	0.093104	0.071429	0.004762	0.02381
0.5	0.075397	0.082585	0.070197	0.010369	0.012857
1	0.071429	0.075498	0.071429	0.017857	0.005495
1.5	0.070197	0.071475	0.074885	0.027056	0.001323
2	0.071429	0.070197	0.080357	0.037815	0
2.5	0.074885	0.071389	0.087662	0.05	0.001232
3	0.080357	0.074813	0.096639	0.063492	0.004762
3.5	0.087662	0.080257	0.107143	0.078185	0.010369
4	0.096639	0.087539	0.119048	0.093985	0.017857
4.5	0.107143	0.096495	0.132239	0.110806	0.027056
5	0.119048	0.106983	0.146617	0.128571	0.037815



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