Chaotic Laser Networks

S. S. Ahmed

Department of Physics, College of Science, University of Baghdad, Baghdad, Iraq

Abstract: We report on chaotic spiking and its synchronization in semiconductor lasers with optical feedback. The effects of the bias current, optical feedback strength and external noise are first studied on the single system by means of bifurcation diagrams. Chaos synchronization is then considered in network of 256 distinct chaotic systems with independent initial conditions when a common Gaussian noise is added. The transition between non synchronization to synchronization states by means of a suitable spatio-temporal representation has been reported.

Keyword: Control, Noise, Synchronization, Optical network

1. Introduction

Synchronization phenomena are widely present in physics, chemistry, biology, social science and many other fields and have attracted much attention for years. The increasing interest in chaotic synchronization is motivated by its potential applications [1]. In particular, synchronization of chaotic oscillators in coupled nonlinear systems is an important issue since its prediction by Pecora and Carroll in 1990[2]. Chaotic optical communications has been an attractive field of research from last couple of decades because of their wide application in telecommunications [3]. One of the most surprising results of the last few decades in the field of the nonlinear dynamics is that a dynamical system and its copies can be synchronized with each other when they are linked by the common excitation only [4]. Optical chaos can be used to hide information so it can be used in privacy and security of optical communication [5]. It requires synchronization between emitter and receiver lasers to encode and decode information. The generated chaos at the emitter it used as carrier to transfer the information which can only be extracted when using the appropriate receiver. The architecture for such a system which requires that the emitter operates chaotically [6-8].

Semiconductor lasers have been widely used in nonlinear optics, optical communication and optical chaotic dynamics. These devices can be easily destabilized by the introduction of extra degrees of freedom, such as optoelectronic feedback [9] or by means of an external optical feedback [10-11].

Noise plays a constructive role in enhancing synchronization [12, 13]. Noise-enhanced synchronization may have meaningful implications in various fields. For example, the circumstance where different systems are not coupled or only weakly coupled but subjected to a common random forcing is of great relevance in life sciences, especially in neuroscience or ecology [14]. [When two non-interacting oscillators are subjected to the same fluctuating forces the trajectories of both systems can actually converge to the same values so when nonlinear oscillators are driven by common noisy signals, they can mutually synchronize this phenomenon called the noise induced synchronization. In my opinion it is a repetition]. Relevant studies on the noise induced synchronization of limit-cycle oscillators were carried out by Termae and Tanaka and by Goldobin and Pikovsky, who analytically showed, by using the phase reduction method that two identical limit-cycle oscillators driven by common weak Gaussian-white noise always synchronize with each other [15].

In this paper the synchronization in an optical network induced by Gaussian white noise of zero means is studied.

2. Model

A system of semiconductor lasers with optical feedback is modeled by delay differential equations. The Semiconductor laser is classified into class B, therefore, the polarization term is adiabatically eliminated and the effect is simply replaced by the linear relation between the field and the polarization. The population inversion for semiconductor lasers is replaced by the carrier density \( N \) produced by electron-hole recombination. The photon number (which is equivalent to the absolute square of the field amplitude) and the carrier density are frequently used as the variables of the rate equations for semiconductor lasers [16]. The dynamic of the single mode semiconductor lasers can be described by Lang-Kobayashi equations [17]. The rate equation for the evolutions of the complex amplitude electric field \( E(t) \) and the carrier number \( N(t) \) read as [18]:

\[
\frac{dE(t)}{dt} = \frac{1}{2} (1 + i\varphi)[G(t) - \gamma]E(t) + KE(t - \tau_e) \exp(-\omega t_f) \tag{1}
\]

\[
\frac{dN(t)}{dt} = \frac{I}{q} - \gamma N(t) - G(t)|E(t)|^2 \tag{2}
\]

\( G(t) \) is the optical gain that is define by:

\[
G(t) = \frac{g[N(t) - N_i]}{1 + s|E(t)|^2}
\]

where \( I \) is the injection current, \( |E(t)|^2 \) is the number of photons inside the cavity.

In the simulation a set of typical semiconductor laser parameters is considered with a line width enhancement factor \( \alpha = 1.5 \), transparency carrier number \( N_t = 1.5 \times 10^8 \), differential gain coefficient \( g = 1.5 \times 10^4 \), gain...
saturation coefficient $s=5\times10^{-7}$, photon decay rate $\gamma = 500 \text{ ns}^{-1}$, carrier decay rate $\gamma_c = 0.5 \text{ ns}^{-1}$, spontaneous emission rate $\beta = 1.6 \times 10^{-4} \text{ ns}^{-1}$, delay time $\tau = 10 \text{ ns}$, feedback strength $K=20 \text{ ns}^{-1}$, Injection strength $\sigma = 20 \text{ ns}^{-1}$. The spontaneous emission processes are considered by introducing independent Gaussian noise sources X with zero mean and unity variance into the rate equation.

Noise effects on laser dynamics

In this study, the effect of noise in dynamical systems has been concerned, the modeling of this behavior is satisfied by programming the theoretical model of the nonlinear system where the bias current and feedback strength fixed at 0.5 mA, 20ns$^{-1}$, respectively and(X) is varied. When X = 0.2mV, the laser output becomes unstable and exhibits a periodic oscillation as in figure 1a.In figure 1b several fixed peaks of frequencies appear in FFT representation. By increasing the noise level X = 0.5mV, the system turn from periodic state to chaotic state as shown in figure 2a, the decay in FFT diagram indicates that the oscillation is chaotic running (figure 2b). Figure 3 shows the bifurcation diagram provides full characterization of the response of our system with Gaussian white noise intensity. At noise equal to 0, the output form the semiconductor laser is steady. When the noise is increased to 0.2mV, a fast scale dynamics behave like a period doubling. As X is increased to 0.5mV, the dynamics of the oscillator becomes chaotic with low amplitude as shown from the bifurcation diagram. After these values the chaotic dynamic of the system increases gradually with the increasing of noise parameter.

**Figure 1 (a):** The time series for dynamical model when X = 0.2 mV, (b) The FFT for dynamical model

**Figure 2 (a):** The time series for dynamical model when X = 0.5 mV, (b) The FFT for dynamical model

**Figure 3:** The bifurcation diagram, feedback strength as a control parameter

**Figure 4:** Time series of $2^8$ oscillators output for (a) non-synchronization, (b) Synchronization conditions.

**Optical Networks**

In the following, Synchronization phenomena in such optical network have been reported and characterized. Numerical results show that the synchronization of 256 chaotic semiconductor lasers is possible using Gaussian noise to couple the system. Figure 4 shows the time series of $2^8$ oscillators (a) the oscillators are running independently(coupling strength X=0), (b) the spiking come at the same time sequences(coupling strength X=2.8).
To describe quantitatively the abrupt change, the degree of order in the system was characterized by mean of spatiotemporal distribution. In this part, figure 5 shows the Space-Time representation of semiconductor lasers intensities. When the coupling is zero, the time of peaks for each oscillator are different from the other as shown in figure 5a, when the coupling is 2.8, the birth of peaks for fixed time difference is observed due to the time correlation between spikes at adjacent sites as shown in figure 5b.

**Figure 5: Spatio-Temporal distribution for (a) non-synchronization, (b) synchronization condition**

### 3. Conclusions

To conclude, a delayed optical feedback is employed to obtain chaotic output from semiconductor laser. Studying the effect of DC bias current, the optical feedback strength and the external noise on chaotic behavior give several aspects of the dynamic response of excitable systems for different values of these parameters, where by changing these parameters the system turn from steady state to unstable periodic state and then to chaotic state and from the bifurcation diagrams we noticed that the system could be controlled by these parameters.

The synchronization in chaotic optical network is possible when add a common Gaussian noise to the optical network that work with different and independent initial condition and leads to perfect synchronization as a consequence of a noise induced change in time scale.

### Acknowledgment

S. S. Ahmed acknowledges Banaz O. Rasheed for her helpful notes.

### References


Volume 6 Issue 6, June 2017

