A Repair Replacement Policy for a Deteriorating Cold Standby System with a Component has Priority in Use and Repair

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Abstract: ***The purpose of this paper is to apply the two monotone process repair model to a two-component cold standby repairable system with one repairman. Now it may be assumed that the component 2 after repair is ''as good as new'' while component 1 follows the geometric process repair, but component 1 has priority in use and repair, and each component after repair is not ''as good as new''. Under these assumptions, by using two monotone processes, we studied a replacement policy N based on the number of repairs of component 1. The problem is to determine an optimal replacement policy N* such that the long-run expected reward per unit time is minimized.***

Keywords: Alpha series process, geometric process, convolution, mean time to failure, renewal processes, renewal reward theorem

1. Introduction

In order to improve the system reliability or raise the availability, a two-component redundant system is often used. In the earlier study some important reliability indices of the system have been derived by using Markov process theory or Markov renewal process theory, under the conditions that the working time and the repair time of the components in the system both have exponential or general distributions. Later, a priority rule for repair or use of a component has been introduced. Nakagawa and Osaki [1975] obtained some interesting reliability indices of the system using Markov renewal theory. It assumed that both the working time and the repair time of the priority component have a general distribution while both the working time and the repair time of the non-priority component have an exponential distribution. In these studies, they assumed that a system (or a component) after repair is ''as good as new''. This is a perfect repair model. However, this assumption is not always true. In practice, most repairable systems are deteriorative because of the ageing effect and the accumulative wear. Barlow and Hunter [1959] first proposed a minimal repair model under which the minimal repair does not change the age of the system. Brown and Proschan [1983] investigated an imperfect repair model in which the repair is perfect repair with probability p or minimal with probability 1-p. For a deteriorating repairable system, it is quite reasonable to assume that the successive working times of the system after repair will become shorter and shorter while the consecutive repair times of the system after failure will become longer and longer. Ultimately, it cannot work any longer, neither can it be repaired.

For such a stochastic phenomenon, Lam [1988a,b] first introduced a geometric process repair model to approach it. Under this model, he studied two kinds of replacement policy for a one-component repairable system with one repairman (called a simple repairable system), one based on the working age T of the system and the other based on the failure number N of the system. The explicit expressions of the long-run average cost per unit time under these two kinds of policy are respectively calculated. Finkelstein [1993] presented a general repair model based on a scale transformation after each repair to generalize Lam's work. Zhang [1994] generalized Lam's work by a bivariate replacement policy (T,N) under which the system is replaced at the working age T or at the time of the Nth failure, whichever occurs first. Many replacement policies have been done by Feldman[1976] ,Stadje and Zuckerman [1992], Stanley [1993], Zhang et al.[2001,2002,2007], Zhang [1994,1999,2002,2004], Lam and Zhang [2003,2004], and others under the geometric process repair model.

In practical applications, a two-component cold standby repairable system with one repairman and priority in use is often used. For example, in the operating room of a hospital, a patient on the operating table has to discontinue his/her operation as soon as the power source is cut (i.e. the power station failures). Usually, there is a standby power station (e.g. a storage battery) in the operating room. Thus, the power station (e.g. write as component 1) and the storage battery (e.g. write as component 2) form a cold standby repairable lighting system. Obviously, it is reasonable to assume that the power station has priority in use due to the operating cost of the power station is cheaper than the operating cost of the storage battery, and the storage battery after repair is as good as new due to its used time being smaller than the power station, and the repair of the storage battery is also convenient.

The purpose of this paper is to apply the two monotone process repair model to a two-component cold standby repairable system with one repairman. Now it may be assumed that the component 2 after repair is ''as good as new'' while component 1 follows the geometric process repair, but component 1 has priority in use and assumed that each component after repair is not ''as good as new''. Under these assumptions, by using two monotone processes, we studied a replacement policy N based on the number of repairs of component 1. The problem is to determine an optimal replacement policy N* such that the long-run expected reward per unit time is minimized. In modeling these deteriorating systems, the definitions according to Lam [1988 a] are considered.

2. The Model

To study problem for a two- dissimilar- component cold standby repairable system with use priority, the following assumptions are considered

1) At the beginning, the two components are both new, and component1 is in a working state while component 2 is in a cold standby state.
2) Both components after repair are not "as good as new" and follow a monotone process repair. When both components are good, component 1 ha use priority. The repair rule is "first- in- first- out". If a component fails during the repair of the other,it must wait for repair and the system is down.
3) The time interval between the completion of the (n-1)th repair and completion of nth repair of of component i (for i=1,2,3….,n) is called nth cycle. Note that a component either begins to work or enters a cold standby state of the next cycle when its repair is completed. Because component 1 has use priority, the repair time of component 2 may be zero in some cycles.
4) A sequence $\{X_n^{(i)}, i=1,2,3,....\}$ and a sequence $\{Y_n^{(i)}, i=1,2,3,....\}$ are independent mutually.
5) Let $\{X_n^{(i)}, i=1,2,3,....\}$ be the sequence of the working times for a decreasing alpha series process while $\{Y_n^{(i)}, i=1,2,3,....\}$ be the sequence of repair times of ith component and form a geometric process.
6) Let the distribution function of $X_n^{(i)}$ and $Y_n^{(i)}$ be $F_n^{(i)}$ *And* $G_n^{(i)}$, are respectively,for i=1,2.
7) Let $X_n^{(i)}$ and $Y_n^{(i)}$ be successive working time follows decreasing a α -series process, the successive repair times form an increasing geometric process respectively and both the processes are exposing to exponential failure law. Where i=1, 2 and n=1, 2, 3….. .
8) Let
$$F_n(i) = F_n^{(i)}(k^{\alpha_i}\lambda_i) \quad and \quad G_n(i) = G_n^{(i)}(b_i^{\,n-1}\mu_i)$$
be the distribution function of $X_n^{(i)}$ and $Y_n^{(i)}$ respectively, and n=1,2,…. where α_i>0 and 0 < b_i < 1,for i=1,2 .
9) $$E(X_n^{(i)}) = \frac{1}{\lambda_i k^{\alpha_i}} \qquad and \qquad E(Y_n^{(i)}) = \frac{1}{\mu_i b_i^{k-1}} \qquad fo$$
10) $$E(X_1^{(i)}) = \lambda_i \;, E(Y_1^{(i)}) = \mu_i \quad for \; i = 1,2.$$
11) A component neither produces the working reward during the cold standby period, nor incurs cost during the waiting time for repair.
12) The replacement policy N based on the number of failures of component 1 is used. The system is replaced by anew and an identical one at the time of N th failure of component 1, and replacement time is negligible.
13) The repair cost rate of component 1 and 2 are C_{r1} and C_{r2} respectively. While working reward of the two components is same C_w. And the replacement cost of the system is C.

3. The Long-run Average Cost Rate Under Policy N

According to the assumption of the model two components appears alternatively in the system. When the failure number of the component 1 reaches N, component 2 is either in the cold standby state of the Nth cycle or in the repair state of the (N-1) th cycle. Normally, a reasonable replacement policy N should be that if component 2 is in former state, it works until failure in the N^{th} cycle. Thus the renewal point under policy is efficiently established.

Let τ be the first replacement time of the system, and $\tau_n (n \geq 2)$ be the time between the (n-1)th replacement and nth replacement of the system under policy N. Thus, $\{\tau_1, \tau_2, \tau_3, ...\}$ form a renewal process, and inter arrival time between two consecutive replacements called a renewal cycle. Let C(N) be average cost rate of the system under policy N. According to renewal reward theorem (see Ross [1970])

$$C(N) = \frac{The\,expected\;\; cost\;\; incurred\; in\, a\;\; renewal\;\; cycle}{The\,expected\;\; length\;\; in\, a\;\; renewal\;\; cycle} \tag{3.1}$$

Because component 1 has use priority, component 1 only exist the working state. Therefore based on the renewal point under policy N we have the following length of renewal cycle.

$$L = \sum_{k=1}^{N} X_k^{(i)} + \sum_{k=1}^{N-1} Y_k^{(i)} + \sum_{k=2}^{N} \left(Y_{k-1}^{(2)} - X_k^{(1)}\right) I_{\left\{Y_{k-1}^{(2)} + X_k^{(1)} > 0\right\}} \, I_{\left\{Y_{k-1}^{(1)} + X_k^{(2)} > 0\right\}} + X_N^{(2)} \tag{3.2}$$

Where the first , second and third term are respectively the length working time, the length of repair time, and the length of waiting for repair of component before the number of failures of component1 reaches N. And I is the indicator function such that

$$I_A = \begin{cases} 1 & \text{if event A occurs.} \\ 0 & \text{if event A does't occurs.} \end{cases}$$

According to the assumption of the model, and definition of the convolution, the probability density functions of $(Y_{k-1}^{(2)} - X_k^{(1)})$ *and* $(X_k^{(2)} - Y_k^{(1)})$ are respectively, $\phi_k(u)$ and $\psi_k(v)$,namely:

$$\phi_k(u) = g_{k-1}^{(2)}(u) * f_k^{(1)}(-u) \qquad (3.3)$$

$$\psi_k(v) = f_k^{(2)}(v) * g_k^{(1)}(-v), \qquad (3.4)$$

Where * denotes the convolution. Therefore

$$E[(Y_{k-1}^{(2)} - X_k^{(1)})\, I_{(Y_{k-1}^{(2)} - X_k^{(1)})>0}\, I_{(Y_{k-1}^{(1)} - X_k^{(2)})>0}] = \phi_{k-1}(0)\int_0^\infty u\, d\phi_k(u), \qquad (3.5)$$

$$E[(X_{k-1}^{(2)} - Y_k^{(1)})\, I_{(X_{k-1}^{(2)} - Y_k^{(1)})>0}] = \int_0^\infty v\, d\psi_k(v) \qquad (3.6)$$

$$E[(Y_k^{(2)})\, I_{(X_{k-1}^{(2)} - Y_k^{(1)})>0}] = E(Y_k^{(2)})\,\psi_k(0) \qquad (3.7)$$

By using equation (3.1)

$$E[L] = \sum_{k=1}^{N} E(X_k^{(i)}) + \sum_{k=1}^{N-1} E(Y_k^{(i)}) + \sum_{k=2}^{N} E\left[(Y_{k-1}^{(2)} + X_k^{(1)})\, I_{\{Y_{k-1}^{(2)} + X_k^{(1)}>0\}}\, I_{\{Y_{k-1}^{(1)} + X_k^{(2)}>0\}}\right] + E(X_N^{(2)})$$

According to assumptions 8 ,9 and equation (3.5), we have

$$E[L] = \sum_{k=1}^{N} \frac{1}{\lambda_1 k^{\alpha_1}} + \sum_{k=1}^{N-1} \frac{1}{\mu_1 b_1^{k-1}} + \sum_{k=2}^{N} \phi_{k-1}(0)\left(\int_0^\infty u\, d\phi_k(u)\right) + \frac{1}{\lambda_2 N^{\alpha_2}} \qquad (3.8)$$

The total working time of the system in a renewal cycle is:

$$U = \sum_{k=1}^{N} X_k^{(i)} + \sum_{k=1}^{N-1} \min\{X_k^{(2)} Y_k^{(i)}\} + X_N^{(2)} \qquad (3.9)$$

$$= \sum_{k=1}^{N} X_k^{(1)} + \sum_{k=1}^{N-1} X_k^{(2)} - \sum_{k=1}^{N-1} (X_k^{(2)} - Y_k^{(1)})\, I_{\{X_{k-1}^{(2)} + Y_k^{(1)}>0\}} + X_N^{(2)}$$

$$= \sum_{k=1}^{N} X_k^{(1)} + \sum_{k=1}^{N} X_k^{(2)} - \sum_{k=1}^{N-1} (X_k^{(2)} - Y_k^{(1)})\, I_{\{X_{k-1}^{(2)} + Y_k^{(1)}>0\}} \qquad (3.10)$$

The total repair time of the system in a renewal cycle is:

$$V = V1 + V2 \qquad (3.11)$$

Where V_1 and V_2 denote, respectively, the repair time of the component1 and 2 in a renewal cycle, namely

$$V_1 = \sum_{k=1}^{N-1} Y_k^{(1)}, \qquad (3.12)$$

$$V_2 = \sum_{k=1}^{N-1} Y_k^{(2)}\, I_{(X_k^{(2)} - Y_k^{(1)})<0}, \qquad (3.13)$$

Now we evaluate expectation of 'U' V_1, and V_2:

$$E[U] = \sum_{k=1}^{N} E(X_k^{(i)}) + \sum_{k=1}^{N} E(X_k^{(2)}) - \sum_{k=1}^{N-1} E\left[(X_k^{(2)} - Y_k^{(1)})\, I_{\{X_{k-1}^{(2)} + Y_k^{(1)}>0\}}\right] \qquad (3.14)$$

According to Equations (5.3.5) and assumption 8, we have:

$$E[U] = \sum_{k=1}^{N} \frac{1}{\lambda_1 k^{\alpha_1}} + \sum_{k=1}^{N} \frac{1}{\lambda_2 k^{\alpha_2}} - \sum_{k=1}^{N-1} \int_0^\infty v\, d\psi_k(v) \qquad (3.15)$$

$$E(V_1) = \sum_{k=1}^{N-1} E(Y_k^{(1)}) = \sum_{k=1}^{N-1} \frac{1}{\mu_1 b_1^{k-1}}, \qquad (3.16)$$

Using equation (3.7) and assumption 9, we have

$$E[V_2] = \sum_{k=1}^{N-1} E\left(Y_k^{(2)}\, I_{(X_k^{(2)} - Y_k^{(1)})<0}\right)$$

$$= \sum_{k=1}^{N-1} E(Y_k^{(2)})\, E\left(I_{(X_k^{(2)} - Y_k^{(1)})<0}\right)$$

$$= \psi_k(0) \sum_{k=1}^{N-1} E(Y_k^{(2)})$$

$$= \psi_k(0) \sum_{k=1}^{N-1} \frac{1}{\mu_2 b_2^{k-1}} \qquad (3.17)$$

Using equatios (3.16) and (3.17), Equation (3.11) becomes

$$E(V) = \sum_{k=1}^{N-1} \frac{1}{\mu_1 b_1^{k-1}} + \psi_k(0) \sum_{k=1}^{N-1} \frac{1}{\mu_2 b_2^{k-1}}, \qquad (3.18)$$

Since, it is assumed that $X_k^{(i)}$ *and* $Y_k^{(i)}$, *for* $i = 1,2.$ are all exponential, then their distribution functions are given by:

$$F_k^{(i)}(x) = F^{(i)}(k^{\alpha_i} x) = 1 - \exp(-k^{\alpha_i} \lambda_i x),\ for\ i = 1,2.$$

$$G_k^{(i)}(y) = G^{(i)}(b_i^{k-1} y) = 1 - \exp(-b_i^{k-1} \mu_i y),\ for\ i = 1,2.$$

,

$$where\ x \geq 0,\ y \geq 0,\ 0 \leq \alpha_i \leq 1,\ 0 \leq b_i \leq 1,$$

According to the assumptions of the model, definition of probability density function, convolution and Jacobian transformations , the probability density functions of $u = (Y_{k-1}^{(2)} - X_k^{(1)})$ *and* $v = (X_k^{(2)} - Y_k^{(1)})$ are respectively, $\phi_k(u)$ and $\psi_k(v)$.

$$\phi_k(u) = \int_0^\infty f(v, u+v)\, dv, \qquad (3.19)$$

Where

$X_k^{(1)} = v,\ Y_{k-1}^{(2)} = u + v,$ such that $u = Y_{k-1}^{(2)} - X_k^{(1)},$

$$\phi_k(u)=\int_0^{\infty} f(v,u+v)\,dv=\begin{cases}\dfrac{k^{\alpha_1}b_2^{k-2}\lambda_1\mu_2}{k^{\alpha_1}\lambda_1+b_2^{k-2}\mu_2}e^{-b_2^{k-2}\mu_2 u} & \text{for } u\geq 0\\[2ex] \dfrac{k^{\alpha_1}b_2^{k-2}\lambda_1\mu_2}{k^{\alpha_1}\lambda_1+b_2^{k-2}\mu_2}e^{-k^{\alpha_1}\lambda_1 u} & \text{for } u<0\end{cases} \tag{3.20}$$

Let $\psi(v)=\int_0^{\infty} f(u+v,u)\,du$ (3.21) $\psi_k(v)=\int_0^{\infty} f(u+v,u)\,du$

where $X_k^{(2)}=u+v$; $Y_k^{(1)}=u$ *such that* $v=X_k^{(2)}-Y_k^{(1)}$.

$$\psi_k(v)=\int_0^{\infty} f(u+v,u)\,du=\begin{cases}\dfrac{k^{\alpha_2}b_1^{k-1}\lambda_2\mu_1}{k^{\alpha_2}\lambda_2+b_1^{k-1}\mu_1}e^{-k^{\alpha_2}\lambda_2 v} & \text{for } v\geq 0\\[2ex] \dfrac{k^{\alpha_2}b_1^{k-1}\lambda_2\mu_1}{k^{\alpha_2}\lambda_2+b_1^{k-1}\mu_1}eb_1^{k-1}\mu_1 v & \text{for } v<0\end{cases} \tag{3.22}$$

Therefore

Where * denotes the convolution. Therefore

$$E[(Y_{k-1}^{(2)}-X_k^{(1)})\,I_{(Y_{k-1}^{(2)}-X_k^{(1)})>0}]=\int_0^{\infty} u\,d\phi_k(u)\,, \tag{3.23}$$

$$=\frac{k^{\alpha_1}\lambda_1}{(k^{\alpha_1}\lambda_1+b_2^{k-2}\mu_2)b_2^{k-2}\mu_2},\quad k\geq 2 \tag{3.24}$$

Let $E[(X_{k-1}^{(2)}-Y_k^{(1)})\,I_{(X_{k-1}^{(2)}-Y_k^{(1)})>0}]=\int_0^{\infty} v\,d\psi_k(v)$ (3.25)

$$=\frac{b_1^{k-1}\mu_1}{(k^{\alpha_2}\lambda_2+b_1^{k-1}\mu_1)k^{\alpha_2}\lambda_2}\,,\ k\geq 1 \tag{3.26}$$

$$\psi_k(0)=p(X_k^{(2)}-Y_k^{(1)}<0)=\frac{k^{\alpha_2}\lambda_2}{(k^{\alpha_2}\lambda_2+b_1^{k-1}\mu_1)},\quad k\geq 1 \tag{3.27}$$

Using equation (3.25), equation (3.15) becomes

$$E[U]=\sum_{k=1}^{N}\frac{1}{\lambda_1 k^{\alpha_1}}+\sum_{k=1}^{N}\frac{1}{\lambda_2 k^{\alpha_2}}-\sum_{k=1}^{N-1}\frac{b_1^{k-1}\mu_1}{(k^{\alpha_2}\lambda_2+b_1^{k-1}\mu_1)k^{\alpha_2}\lambda_2} \tag{3.28}$$

Using equations (3.23) and (3.27), equation (3.8) becomes:

$$E[L]=\sum_{k=1}^{N}\frac{1}{\lambda_1 k^{\alpha_1}}+\sum_{k=1}^{N-1}\frac{1}{\mu_1 b_1^{k-1}}+\sum_{k=2}^{N}\left(\frac{(k-1)^{\alpha_2}\lambda_2}{((k-1)^{\alpha_2}\lambda_2+b_1^{k-2}\mu_1)}\right)\left(\frac{k^{\alpha_1}\lambda_1}{(k^{\alpha_1}\lambda_1+b_2^{k-2}\mu_2)b_2^{k-2}\mu_2}\right)+\frac{1}{\lambda_2 N^{\alpha}} \tag{3.29}$$

Therefore, substituting the equations (3.16), (3.17), (3.28), and (3.29) into the expression (3.1),then the average cost rate of the system under policy N is given by

$$C(N)=\frac{C_r^{(1)}E(V_1)+C_r^{(2)}E(V_2)+C-C_wE(U)}{E(L)} \tag{3.30}$$

$$C(N)=\frac{\left[\dfrac{C_r^{(1)}}{\mu_1}\sum_{k=1}^{N-1}\left(\dfrac{1}{b_1^{k-1}}\right)+C_r^{(2)}\sum_{k=1}^{N-1}\left(\dfrac{\lambda_2 k^{\alpha_2}}{\mu_2 b_2^{k-1}\left(\mu_2 b_2^{k-1}+\lambda_2 k^{\alpha_2}\right)}\right)+C \\ -C_w\left(\sum_{k=1}^{N}\dfrac{1}{\lambda_1 k^{\alpha_1}}+\sum_{k=1}^{N}\dfrac{1}{\lambda_2 k^{\alpha_2}}-\sum_{k=1}^{N-1}\dfrac{b_1^{k-1}\mu_1}{\left(k^{\alpha_2}\lambda_2+b_1^{k-1}\mu_1\right)k^{\alpha_2}\lambda_2}\right)\right]}{\sum_{k=1}^{N}\dfrac{1}{\lambda_1 k^{\alpha_1}}+\sum_{k=1}^{N-1}\dfrac{1}{\mu_1 b_1^{k-1}}+\sum_{k=2}^{N}\left(\dfrac{(k-1)^{\alpha_2}\lambda_2}{\left((k-1)^{\alpha_2}\lambda_2+b_1^{k-2}\mu_1\right)}\right)\left(\dfrac{k^{\alpha_1}\lambda_1}{\left(k^{\alpha_1}\lambda_1+b_2^{k-2}\mu_2\right)b_2^{k-2}\mu_2}\right)+\dfrac{1}{\lambda_2 N^{\alpha}}} \tag{3.31}$$

which is an expression for the long run average cost per unit time.

The next section provides Numerical results to highlight the theoretical results.

4. Numerical Results and Conclusions

For the given hypothetical values of **C=3500, C_w=40, C^1_r=8, Cr2=40, λ_1=0.01, μ_1=1.5, λ_2=0.2, and μ_2=0.15 the average cost rate is given by:**

Table 4.1: The long-run average cost rate values under policy N

α1=0.75,α2=0.6,β1=0.65,β2=0.35,			
N	C(N)	N	C(N)
2	-17.9802	11	-23.8357
3	-22.0263	12	-23.2149
4	-23.9363	13	-22.5372
5	-24.8781	14	-21.8142
6	-25.2797	15	-21.055
7	**-25.3417**	16	-20.2668
8	-25.1732	17	-19.4553
9	-24.8399	18	-18.6251
10	-24.3843	19	-17.7801
		20	-16.9235

Table 4.2: The long-run average cost rate values under policy N

α1=0.95,α2=0.85, β1=0.65,β2=0.35,			
N	C(N)	N	C(N)
2	-16.7568	11	-18.8414
3	-20.3264	12	-17.7658
4	-21.8627	13	-16.6314
5	-22.4333	14	-15.4512
6	**-22.4463**	15	-14.2357
7	-22.1011	16	-12.9931
8	-21.5094	17	-11.7303
9	-20.7406	18	-10.4527
10	-19.8407	19	-9.16485
		20	-7.87071

5. Conclusions

a) From the table 4.1 and graph 4.1, We can examine that the long-run average cost per unit time at the time C (7) = **-25.3417** is minimum .We should replace the system at the time of 7th failure.
b) From the table 4.2 and graph 4.2, we observe that the long-run average cost per unit time at the time C (6) = **-22.4463** is minimum**.** We should replace the system at the time of 6th failure.

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