# On Normality in Soft Topological Spaces

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**Abstract:** The concept Soft Sets was introduced by Molodtsov in the year 1999 to deal with problems of incomplete information. Due to the parameterization tools the used soft sets enhanced applicability of various generalizations of crisp sets. Soft topological space was formulated by Shabir et. al. and Cagman et al. separately in 2011. In this paper we study normality and related problems in soft topological spaces. We prove a form of Urysohn Lemma in this setting.

Keywords: Soft point, soft neighborhood, soft continuity, soft topology, soft normality

#### 1. Introduction

In 1999, Molodtsov [6] proposed a new concept called soft set theory to model uncertainty, which associates a set with a set of parameters. The soft set theory has been applied to many different fields. In 2010 M. Shabir, Munazza, Naz [4] used soft sets to define a topology namely Soft topology and studied soft neighborhoods of a point, soft separation axioms and their basic properties. The same concept was introduced and studied independently by Cagman et. al. [3] independent ally in 2011. In [1], Aygunoglu- Aygun introduced the soft product topology and defined the version of compactness in soft spaces named soft compactness.

#### 2. Prelimanaries

In this section, we give some basic definitions and results of soft set theory .

In this paper, U refers to an initial universe, E is a set of parameters, P(U) is the power set of U, and  $A \subseteq E$ .

**Definition 2.1.** A soft set  $F_A$  on the universe U is defined by the set of ordered pairs

 $F_A = \{ (x, f_A(x)) : x \in E, f_A(x) \in P(U) \},\$ where  $f_A : E \to P(U)$  such that  $f_A(x) = \emptyset$  if  $x \notin A$ 

Here,  $f_A$  is called an approximate function of the soft set  $F_A$ . The value of  $f_A(x)$  may be arbitrary. Some of them may be empty, some may have nonempty intersection. Note that the set of all soft sets over U will be denoted by S(U).

**Example 2.1.** Let a soft set  $F_A$  describe the attractiveness of the shirts with respect to the parameters, which Mr. A is going to wear. Suppose that there are four shirts in the universe  $U = \{x_1, x_2, x_3, x_4\}$  under consideration and  $E = \{e_1 = cheap, e_2 = warm, e_3 = colorful\}$  is the set of parameters. To define a soft set means to point out cheap shirts, expensive shirts and colorful shirts. Suppose that  $f_A(e_1) = \{x_1, x_2\}, f_A(e_2) = \{x_3, x_4\}, f_A(e_3) = \{x_1, x_3, x_4\}$ . Then the family  $\{A(e_i) : i = 1, 2, 3\}$  of  $2^X$  is a soft set  $F_A$ .

For two soft sets  $F_A$  and  $G_B$  over common universe U, we say that  $F_A$  is a *soft subset*  $G_B$  if  $A \subseteq B$  and  $F(e) \subseteq G(e)$ , for all  $e \in A$ . In this case, we write  $F_A \subseteq G_B$  and  $G_B$  is said to be a *soft superset* of  $F_A$ . Two soft sets  $F_A$  and  $G_B$  over a

common universe U are said to be *soft equal* if  $F_A \subseteq G_B$  and  $G_B \subseteq F_A$ 

A soft set  $F_A$  over U is called a *null soft set*, denoted by  $\Phi_A$ , if for each  $\in A$ ,  $F(e) = \phi$ ; Similarly, it is called *absolute soft set*, denoted by  $\widetilde{U}$ , if for each  $e \in A$ , F(e) = U.

The *union* of two soft sets  $F_A$  and  $G_B$  over the common universe *U* is the soft set  $H_C$ , where  $C = A \cup B$  and for each  $e \in C$ 

$$H(e) = \begin{cases} F(e) & for \ e \in A \\ G(e) & for \ e \in B \\ F(e) \cup G(e) & for \ e \in A \cap B \end{cases}$$

We write  $F_A \cup G_B = H_C$ . Moreover, the *intersection*  $H_C$  of two soft sets  $F_A$  and  $G_B$  over a common universe U, denoted by  $F_A \cap G_B$ , is defined as  $C = A \cap B$  and  $H(e) = F(e) \cap G(e)$  for each  $e \in C$ .

The difference  $H_E$  of two soft sets  $F_E$  and  $G_E$  over X, denoted by  $F_E \setminus G_E$ , is defined as  $H(e) = F(e) \setminus G(e)$ , for each  $e \in E$ .

Let Y be a nonempty subset of U. Then  $\tilde{Y}$  denote the soft set  $Y_E$  over U where (e) = Y, for each  $e \in E$ . In particular,  $U_E$  will be denoted by  $\tilde{U}$ .

Let  $F_E$  be a soft set over U and  $x \in U$ . We say that  $x \in F_E$ , whenever  $x \in F(e)$ , for each  $e \in E$ .

The relative complement of a soft set  $F_A$  is denoted by  $F_A^c$ and is defined as a mapping given  $F^c: A \to \mathbb{P}(U)$  by  $F^c(e) = U \setminus F(e)$ , for each  $e \in A$ .

**Definition2.2**: Let  $\mathcal{T}$  be the collection of soft sets over U. Then  $\mathcal{T}$  is called a soft topology on U if  $\mathcal{T}$  satisfies the following axioms: (i)  $\Phi_E, \widetilde{U}$  belong to  $\mathcal{T}$ . (ii) The union of any number of soft sets in  $\mathcal{T}$  belong to  $\mathcal{T}$ . (iii) The intersection of any two soft sets in  $\mathcal{T}$  belong to  $\mathcal{T}$ .

The triple  $(U; \mathcal{T}; E)$  is called a soft topological space over *X*. The member of  $\mathcal{T}$  are said to be soft open in *X*, and the soft set  $F_E$  is called soft closed in *X* if its relative component  $F_E^{\ C}$  belongs to  $\mathcal{T}$ .

Let SS(U)E be the collection of all soft sets with set of  $M_{OV} = 5$  Mov 2017

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parameter *E*, over *U*. The Cartesian product of soft sets  $F_A \in SS(U)A$  and  $G_B \in SS(V)B$  is a soft set  $F \times G_{A \times B}$  in  $SS(U \times V)A \times B$  where

 $F \times G : A \times B \rightarrow P(U) \times P(V)$  is a mapping given by  $(F \times G)(a; b) = F(a) \times G(b)$  for each  $(a; b) \in A \times B$ .

**Definition2.3:** The soft set  $F_A \in SS(U)_A$  is called a soft point in  $U_A$ , denoted by  $e_F$ , if for the element  $e \in A, F(e) \neq \varphi$ and  $F(e') = \varphi$ , for all  $e \in A - \{e\}$ .

**Definition2.4:** The soft point  $e_F$  is said to be in the soft set  $G_A$ , denoted by  $e_F \in G_A$ , if for the element  $e \in A, F(e) \subseteq G(e)$ .

**Definition2.5:** Let  $(U; \tau; A)$  be a soft topological space over U and  $F_A$  a soft set in  $SS(U)_A$ . The soft point  $e_F \in U_A$  is called a soft interior point of a soft set  $F_A$ , if there exists a soft open set  $H_A$  such that  $e_F \in H_A \subseteq F_A$ . The soft interior of a soft set  $F_A$  is denoted by  $(F_A)^0$  and is defined as the union of all soft open sets contained in  $F_A$ . Clearly  $(F_A)^0$  is the largest soft open set contained in  $F_A$ .

**Definition2.6:** Let  $(U; \tau; A)$  be a soft topological space. Then a soft set  $G_A$  in  $SS(U)_A$  is called a soft neighborhood of the soft point  $e_F \in U_A$ , if there exists a soft open set  $H_A$  such that  $e_F \in H_A \subseteq G_A$ .

The soft neighborhood system of a soft point  $e_F$ , denoted by  $N_\tau(e_F)$ , is the family of all its soft neighborhoods.

**Definition2.7:** Let  $(U; \tau; A)$  be a soft topological space over U and  $F_A$  a soft set over U. Then the soft closure of  $F_A$ , denoted by  $\overline{F_A}$ , is the intersection of all soft closed supersets of  $F_A$ . Clearly  $\overline{F_A}$ , is the smallest soft closed set in  $(U; \tau; A)$  which contains  $F_A$ .

**Definition2.8:** Let  $(U; \tau; A)$  be a soft topological space over U and  $Y \subseteq U$ . Then  $\tau_Y = \{(F_Y; A) = Y_A \cap F_A | F_A \in \tau\}$  is said to be the soft relative topology on Y, where  $F_Y(e) = Y \cap F(e)$ , for all  $e \in A$ .  $(Y; \tau_Y; A)$  is called a soft subspace of  $(U; \tau; A)$ .

We can easily verify that  $\tau_Y$  is, in fact, a soft topology on Y.

**Theorem2. 1:**Let  $(Y; \tau_Y; A)$  be a soft subspace of a soft topological space  $(U; \tau; A)$  and  $F_A$  a soft set over U. Then (1)  $F_A$  is soft open in  $(Y; \tau Y; A)$  if and only if  $F_A = Y_A \cap G_A$ , for some soft open set  $G_A$  in  $(U; \tau; A)$ . (2)  $F_A$  is soft closed in  $(Y; \tau_Y; A)$  if and only if  $F_A = Y_A \cap G_A$ , for some soft closed set  $G_A$  in  $(U; \tau; A)$ .

**Proposition2.1:** Soft set  $F_A$  over U is soft open  $\Leftrightarrow F_A$  is a soft nbd of each of its soft elements

**Proposition2.2:** Let  $F_A, G_A \in SS(U)_A$ . Then following are true

(1) If  $F_A \cap G_A = \emptyset_A$  then  $F_A \nsubseteq (G_A)^c$ (2)  $F_A \subseteq G_A \Leftrightarrow (G_A)^c \subseteq (F_A)^c$ 

**Theorem2.2** : Let  $(U, A, \tau)$  be a soft space over U. Let  $F_A$  and  $G_A$  are soft sets over U. Then

(1)  $F_A \subseteq \overline{F_A}$ (2)  $F_A$  is a soft closed set

(2)  $F_A$  is a soft closed set  $\Leftrightarrow F_A = \overline{F_A}$ 

$$(3) F_A \subset G_A \implies \overline{F_A} \subset \overline{G_A}$$

**Definition2.9:** Let (U, A,  $\tau$ ) and (V, A,  $\tau$ ') be two soft topological spaces. f :( U, A,  $\tau$ )  $\rightarrow$  (V, A,  $\tau$ ') be a mapping. For each soft neighborhood  $H_A$  of f  $(e_X)_A$ , if there exists a soft neighborhood  $F_A$  of  $(e_X)_A$  such that f  $(F_A) \subset H_A$ . Then f is said to be soft continuous mapping at  $(e_X)_A$ . If f is soft continuous mapping for all  $(e_X)_A$  then f is called *soft continuous mapping*.

**Theorem 2.3**: Let  $(U, A, \tau)$  and  $(V, A, \tau')$  be two soft topological spaces.

 $f: (U, A, \tau) \rightarrow (V, A, \tau')$  be a mapping. Then following conditions are equivalent:

(1)  $f : (U, A, \tau) \rightarrow (V, A, \tau')$  is a soft continuous mapping (2) For each soft open set  $G_A$  over V,  $f^{-1}(G_A)$  is a soft open set over U.

**Proposition2.3**: Let U, V be two non-empty sets and f:  $U \rightarrow V$  be a mapping.

If  $F_A \in S(U)$  then (i)  $F_A \subseteq f^{-1} f(F_A)$ (ii)  $f^{-1} f(F_A) = F_A$  if f is injective.

**Proposition 2.4:** Let U, V be two non-empty subsets and f: U  $\rightarrow$  V be a mapping. If  $G_{1_A}, G_{2_A} \in S(V)$  then (i)  $G_{1_A} \subseteq G_{2_A} \Longrightarrow f^{-1}(G_{1_A}) \subseteq f^{-1}(G_{2_A})$ (ii)  $f^{-1}[G_{1_A} \cup G_{2_A}] = f^{-1}(G_{1_A}) \cup f^{-1}(G_{2_A})$ (iii)  $f^{-1}[(G_{1_A}) \cap (G_{2_A})] = f^{-1}(G_{1_A}) \cap f(G_{2_A})$ 

#### 3. Soft Normal Spaces

In this section we define soft regular spaces, soft normal spaces and discuss their properties and relationship with other Ti spaces.

**Definition3.1:** Two soft sets  $G_A$ ,  $H_A$  in  $SS(U)_A$  are said to be soft disjoint, written  $G_A \cap H_A = \Phi_A$ , if  $G(e) \cap H(e) = \varphi$ , for all  $e \in A$ 

**Definition3.2:** Two soft points  $e_G$ ,  $e_H$  in  $U_A$  are distinct, written  $e_G \neq e_H$ , if there corresponding soft sets  $G_A$  and  $H_A$  are disjoint.

**Definition3.3:** Let  $(U; \tau; A)$  be a soft topological space over U and  $e_G$ ;  $e_H \in U_A$  such that  $e_G \neq e_H$ . If there exist at least one soft open set  $F1_A$  or  $F2_A$  such that  $e_G \in F1_A$ ,  $e_H \notin F1_A$  or  $e_H \in F2_A$ ,  $e_H \notin F2_A$ , then  $(U; \tau; A)$  is called a soft  $T_0$ -space.

**Definition3.4:** Let  $(U; \tau; A)$  be a soft topological space over U and  $e_G$ ;  $e_H \in U_A$  such that  $e_G \neq e_H$ . If there exist soft open sets  $F1_A$  and  $F2_A$  such that  $e_G \in F1_A$ ,  $e_H \notin F1_A$  and  $e_H \in F2_A$ ,  $e_G \notin F2_A$ , then  $(U; \tau; A)$  is called a soft  $T_1$  space.

**Definition3.5:** Let  $(U; \tau; A)$  be a soft topological space over U and  $e_G$ ;  $e_H \in U_A$  such that  $e_G \neq e_H$ . If there exist soft open sets  $F1_A$  and  $F2_A$  such that  $e_G \in F1_A, e_H \in F2_A$  and  $F1_A \cap F2_A = \Phi_A$ , then  $(U; \tau; A)$  is called a soft  $T_2$  -space.

Now we define soft regular space as:

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**Definition3.6:** Let  $(U; \tau; A)$  be a soft topological space over *X*, *G*<sub>A</sub> a soft closed set

in  $(U; \tau; A)$  and  $e_F \in X_A$  such that  $e_F \notin G_A$ . If there exist soft open sets  $F1_A$  and  $F2_A$  such that  $e_F \in F1_A$ ,  $G_A \subseteq F2_A$ and  $F1_A \cap F2_A = \Phi_A$ , then  $(U; \tau; A)$  is called a soft regular space.

**Theorem3.1:** Let  $(U; \tau; A)$  be a soft topological space over U. Then the following statements are equivalent:

(1) (*U*;  $\tau$ ; *A*) is soft regular.

(2) For any soft open set  $F_A$  in  $(U; \tau; A)$  and  $e_G \in F_A$ , there is a soft open set  $G_A$  containing  $e_G$  such that  $e_G \in \overline{G_A} \subseteq F_A$  (3) Each soft point in  $(U; \tau; A)$  has a soft nbd base consisting of soft closed sets.

**Proof:** ((1)  $\Rightarrow$  (2)) Let  $F_A$  be a soft open set in (U;  $\tau$ ; A) and  $e_G \in F_A$ . Then  $(F_A)^c$  is a soft closed set such that  $e_G \notin (F_A)^c$  By the soft regularity of (U;  $\tau$ ; A), there are soft open sets  $F1_A, F2_A$  such that  $e_G \in F1_A, (F_A)^c \subseteq F2_A$  and  $F1_A \cap F2_A = \Phi_A$ . Clearly  $(F2_A)^c$  is a soft closed set contained in  $F_A$ . Thus  $F1_A \subseteq (F2_A)^c \subseteq F_A$ . This gives  $\overline{F1_A} \subseteq (F2_A)^c \subseteq F_A$ . Put  $F1_A = G_A$ . Consequently,  $e_G \in G_A$  and  $G_A \subseteq F_A$ .

 $((2) \Rightarrow (3))$  Let  $e_G \in U_A$ . For soft open set  $F_A$  in  $(U; \tau; A)$ , there is a soft open set  $G_A$  containing  $e_G$  such that  $e_G \in G_A$ ,  $\overline{G_A} \subseteq F_A$ . Thus for each  $e_G \in X_A$ , the sets  $G_A$  form a soft nbd base consisting of soft closed sets of  $(U; \tau; A)$ .

 $((3) \Rightarrow (1))$  Let  $F_A$  be a soft closed set such that  $e_G \notin F_A$ . Then  $(F_A)^c$  is a soft open nbd of  $e_G$ . By (3), there is a soft closed set  $F1_A$  which contains  $e_G$  and is a soft nbd of  $e_G$  with  $1_A \subseteq (F_A)^c$ . Then  $e_G \notin (F1_A)^c$ ,  $F_A \subseteq (F1_A)^c = F2_A$  and  $F1_A \cap F2_A = \Phi_A$ . Therefore  $(X; \tau; A)$  is soft regular.

**Theorem3.2:** Let  $(U; \tau; A)$  be a soft regular space over U. Then every soft subspace of  $(U; \tau; A)$  is soft regular.

**Proof**. Let  $(Y; \tau_Y; A)$  be a soft subspace of a soft regular space  $(U;\tau; A)$ . Suppose  $H_A$  is a soft closed set in  $(Y; \tau_Y; A)$  and  $e_F \in Y_A$  such that  $e_F \notin H_A$ . Then  $H_A = G_A \cap Y_A$ , where  $G_A$  is soft closed in  $(U; \tau; A)$ . Then  $e_F \notin G_A$ . Since  $(U; \tau; A)$  is soft regular, there exist soft disjoint soft open sets  $F1_A$ ,  $F2_A$  in  $(U; \tau; A)$  such that  $e_F \in F1_A$ ,  $G_A \subseteq F2_A$ . Clearly  $e_F \in F1_A \cap Y_A = F1Y_A$  and  $H_A \subseteq F2_A \cap Y_A = F2Y_A$  such that  $F1Y_A \cap F2Y_A = \Phi_A$ . This proves that  $(Y; \tau_Y; A)$  is a soft regular subspace of  $(U; \tau; A)$ .

**Definition3.7:** Let  $(U; \tau; A)$  be a soft topological space over U,  $F_A$  and  $G_A$  soft closed sets over U such that  $F_A \cap G_A = \phi_A$ . If there exist soft open sets  $F1_A$  and  $F2_A$  such that  $F_A \subset F1_A$ ,  $G_A \subset F2_A$  and  $F1_A \cap F2_A = \phi_A$ , then  $(U; \tau; A)$  is called a soft normal space.

**Theorem3.3** : A soft topological space  $(U; \tau; A)$  is soft normal if and only if for any soft closed set  $F_A$  and soft open set  $G_A$  such that  $F_A \subset G_A$ , there exists at least one soft open set  $H_A$  containing  $F_A$  such that  $F_A \subset H_A \subset \overline{H_A} \subset G_A$ .

**Proof.** Suppose that  $(U; \tau; A)$  is a soft normal space and  $F_A$  is any soft closed subset of  $(U; \tau; A)$  and  $G_A$  a soft open set such that  $F_A \subset G_A$ . Then  $(G_A)^c$  is soft closed,  $F_A \cap (G_A)^c = \phi_A$ 

By supposition, there are soft open sets  $H_A$  and  $K_A$  such that  $F_A \subset H_A$ ,  $(G_A)^c \subset K_A$  and  $H_A \cap K_A = \phi_A$ . Since  $H_A \cap K_A = \phi_A$ ,  $H_A \subset (K_A)^c$ . But  $(K_A)^c$  is soft closed, so that  $F_A \subset H_A \subset \overline{H_A} \subset (K_A)^c \subset G_A$ . Hence  $F_A \subset H_A \subset \overline{H_A} \subset G_A$ . . Conversely, suppose that for every soft closed set  $F_A$  and a soft open set  $G_A$  such that  $F_A \subset G_A$ , there is a soft open set  $H_A$ such that  $F_A \subset H_A \subset \overline{H_A} \subset \overline{G_A}$ . Let  $F1_A, F2_A$  be any two soft disjoint soft closed sets. Then  $F1_A \subset (F2_A)^c$ , where  $(F2_A)^c$ is soft open. Hence there is a soft open set  $H_A$  such that  $F1_A \subset H_A \subset \overline{H_A} \subset (F2_A)^c$ . But then  $F2_A \subset (H_A)^c$  and  $(H_A) \cap (H_A)^c = \phi$ .

So,  $F1_A \subset H_A, F2_A \subset (H_A)^c$  with  $H_A \cap (H_A)^c = \phi_A$ . Hence  $(U; \tau; A)$  is soft normal.

**Proposition3.1:** Let  $(Y; \tau_Y; A)$  be a soft subspace of a soft topological space  $(U; \tau; A)$  and  $F_A$  be a soft open (closed) in  $(Y; \tau_Y; A)$ . If  $Y_A$  is soft open(closed) in  $(U; \tau; A)$ , then  $F_A$  is soft open(closed) in  $(U; \tau; A)$ , then  $F_A$  is soft open(closed) in  $(U \tau; A)$ .

**Theorem3.4:** A soft closed subspace of a soft normal space is soft normal

**Proof.** Let  $(Y; \tau_Y; A)$  be soft subspace of soft normal space  $(U; \tau; A)$  such that  $Y_A \in \tau^c$ .

Let  $F1_A$ ,  $F2_A$  be two disjoint soft closed subsets of  $(Y; \tau_Y; A)$ . Then there exists soft closed sets  $F_A$ ,  $G_A$  in  $(X; \tau; A)$  such that  $F1_A = Y_A \cap F_A$  and  $F2_A = Y_A \cap G_A$ . Since  $Y_A$  is soft closed in  $(U; \tau; A)$ , therefore  $F1_A, F2_A$  are soft disjoint soft closed in $(U; \tau; A)$ . Then  $(U; \tau; A)$  is soft normal implies that there exist soft open sets  $F3_A, F4_A$  in  $(U; \tau; A)$  such that  $F1_A \subset F3_A, F2_A \subset F4_A$  and  $F3_A \cap F4_A = \phi_A$ . But then  $F1_A \subset Y_A \cap F3_A, F2_A \subset Y_A \cap F4_A$ , where  $Y_A \cap F3_A, Y_A \cap F4_A$  are soft disjoint soft open subsets of  $(Y; \tau_Y; A)$ . This proves that  $(Y; \tau_Y; A)$  is soft normal.

# 4. The Urysohn Lemma

In general topology Urysohn lemma states that in every normal topological space two disjoint closed subsets may be separated by a real-valued function. Here we prove this lemma is true in soft topological spaces.

**Theorem4.1 (Urysohn's lemma):** Let  $(U; \tau; A)$  be a soft normal space; let  $F_A$  and  $G_A$  be disjoint soft closed subsets of  $(U; \tau; A)$ . Then there exists a soft continuous map  $f: (U; \tau; A) \rightarrow [0, 1]$  such that:  $f(e_x) = 0$  for every  $x \in F_A$  and  $f(e_x) = 1$  for every  $x \in G_A$ .

**Proof:** Let Q be the set of rational numbers in [0, 1]. Clearly the set Q is countable. Define, for each  $p \in Q$  a soft open set  $Up_A$  of the soft normal space(U;  $\tau$ ; A). such that if p, q $\in$ Q with p< q then  $\overline{Up_A} \subseteq Uq_A$ .-----(1)

Construct a sequence of soft open sets in  $(U; \tau; A)$  as follows.

First define  $U1_A = (U; \tau; A) - G_A$ . Here  $F_A$  is a soft closed set contained in the soft open set  $U1_A$ . Using soft normality of  $(U; \tau; A)$  and by theorem there must exist a soft open set which contains the soft closed set  $F_A$  and its closure is contained in  $U1_A$ . Let this soft open set be  $U0_A$ . In general let Qn denote the set consisting of the first n rational

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numbers in the sequence. Define  $Ur_A$  where r is the next rational number in the sequence.

Consider the set  $Q_{n+1} = Q_n \cup \{r\}$ . It is a finite subset of the interval [0, 1]. In a finite simply ordered set every element has an immediate predecessor and an immediate successor. Let the immediate predecessor of r be p and immediate successor of r be q. The sets  $Up_A$  and  $Uq_A$  are already defined and by induction hypothesis  $\overline{Up_A} \subseteq Uq_A$ 

Here  $Uq_A$  is a soft open set containing a soft closed set  $(\overline{Up_A})$ . Using theorem there must exist a soft open set containing  $(\overline{Up_A})$  and its closure is contained in  $(Up_A)$ .

Let this soft open set be  $(Ur_A)$ .

 $\therefore (\overline{Up_A}) \subseteq Ur_A \subseteq \overline{Ur_A} \subseteq Uq_A$  It can be concluded that (1) holds for every pair of elements  $Q_{n+1}$ . If both the elements lie in  $Q_n$  then (1) holds by induction hypothesis.

Let r, s be such a pair from Qn. Then either  $s \leq p$  or  $s \geq q$ . So it will lead to

 $(\overline{Us_A}) \subseteq \overline{Up_A} \subseteq Ur_A$  and  $(\overline{Ur_A}) \subseteq Uq_A \subseteq Us_A$  respectively.

The relation (1) is still true for any pair of rational numbers with p < q.

Let  $e_x \in (U; \tau; A)$ . Define  $L(e_x) = \{p | e_x \in Up_A\}$ . From (2),  $L(e_x) = \emptyset$ ; p < 0 and  $= Z_+ - \{1\}$ ; p > 1So,  $L(e_x)$  is bounded below and its g.l.b. is a point in [0, 1] say  $f(e_x)$ . We consider two cases

Case (i):  $e_x \in F_A$  then  $e_x \in Up_A$  for every  $p \ge 0 \implies$   $L(e_x) = \{p | p \in Z_+ \cup \{0\}\} \implies f(e_x) = 0.$ Case (ii):  $e_x \in G_B$  then  $e_x \notin Up_A$ ,  $p \le 1$ ;  $\implies e_x \notin U_A$ ,  $p > 1 \implies L(e_x) = \{p/p \in Z_+ - \{1\}\} \implies f(e_x) = 1.$ 

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