Modeling and Simulation of the Analogous Mechanical and Electrical Systems

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Abstract: ***In this paper a brief review of the analogous between mechanical and electrical system has been introduced. The analogy electrical circuit contains series combination of electrical elements such as resistor, inductor and capacitor as well as the voltage source. The (RLC) circuit was implemented to carry out the analogous model of the mechanical system consists of damper mass spring systems. The review includes derived the required mathematical equations and implementation of the controller model of the analogy systems. From the mathematical model the relative transfer functions of the two selective systems have been obtained and the simulation has been done in Matlab environment. The classical (PID) controller schemes with the combination of the proportional, integral and derivative controller models has been used to investigate the best performance of the simulated electrical system.***

Keywords: Analogous Systems, PID controller, Matlab

1. Introduction

Conceptually the term analogous may be used to describe the different types of engineering systems which can be represented by the same mathematical model, but have different physical construction. Therefore the analogous systems can be applied to provide a proper approach for describing one physical system in relation to another system and finding the corresponding mathematical equations [1]. This approach was implemented for different engineering system such as hydraulic or pneumatic systems depending on analyzing their electrical analog model. In other words the analogy system provides an alternative method to deal with the system experimentally instead of building and studying the specified mechanical system. In order to consider the behavior of many mechanical systems it may be required to make some essential mathematical analysis of these systems to obtain their mathematical models and to realize their performance. In the analogous systems the same mathematical equations or transfer functions are mathematically represented [2].

In fact the mechanical to electrical analogies can be defined as the representation of the mechanical systems using electrical networks. Such analogies were firstly utilized in reverse engineering to understand different electrical phenomena in familiar mechanical terms. The analogies were introduced as an easily method to solve certain mechanical problems through an electrical analogy. Through the developments in the electrical domain, the analysis of the specified mechanical system using analogy approach was became as a standard tool and has the major advantage in designing the entire system. Different area of application of the electrical analogies was used such as in controlengineering systems that contains sensors and actuators although a given system have no electrical parts at all but it can be represented by an electrical analogy. For this reason the analogous system is preferred whendeveloping network diagrams for control systems [3].In this work a mechanical system that represented by damper mass spring system and its analogy electrical system has been introduced with using force voltage analogy method. The required mathematical equations for the two systems are derived and the corresponding transfer functions are obtained using Laplace transformation. The closed loop controlled model of the (*RLC*) system is established in Matlab/Simulink software package with using (*PID*) controller to reduce the error between the output and input signals and to improve the response of the system.

2. Analogies of Electrical and Mechanical Systems

In general the mechanical to electrical analogies method can be considered as a useful analytical approach for the system that includes different energy domains. This approach is based on using the corresponding electrical laws and parameters in the analogy electrical network. In analogy the circuit diagram does not represent the actual physical dimensions of the components in the system but it can be used to describe the topology of the elements with specialized graph notation [3].

For example, the solenoid that used in different industrial applications to employ many control process contains electrical as well as mechanical parts. The electrical part of the solenoid can be represented in the circuit diagram as a coil consists of series combination of resistance an inductance. If the electrical current passes through the solenoid coil, it will produce magnetic field leading to actuate the armature in the mechanical part of the solenoid [1].

The magnetic flux that passed in the coil of the solenoid can be expressed according to the ohm’s law as in the following:-

$$\phi = \frac{f}{R} \qquad (1)$$

So the analogy circuit of the solenoid coil can be modeled according to the Ampere’s circuital law as shown in the table (1).

Table 1: The Analogy between the Magnetic and Electrical Circuit [4]

Electrical Circuit		Magnetic Circuit	
Quantity	Symbol	Quantity	Symbol
Electrical Current	I	Magnetic Flux	Φ
Electrical Resistance	R	Magnetic Reluctance	R
Electrical Voltage	E	Magnetomotive Force	F

The first step in forming the analogy between the mechanical systems and the electrical systems is to describe them as a topological graph of ideal elements and obtained their mathematical models in a similar way to the electrical system. There are two electrical analogies for mechanical systems, the force to voltage analogy and the force to current analogy [3], [5].In the force to voltage analogy method the parameters of the mechanical and corresponding electrical systems can be represented as shown in the table (2).

Table 2: Force Voltage Analogy between Electrical and Mechanical Circuits

Electrical circuit		Mechanical Circuit	
Quantity	Symbol	Quantity	Symbol
Voltage	V	Force	f
Current	I	Velocity	v
Resistance	R	Friction Coefficient	b
Inductance	L	Mass	m

3. The Mathematical Model of the Mechanical and Electrical Analogies System

In the most of electrical systems the different parameters such as electrical current and voltage level represent the primary variables that are used in describing the mathematical behavior of the electrical elements in the circuit diagram. Basically in electrical circuit there are passive electrical elements such as resistance, inductance, and capacitance as well as active elements such as voltage, and current sources [2]. In mechanical system, like electrical system, there are several passive and linear elements such as the spring and the mass which are energy-storage elements while the viscous damper, dissipates energy. Starting with the two energy-storage elements in mechanical system, it can be formulate the analogous to the two electrical energy-storage elements, the inductor and capacitor. The simple linear damper mass spring systems has been taken as an example of linear mechanical system to mathematically analyzed with its analogy (*RLC*) series electrical circuit as shown in figure(1)[3],[5].

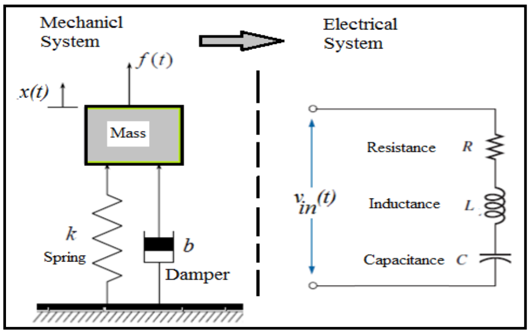

Figure 1: The Damper Mass Spring Model and (*RLC*) Electrical Circuit

In figure (1) the mass is represented by the symbol (*m*), while the coefficient of the viscous friction was represented by (*b*) and the spring constant was represented by the symbol (*k*). In comparison with the analogy electrical systems, the corresponding series combination (*RLC*) circuit used as shown in figure (1), in which the resistance (*R*) provides a direct relationship between current and voltage while the capacitance (*C*) represent the storage element for the electrical energy and provides a relationship between charge and voltage while the inductance (*L*) provides a relationship between current and magnetic flux. The voltage source is connected in series with these elements to provide the electrical circuit with the required electrical current level. The mathematical model of the damper mass spring system can be obtained with application of kinematics and Newton's laws as in the following:-

$$\sum_{i=1}^{3} F_i = F_1 + F_2 + F_3 \tag{2}$$

Where F_1, F_2 and $F_3$are representing the inertial force, viscous force spring and restoring force respectively. The spring, damper and the mass have positions denoted as the reference positions. in order to consider the linear displacement of the motion, the sum of the mechanical forces of the damper mass spring system will be obtained as in the following:-

$$F_1 = m\frac{d^2x}{dt^2} \tag{3}$$

$$F_2 = b\frac{dx}{dt} \tag{4}$$

$$F_3 = kx \tag{5}$$

Then

$$F_t = m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx \tag{6}$$

The equation (6) represents the mathematical model of the damper mass spring system under consideration in which the parameter (*x*) represents the displacement of mass that measured from equilibrium position. Taking the Laplace transform the equation (6) under assumption of zero initial condition to obtain the transfer function of the mechanical system as in equation (7).

$$\frac{X(s)}{F(s)} = \frac{1}{ms^2 + bs + k} \tag{7}$$

To obtain the mathematical model of the electrical circuit in figure (1), it is required to calculate the voltage across each element in the circuit diagram based on the of Kirchhoff's voltage law (*KVL*) and the mathematical equation will be expressed as in the following:-[4]

$$\sum_{i=1}^{n} V_i = V_1 + V_2 + ------ + V_n \tag{8}$$

Hence

$$V_t = V_R + V_L + V_C \tag{9}$$

Where V_t,V_R ,V_L and V_C represent the total applied voltage, the voltage across the resistor,the voltage across the Inductor and the voltage across the capacitor, then:-

$$V_t = i(t)R + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt \tag{10}$$

Taking the Laplace transform of equation (10) with assuming the initial condition equal to zero, then

$$V_t(s) = RI(s) + sLI(s) + \frac{1}{sC} I(s) \tag{11}$$

$$V_t(s) = I(s)[R + sL + \frac{1}{sC}] \tag{12}$$

$$V_t(s) = I(s)\frac{[sCR + s^2LC + 1]}{sC} \tag{13}$$

Since the output voltage is taken across the capacitor element in the (*RLC*) circuit and (V_C=V_O), then

$$V_C = I(s)\frac{1}{Cs} \tag{14}$$

And equation (14) will be re-written as in the following:-

$$V_{in}(s) = V_O[CRs + s^2LC + 1] \tag{15}$$

The transfer function of the (*RLC*) circuit will be obtained as in equation (16)

$$\frac{V_O(s)}{V_{in}(s)} = \frac{1}{LCs^2 + CRs + 1} \tag{16}$$

Equation (16) represents the transfer function of the(*RLC*) electrical system and it can be seen that it is quite similar to the transfer function that has previously obtained in equation (7) for the damper mass spring system.

4. Simulation and Results

The simulation model of the electrical analogy circuit has been set up using matlab /Simulink software package with using the selected numerical values of the resistance (*R*), the inductance (*L*) and the capacitance (*C*). The open-loop control model was using the transfer function in equation (16) of the analogy electrical circuit as shown in the table (3).

Table 3: The Transfer Function of the Analogy Electrical circuit

step	*R* (Ω)	*L* (*mH*)	*C* (*µF*)	*Transfer Function* $\frac{V_O(s)}{V_{in}(s)} = \frac{1}{LCs^2 + CRs + 1}$
1	1	4	400	$\frac{1}{1.6\times10^{-6}s^2 + 0.0004s + 1}$
2	4	6	600	$\frac{1}{3.6\times10^{-6}s^2 + 0.0024s + 1}$
3	4	8	800	$\frac{1}{6.4\times10^{-6}s^2 + 0.0032s + 1}$

The response of the analogy system in open loop control model will be displayed in Matlab as shown in figures(2),(3) and(4) respectively.

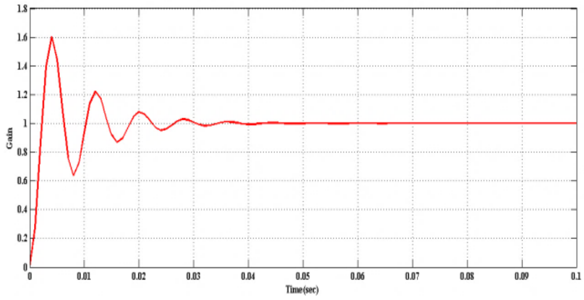

Figure 2: The Open Loop Response of the AnalogySystem(*R*=1Ω, *L*= 4mH and *C*=400 µF)

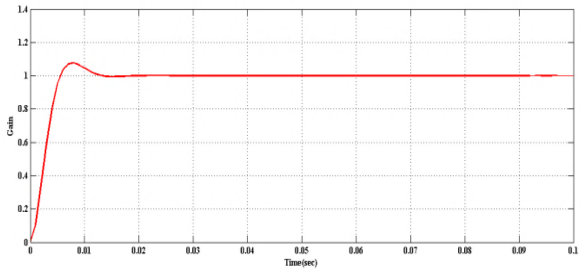

Figure 3: The Open Loop Response of the Analogy System(*R*=4Ω, *L*= 6mH and *C*=600 µF**)**

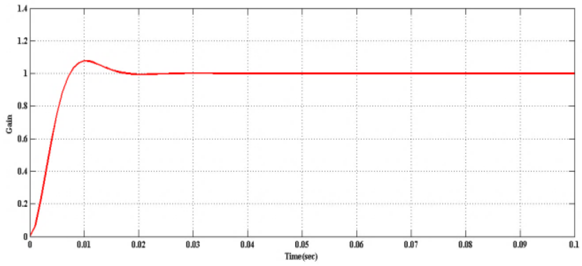

Figure 4: The Open Loop Response of the Analogy System(*R*=4Ω, *L*= 8mH and *C*=800 µF)

In the simulation process of the analogy system a closed loop control mode have also been used as shown in figure (5)with taking values of (R= 1Ω), (*L*= 4mH) and (*C*=400 µF) as testify values to detect the best performance of the system.

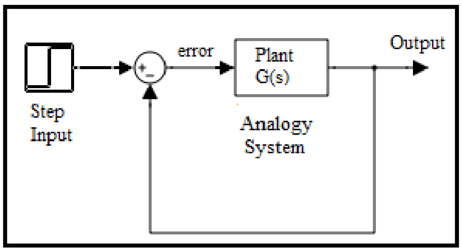

Figure 5: The Closed-Loop Control Model of the Analogy System

The response of the analogy system in closed loop model with unity feedback will be displayed as shown in figure (6).

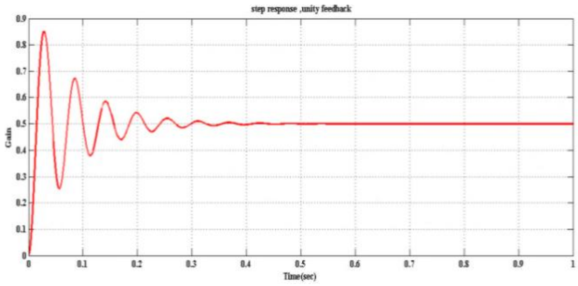

Figure 6: The Closed Loop Response of the Analogy System(*R*=4Ω, *L*= 8mH and *C*=800 µF)

In the closed loop model of the analog system a compensator such as classical (*PID*) controller can be added as shown in figure (7). The transfer function of the (*PID)* controller will be as in equation (17).

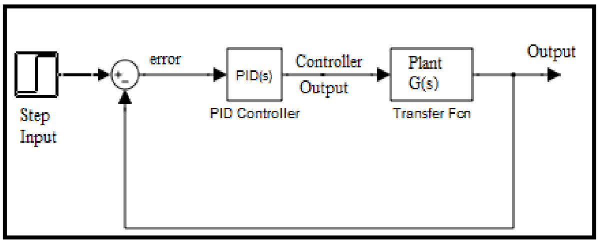

Figure 7: The Closed -Loop Model of the Analogy System with PID Controller

$$G_{PID}(s) = K_P + \frac{K_I}{s} + K_D s = \frac{K_D s^2 + K_P s + K_I}{s} \qquad (17)$$

Where (K_P),$(K_I)$and (K_D) are proportional gain, integral gain and derivative gain of the (*PID*)controllerrespectively.

As mentioned in figure (9), the output control signal (u) from the (*PID*) controller will be input to the plant and it is equal to production of the proportional gain (K_P) times the magnitude of the error plus the integral gain (K_I) times the integral of the error plus the derivative gain (K_D) times the derivative of the error [6]. The parameters (K_P, K_I and K_D) of the (*PID*) controller can be tuned via Matlab/ Simulink environment and the transfer function of the tuned controller will be as shown in the following:-

$$G_{PID}(s) = \frac{0.009829s^2 + 1104.196s + 7.8381}{s}$$

The response results with tuned (*PID*) controller will be displayed as shown in figure (8) with rise time = 250 µsec, settling time=0.0141 sec and overshoot equal to 10.2 %.

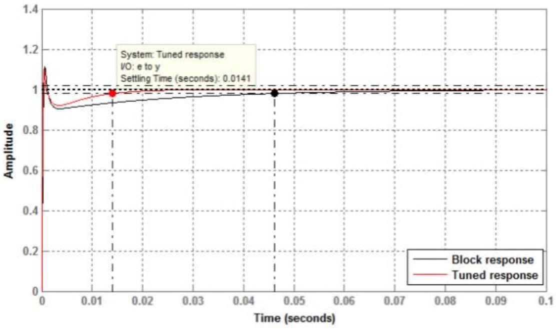

Figure 8: The Response of the Tuned PID Controller

5. Conclusions

In this work an attempt to employ design exercise of the analogy (*RLC*) electrical system to the damper- mass -spring mechanical system has been carried out. The force to voltage analogous method was illustrated and according to the obtained results the following conclusions can be remarked:-

1- The analogies electrical system can be easily used as simulation model to the mechanical system. This can be done with taking the mass, dampers and springs in the damper mass spring system to the corresponding capacitance, resistors and inductive elements in the analogy electrical (*RLC*) circuit. Thus, the summing forces in damper mass spring system were analogous to summing voltages written in (*RLC*) electrical system.

2- Performing mathematical approach involving mechanical and electrical laws such as Newton's second law and Kirchhoff's voltage law to obtain the mathematical model for each system, taking into account of the forces and voltages parameters.

3- Mathematical model of the analogous systems will take the form of second order differential equations and the resulting mechanical differential equations are analogous to electrical equations. The corresponding transfer functions between input and output variables of the two systems were obtained and simulation in Matlab environment of the analogous system has been done including the most popular (*PID*) controllers to improve the response of the system.

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