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From Concrete Manipulatives to Digital Manipulatives: A Paradigm Shift in Mathematics Education

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Abstract: ***The use of manipulatives has a long tradition in the teaching of Mathematics. Manipulatives have evolved over the years with the rapid advancements in the field of technology. The paper discusses the evolution of manipulatives and their importance in Mathematics education with the support of research findings. The cognitive theories which form the basis for use of digital manipulatives are also discussed.***

Keywords: Virtual Manipulatives, Digital Manipulatives, Embodied Cognition, Distributed Cognition, Dual Coding Theory

1. Introduction

An effective mode of pedagogy is expected to walk a student through a series of learning experiences in the process of developing understanding and subsequently to address her/his questions about the same. This mode of pedagogy would also be more effective, if the experiences provided to the student are multi-sensory, i.e. the student perceive the object of study through multiple senses viz. sight, touch, smell, sound etc. Manipulatives are a broader class of learning tools that facilitate this multi-sensory approach to learning. They are objects - physical or virtual (computer based) - that can be interacted with, by the student and teacher, in the process of learning. The use of manipulative materials in the teaching of mathematics has a long tradition. The basis for the use of manipulatives can be traced back to the learning theories put forward by Piaget, Dewey, Bruner and Dienes (Post, 1981). Jean Piaget suggested that concepts are formed by children through a reconstruction of reality, not through an imitation of it; John Dewey argued for the provision of firsthand experiences in a child's educational program; Jerome Bruner indicated that knowing is a process, not a product; and Zoltan Dienes, whose work specifically relates to mathematics instruction, suggested that children need to build or construct their own concepts from within rather than having those concepts imposed upon them.

2. Manipulatives in Mathematics Education

Manipulatives play an important role in the teaching of mathematics. Heddens (2005) argued that using manipulative materials in teaching mathematics will help students learn:

- To relate real world situations to mathematics symbolism.
- To work together cooperatively in solving problems.
- To discuss mathematical ideas and concepts.
- To verbalize their mathematics thinking.
- to make presentations in front of a large group.
- That there are many different ways to solve problems.
- that mathematics problems can be symbolized in many different ways.
- That they can solve mathematics problems without just following teachers' directions.

Further, Laski, Jordan, Daoust and Murray (2015), on synthesizing the findings relevant to the use of manipulatives in early childhood math instruction, arrived at the following four general principles for maximising the effective use of mathematical manipulatives: (a) use a manipulative consistently, over a long period of time; (b) begin with highly transparent concrete representations and move to more abstract representations over time; (c) avoid manipulatives that resemble everyday objects or have distracting irrelevant features; and (d) explicitly explain the relation between the manipulatives and the math concept.

3. Evolution of Manipulatives

With the ubiquitous and rapid evolution of information technology, manipulatives have evolved considerably over the years – starting with concrete manipulatives, progressing into virtual manipulatives and further into digital manipulatives. The following sections deal with each form of manipulative in detail and give research findings on their importance in the development of mathematical concepts.

a. Concrete Manipulatives

Concrete manipulatives generally refer to physical objects that are used as teaching tools to engage students in the hands-on learning of mathematics. Some of the manipulatives that are commonly used at the elementary level are Counters, Base-10 blocks, fraction strips etc. Clements (1999) has pointed out the inappropriateness of using the word 'concrete' for this class of manipulatives. . By 'concrete', many refer to physical objects that the students can hold. But concrete may not equal physical. Physical manipulation with objects need not always give a concrete learning experience to the students. But the word concrete has come to be so accepted in terms of the real learning experiences provided by them.

A large number of research studies have focused on the effectiveness of using concrete manipulatives in mathematics instruction. Meta-analytic studies have been carried out by researchers during the past decades. Suydam and Higgins (1977) conducted studies on the use of manipulative materials in grades K-8 and reported that lessons-using manipulative materials have a higher probability of producing greater mathematical achievement than do-non-manipulative lessons. The use of materials was found to be effective with children at all achievement levels, ability levels, and socioeconomic levels. Sowell (1989) combined the results of 60 studies to determine the effectiveness of mathematics instruction using manipulative materials. Students ranged in age from kindergarteners to college-age adults and studied a variety of mathematics topics. Results indicated that the long-term use of concrete manipulative materials resulted in an increase in academic achievement. Treatments of shorter duration did not produce statistically significant results. Also, instruction with pictures and diagrams did not appear to differ in effectiveness from instruction with symbols.

b. Virtual Manipulatives

Virtual manipulatives are modelled after existing concrete manipulatives such as base ten blocks, coins, blocks, rulers, fraction bars, algebra tiles, geoboards, geometric plane, and solids figures.

Moyer, Bolyard and Spikell (2002) defined a virtual manipulative as "an interactive, web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge". This definition has been widely used in the educational community and has been referenced and cited by researchers. But with the introduction of new technologies, there arose a need to revisit the definition of virtual manipulative and hence the definition of a virtual manipulative was updated by Moyer and Bolyard (2016) as "an interactive, technology-enabled visual representation of a dynamic mathematical object, including all of the programmable features that allow it to be manipulated, that presents opportunities for constructing mathematical knowledge". This updated definition preserves the term "interactive" in the definition because this is a defining characteristic of a virtual manipulative. The updated definition takes into account that all virtual manipulatives do not have to be "web-based", and replaces this terminology with the term "technology-enabled". The updated definition also preserves the terms "visual representation of a dynamic object" and adds the term "mathematical" to clarify that we are referring to a representation of a mathematical object. Further, it clarifies that the visual representation of a dynamic object is accompanied by all of its programmable features, because without these features it would not be interactive and dynamic. Implied in this updated definition is that a virtual manipulative may: (a) appear in many different technology-enabled environments; (b) be created in any programming language; and (c) be delivered via any technology-enabled device.

Virtual manipulatives are usually in the form of Java or Flash applets. These tools can be especially helpful for all students, including those with disabilities because they can improve their understanding of the abstract symbolic language of mathematics. Students who struggle in mathematics often find it hard to connect visual and symbolic representations, but virtual manipulatives can help make these connections clear. In addition, virtual objects can be altered in ways that concrete ones cannot (for example, the size, shape, and colour of a block can be changed). In many cases, this enables students to create more examples than they could with physical objects. Many virtual manipulatives also have the added benefit of providing students with hints and feedback, allowing them to practice on their own without teacher assistance. Apps aiding the use of Virtual manipulatives are also available online. The National Library of Virtual Manipulatives (NLVM), an NSF supported project that began in 1999 at Utah State University has developed a library of uniquely interactive, web-based virtual manipulatives or concept tutorials, mostly in the form of Java applets, for mathematics instruction.

Moyer and Westenskow (2013) conducted a meta-analysis that synthesized the findings from 66 research reports examining the effects of virtual manipulatives on student achievement. Of the sixty six reports, thirty two contained data yielding eighty two effect size scores with effects of virtual manipulatives on student achievement. The sixty six reports also contributed to a conceptual analysis of affordances that promote mathematical learning. The results of the averaged effect size scores yielded a moderate effect for virtual manipulatives compared with other instructional treatments. There were additional large, moderate, and small effects when virtual manipulatives were compared with physical manipulatives and textbook instruction, and when the effects were examined by mathematical domains, grade levels, and study duration.

c. Digital Manipulatives

Another major development that has happened in the field is the evolution of digital manipulative, where a computational / communication element is embedded in a concrete physical manipulative through electronic means, to make the physical manipulative digitally interactable. This can be used further to enhance the learning experience by combining the best of both worlds – concrete and virtual.

The position statement issued by the National Council of Supervisors of Mathematics (NCSM) in 2013 states that "in order to develop every student's mathematical proficiency, leaders and teachers must systematically integrate the use of concrete and virtual manipulatives into classroom instruction at all grade levels." The statement further adds that though virtual manipulatives are important tools for teacher modelling and demonstration, they do not replace the power of physical objects in the hands of learners. Digital manipulatives attempt to address this scenario by providing a physical object that the student/teacher can interact with to provide a sensory experience, but also contains a computational / communication element that could enable a multi-sensory interaction with the object. Physical objects could also be interfaced to virtual worlds to obtain the best of concrete and virtual manipulatives.

The following table gives the comparison between the key perceptual properties of manipulatives (e.g., colour); how this information tends to be perceived (visual/tactile/audio); and whether it can be adapted (stable/adaptable).

	Perceptual property	Form of manipulative		
		Physical	Virtual	Digital
Visuo-spatial properties of objects	Shape	Visual/ Tactile Stable	Visual (2D) Adaptable	Visual/Tactile Stable
	Size	Visual/ Tactile Stable	Visual Adaptable	Visual/Tactile Stable
	Colour	Visual Stable	Visual Adaptable	Visual Adaptable
	Spatial arrangement	Visual/ Tactile Adaptable	Visual Adaptable(single point interaction, 2D screen space)	Adaptable (multipoint interaction, 3D screen space)
Additonal information	e.g. symbols, text, images	Visual (tactile if inscribed) Stable	Visual/Audio Adaptable	Visual/Audio Adaptable (typically linked to spatial manipulation)

Source: Manches (2011)

To construct digital manipulatives, it is important to embed computational / communicational elements into concrete manipulatives. The electronics industry is seeing a large revolution, with the advent of Open Source Technology, namely Open Source Hardware. Electronics equipment which were previously accessible and usable to trained engineers are now becoming easily accessible and deployable to a wide array of professionals including artists, medical professionals, educators etc. There is a very vast array of information available online that enable the user to construct tools on their own. There are also a wide array of platforms available online, including Arduino, Raspberry Pi and others that allow interfacing a wide array of sensors, actuators, communication interfaces etc to them, and allow them to be programmed by very simple interfaces and programming languages, so that non-engineering-professionals can create tools that will greatly help them in their respective careers. This has facilitated the ease of creation of digital manipulatives for learning applications.

A relative low cost technology that facilitates the creation of digital manipulatives is Radio Frequency Identification (RFID). They are highly cost-effective and provide a relatively easier mode to embed an identification/communication element in an everyday object. Though digital manipulatives have got a huge scope in the field of education, a review of literature shows that they have not been explored much. More research is needed towards this end in order to help realise the exciting potential presented by digital manipulatives for transforming the learning experiences of children at risk in mathematics.

4. Cognitive Theory Base for Digital Manipulatives

a. Embodied Cognition

Embodied Cognition is a relatively new topic in cognitive science which proposes that cognitive processes are deeply rooted in the body's interactions with the world. In the field of tangible computing, embodiment refers to the kind of interaction used to manipulate digital content through the use of physical objects (Ainsworth, 1999). Instead of placing the emphasis on the tool itself, the interaction provided by tangible interfaces focus primarily on the manipulation of the objects. O'Malley and Fraser (2005) have pointed out that it is the physical activity itself, which takes place when manipulating digital information using objects that helps to build representational mappings, which facilitates the understanding of more symbolically mediated activity. The point of view of Embodied Cognition is well supported by Digital manipulatives, which combine physical and digital interactivity in providing a multisensory approach to learning.

b. Distributed Cognition

Distributed cognition is a branch of cognitive science which proposes that cognition and knowledge are not confined to an individual; rather, it is distributed across objects, individuals, artefacts, and tools in the environment. It is an important theory in the field of Human Computer Interaction and Instructional Design. Using digital manipulatives, children learn or acquire concepts through physical activity and collaborative learning.

c. Dual Coding Theory

The conceptual model of multimodal communication proposed by Paivio (1971) is well supported by digital manipulatives. In his Dual Coding Theory, Pavio refers that children codify information in two different systems, one system codifies icons/images, and the other codifies words. The simultaneous use of both symbolic systems facilitates and strengthens the understanding of the conveyed content (Sylla, 2014).

5. Conclusion

Manipulatives play a very crucial role in the acquisition of knowledge. With the rapid evolution of information technology, manipulatives have evolved considerably over the years – starting with concrete manipulatives, progressing into virtual manipulatives and further into digital manipulatives. On the flip side, there have been studies which are not very encouraging to the proponents of manipulative materials strategies (Friedman, 1978). What is required is a very meaningful and appropriate use of manipulatives appropriate for the concept being developed and according to the developmental level of the students. The evolution of manipulatives does not mean that one type of manipulative is better than the other. There may be instructional settings where digital manipulatives would be the best to be used and at the same time, concrete manipulatives may be beneficial in some other cases. Further, as Clements (1999) has rightly pointed out, teachers and students should avoid using manipulatives as an end rather than as a means to that end. The manipulability and meaningfulness provided by the manipulative should be given utmost importance than the mere use of any particular type of manipulative.

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