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A Semi-Analytical Iterative Method for Solving Linear and Nonlinear Partial Differential Equations

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Abstract: ***This study presents an iterative method suggested by Temimi and Ansari namely (TAM) for solving linear and nonlinear wave, wave-like, heat and heat-like equations. This method is characterized by applying it without any restrictive assumptions for non-linear terms and provide the exact solution. The software used for the calculations in this study was Mathematica®9.***

Keywords: Wave equation, Wave-like equation, Heat and Heat-like equations, Iterative method

1. Introduction

Many applications in various fields of engineering and physical can be modeled by partial differential equations, such as the study of heat transfer in metals, the study of fluid flow, static electricity and flexibility.

Several efforts have been made to implement either analytical or approximate methods to solve the linear and nonlinear equations of wave and heat equations such as ADM [1], [2], HPM [3], [4], DGJ [5], [6], VIM [7], [8], these methods have been successfully implemented but some difficulties have arisen, for instance, in finding Adomian polynomials to deal with the nonlinear terms in ADM, construct a homotopy and solve the corresponding algebraic equations in HPM, evaluating the Lagrange multiplier in VIM [7,8].

Recently, Temimi and Ansari [9] have suggested a new iterative method for solving linear and nonlinear functional equations namely (TAM). The TAM has been successfully applied by many researchers for solving linear and nonlinear ordinary and partial differential equations see [10], [11], [12], [13], [14], [15], [16].

In this study, the TAM will be applied to solve the linear and nonlinear wave, wave-like, heat and heat-like equations and the exact solutions will be achieved.

The present study has been arranged as follows: in section two the description of TAM will be introduced. In section three solving linear and nonlinear wave, wave-like equations by TAM will be discussed. In section four the linear and nonlinear heat and heat-like equations will be solved by TAM and lastly in section five the conclusion will be been given.

2. The basic idea of TAM

To explain the basic idea of TAM, let us consider the general equation

$$\mathrm{L}(\mathrm{y}(\mathrm{s},\mathrm{t}))+\mathrm{N}(\mathrm{y}(\mathrm{s},\mathrm{t}))+\mathrm{h}(\mathrm{s},\mathrm{t})=0, \quad (1)$$

with boundary conditions $\mathrm{B}\left(\mathrm{y},\frac{\partial y}{\partial t}\right)=0,$

where y is unknown function, L is the linear operator, N is the nonlinear operator and h is a known function.

Assuming that $\mathrm{y}_0(s,t)$ is a solution of equation (2.1) of the initial condition

$$\mathrm{L}(\mathrm{y}_0(s,t))+\mathrm{h}(\mathrm{s},\mathrm{t})=0,\ \text{with}\ \mathrm{B}\left(\mathrm{y}_0,\frac{\partial y_0}{\partial t}\right)=0\,. \quad (2)$$

To find the next iteration we resolve the following equation:

$$\mathrm{L}(\mathrm{y}_1(s,t))+\mathrm{N}(\mathrm{y}_0(s,t))+\mathrm{h}(\mathrm{s},\mathrm{t})=0,\mathrm{B}\left(\mathrm{y}_1,\frac{\partial y_1}{\partial t}\right)=0.$$

Thus, an iterative procedure can be effective solution of the following problem,

$$\mathrm{L}(\mathrm{y}_{n+1}(s,t))+\mathrm{N}(\mathrm{y}_n(s,t))+\mathrm{h}(\mathrm{s},\mathrm{t})=0,\mathrm{B}\left(\mathrm{y}_{n+1},\frac{\partial y_{n+1}}{\partial t}\right)=0. \quad (3)$$

Each of y_i are solutions to equation (1) [9].

3. Solving wave and wave-like equations by TAM

In this section the TAM will be used to solve linear and nonlinear wave and wave-like equations.

3.1. Wave and wave-like equations

The linear and nonlinear wave and wave-like equations are given [7], [17]:

$$y_{tt}=y_{ss}+\mathrm{f}(\mathrm{y}),0<x<M\,,t>0\,,$$

$$y_{tt}=y_{ss}+\mathrm{F}(\mathrm{y})+h(s,t),0<s<M\,,t>0.$$

$$y_{tt}=\frac{s^2}{2}y_{ss},0<s<M\,,t>0,$$

$$y_{tt}=y_{ss},-\infty<s<\infty\,,t>0.$$

The functions f(y), F(y), and h(s, t) are known functions which are linear, nonlinear and source functions, respectively. The TAM will be applied to obtain the exact solutions for these problems.

3.2 Linear Wave Problem

Consider the following linear wave equation [7]

$$y_{tt}=y_{ss}-3y\,,0<s<\pi\,,t>0, \quad (4)$$

with initial conditions $y(s,0)=0\,,y_t(s,0)=2\cos s$,

and boundary conditions

$$y(0,t)=\sin 2t\ \,,y(\pi,t)=-sin2t.$$

The TAM can be applied by choosing the following

$\mathrm{L}(\mathrm{y})=y_{tt}\,,\mathrm{N}(\mathrm{y})=y_{ss}-3\mathrm{y}$ and $\mathrm{h}(\mathrm{s},\mathrm{t})=0,$

Thus, the primary problem is

$$L(y_0) = 0, \text{ with } y_0(x,0) = 0, (y_0)_t(s,0) = 2\cos s. \quad (5)$$

A general iterative problem can be written as

$$L(y_{n+1}) + N(y_n) + h(s,t) = 0, y_{n+1}(s,0) = 0, (y_{n+1})_t(s,0) = 2\cos s. \quad (6)$$

By solving the primary problem (5), we get

$y_0(s,t) = 2t\cos s.$

The first iteration can be done through and given as

$$(y_1)_{tt} = (y_0)_{ss} - 3y_0 \text{ with } y_1(s,0) = 0, (y_1)_t(s,0) = 2\cos s.(7)$$

Then the solution of equation (7) is

$y_1(s,t) = \left(2t - \frac{4}{3}t^3\right)\cos s.$

The second iteration is

$$(y_2)_{tt} = (y_1)_{ss} - 3y_1 \text{ with } y_2(s,0) = 0, (y_2)_t(s,0) = 2\cos s \ .(8)$$

Then the solution of equation (8) is

$y_2(s,t) = \left(2t - \frac{4}{3}t^3 + \frac{4}{15}t^5\right)\cos s.$

Similarly, we get,

$y_3(s,t) = (2t - \frac{4}{3}t^3 + \frac{4}{15}t^5 - \frac{8}{315}t^7)\cos s,$

$y_4(s,t) = \left(2t - \frac{4}{3}t^3 + \frac{4}{15}t^5 - \frac{8}{315}t^7 + \frac{2}{2835}t^9\right)\cos s.$

This has the closed form:

$y(s,t) = \sin 2t \cos s.$ (9)

which is the exact solution of the problem [7].

3.3. The nonlinear wave equation

Consider the nonlinear wave equation

$$y_{tt} = yy_{ss} - y^2 - y, 0 < s < 1, t > 0, \quad (10)$$

with initial conditions $y(s,0) = e^s, y_t(s,0) = 0$,
and boundary conditions $y(0,t) = \cos t$.
The TAM will be applied as follows

$$L(y) = y_{tt}, N(y) = yy_{ss} - y^2 - y \text{ and } h(s,t) = 0, \quad (11)$$

Thus, the primary problem is

$$L(y_0) = 0, \text{ with } y_0(s,0) = e^s, (y_0)_t(s,0) = 0. \quad (12)$$

A general iteration problem can be represented as

$$L(y_{n+1}) + N(y_n) + h(s,t) = 0, y_{n+1}(s,0) = e^s, (y_{n+1})_t(s,0) = 0. \quad (13)$$

By solving the primary problem, we obtain

$y_0(s,t) = e^s.$

The first repetition can be done through the following

$$(y_1)_{tt} = y_0(y_0)_{ss} - {y_0}^2 - y_0 \text{ with } y_1(s,0) = e^s, (y_1)_t(s,0) = 0. \quad (14)$$

Then the solution of equation (14) is

$y_1(s,t) = e^s(1 - \frac{t^2}{2!}).$

The second iteration is

$$(y_2)_{tt} = y_1(y_1)_{ss} - {y_1}^2 - y_1 \text{ with } y_2(s,0) = e^s, (y_2)_t(s,0) = 0. \quad (15)$$

Then the solution of equation (15) is

$y_2(s,t) = e^s(1 - \frac{t^2}{2!} + \frac{t^4}{4!}).$

Similarly, we get,

$y_3(s,t) = e^s(1 - \frac{t^2}{2!} + \frac{t^4}{4!} - \frac{t^6}{6!}),$

$y_4(s,t) = e^s(1 - \frac{t^2}{2!} + \frac{t^4}{4!} - \frac{t^6}{6!} + \frac{t^8}{8!}).$

This has the closed form:

$$y(s,t) = e^s \cos t. \quad (16)$$

which is the exact solution of the problem.

3.4 The wave equation in an unlimited field

Consider the wave equation in an unlimited field [7]

$y_{tt} = y_{ss}, -\infty < s < \infty, t > 0$, (17) (3.15)

with initial conditions $y(s,0) = \sin s \ , y_t(s,0) = 0.$

The TAM will be applied as follows

$$L(y) = y_{tt}, N(y) = y_{ss} \text{ and } h(s,t) = 0, \quad (18)$$

Thus, the primary problem is

$$L(y_0) = 0, \text{ with } y_0(s,0) = \sin s, (y_0)_t(s,0) = 0. \quad (19)$$

A general relation can be generated

$$L(y_{n+1}) + N(y_n) + h(s,t) = 0, y_{n+1}(s,0) = \sin s, (y_{n+1})_t(s,0) = 0. \quad (20)$$

By solving the primary problem (19) we have got

$y_0(s,t) = s + ts^2.$

The first repetition can be done through the following

$(y_1)_{tt} = (y_0)_{ss}$ with $y_1(s,0) = \sin s \ , (y_1)_t(s,0) = 0.$ (21)

Then the solution of equation (21) is

$y_1(s,t) = \left(1 - \frac{t^2}{2!}\right)\sin s.$

The second iteration is

$(y_2)_{tt} = (y_1)_{ss}$ with $y_2(s,0) = \sin s \ , (y_2)_t(s,0) = 0.$ (22)

Then the solution of equation (22) is

$y_2(s,t) = \left(1 - \frac{t^2}{2!} + \frac{t^4}{4!}\right)\sin s.$

Similarly, we get,

$y_3(s,t) = \left(1 - \frac{t^2}{2!} + \frac{t^4}{4!} - \frac{t^6}{6!}\right)\sin s,$

$y_4(s,t) = \left(1 - \frac{t^2}{2!} + \frac{t^4}{4!} - \frac{t^6}{6!} + \frac{t^8}{8!}\right)\sin s.$

This has the closed form:

$$y(s,t) = \sin s \cos t. \quad (23)$$

which is the exact solution of the problem [7].

3.5. Linear wave-like problem

Consider the following linear wave-like equation [5], [17]

$$y_{tt} = \frac{s^2}{2}y_{ss}, 0 < s < 1, t > 0, \quad (24)$$

with initial conditions $y(s,0) = s, y_t(s,0) = s^2$,
and boundary conditions $y(0,t) = 0, y(1,t) = 1 + \sinh t$.

The TAM will be applied as follows

$$L(y) = y_{tt}, N(y) = \frac{s^2}{2}y_{ss} \text{ and } h(s,t) = 0, \quad (25)$$

Thus, the primary problem is

$$L(y_0) = 0, \text{ with } y_0(s,0) = s, (y_0)_t(s,0) = s^2. \quad (26)$$

Then, the general iterative problem

$$L(y_{n+1}) + N(y_n) + h(s,t) = 0, y_{n+1}(s,0) = s, (y_{n+1})_t(s,0) = s^2. \quad (27)$$

By solving the primary problem we have got

$y_0(s,t) = s + ts^2.$

The first iteration can be done through and given as

$(y_1)_{tt} = \frac{s^2}{2}(y_0)_{ss}$ with $y_1(s,0) = s$, $(y_1)_t(s,0) = s^2$, (28)

Which has a solution

$y_1(s,t) = s + (t+\frac{t^3}{3!})s^2.$

The second iteration is

$(y_2)_{tt} = \frac{s^2}{2}(y_1)_{ss}$ with $y_2(s,0) = s$, $(y_2)_t(s,0) = s^2$, (29)

Which has a solution

$y_2(s,t) = s + (t+\frac{t^3}{3!}+\frac{t^5}{5!})s^2.$

Similarly, we get,

$y_3(s,t) = s + (t+\frac{t^3}{3!}+\frac{t^5}{5!}+\frac{t^7}{7!})s^2,$

$y_4(s,t) = s + (t+\frac{t^3}{3!}+\frac{t^5}{5!}+\frac{t^7}{7!}+\frac{t^9}{9!})s^2.$

This has the closed form:

$$y(s,t) = s + s^2 \sinh t. \quad (30)$$

which is the exact solution to the problem [5], [17].

3.6. The nonlinear wave-like problem

Consider the nonlinear wave-like equation

$$y_{tt} = syy_{ss} , 0 < s < 1 , t > 0, \quad (31)$$

with initial conditions $y(s,0) = 0$, $y_t(s,0) = s$,
and boundary conditions $y(0,t) = 0$, $y(1,t) = t$.

The TAM will be applied as follows

$$\mathrm{L(y)} = y_{tt} , \mathrm{N(y)} = y_{ss} \text{ and } \mathrm{h(s,t)} = 0, \quad (32)$$

Thus, the primary problem is

$$\mathrm{L(y_0)} = 0, \text{ with } y_0(s,0) = 0, (y_0)_t(s,0) = s. \quad (33)$$

A general iterative problem can be written in the following form

$$\mathrm{L}(y_{n+1}) + \mathrm{N}(y_n) + \mathrm{h(s,t)} = 0, y_{n+1}(s,0) = 0, (y_{n+1})_t(s,0) = s \quad (34)$$

By solving the primary problem, we get

$y_0(s,t) = st.$

The first iteration can be done through and given as

$$(y_1)_{tt} = (y_0)_{ss} \text{ with } y_1(s,0) = 0 , (y_1)_t(s,0) = s. \quad (35)$$

Then, the solution of equation (35) is

$y_1(s,t) = st.$

The second iteration is

$$(y_2)_{tt} = (y_1)_{ss} \text{ with } y_2(s,0) = 0 , (y_2)_t(s,0) = s. \quad (36)$$

Then, the solution of equation (36) is

$y_2(s,t) = st.$

Similarly, we get,

$y_3(s,t) = st,$

$y_4(s,t) = st.$

The solution is

$y(s,t) = st.$ (37)

which is the exact solution to the problem.

4. Solving heat and heat-like equations by using TAM

In this section, the TAM will be used to solve linear and nonlinear heat and heat-like equations. The linear and nonlinear heat and heat-like equations are given [17], [18]

$$y_t = y_{ss} , 0 < s < M , t > 0,$$

$$y_t = y_{ss} + h(s,t), 0 < s < M , t > 0,$$

$$y_t = \frac{s^2}{2} y_{ss} , 0 < s < M , t > 0.$$

The function h(s, t) is heat source. The TAM will be applied to obtain the exact solutions for these problems.

4.1. The homogeneous linear heat problem

Consider the following linear heat equation [18]

$$y_t = y_{ss} , 0 < s < \pi, t > 0, \quad (38)$$

with initial condition $y(s,0) = \sin s$,
and boundary conditions $y(0,t) = 0$, $y(\pi,t) = 0$.

The TAM will be applied as follows

$$\mathrm{L(y)} = y_t , \mathrm{N(y)} = y_{ss} \text{ and } \mathrm{h(s,t)} = 0, \quad (39)$$

Thus, the primary problem is

$$\mathrm{L(y_0)} = 0, \text{ with } y_0(s,0) = \sin s, \quad (40)$$

A general relation can be given as

$$\mathrm{L}(y_{n+1}) + \mathrm{N}(y_n) + \mathrm{h(s,t)} = 0, y_{n+1}(s,0) = \sin s. \quad (41)$$

By solving the primary problem, we get

$y_0(s,t) = \sin s.$

The first iteration can be done through and given as

$$(y_1)_t = (y_0)_{ss} \text{ with } y_1(s,0) = \sin s. \quad (42)$$

Then, the solution of equation (42) is

$y_1(s,t) = (1-t)\sin s.$

The second iteration is

$$(y_2)_t = (y_1)_{ss} \text{ with } y_2(s,0) = \sin s. \quad (43)$$

Then, the solution of equation (43) is

$y_2(s,t) = \left(1 - t + \frac{t^2}{2!}\right)\sin s.$

Similarly, we get,

$y_3(s,t) = \left(1 - t + \frac{t^2}{2!} - \frac{t^3}{3!}\right)\sin s,$

$y_4(s,t) = \left(1 - t + \frac{t^2}{2!} - \frac{t^3}{3!} + \frac{t^4}{4!}\right)\sin s.$

This has the closed form:

$$y(s,t) = e^{-t}\sin s. \quad (44)$$

which is the exact solution to the problem [18].

4.2 The inhomogeneous linear heat problem

Consider the following inhomogeneous linear heat equation [6], [8], [18]

$$y_t = y_{ss} + \cos s , 0 < s < \pi , t > 0, \quad (45)$$

with initial condition $y(s,0) = 0$,
and boundary conditions $y(0,t) = 1 - e^{-t}$, $y(\pi,t) = -1 + e^{-t}$.

The TAM will be applied as follows

$$\mathrm{L}(y) = y_t \text{ , } \mathrm{N}(y) = y_{ss} \text{ and } \mathrm{h}(s,t) = \cos s, \quad (46)$$

Thus the primary problem is

$$\mathrm{L}(y_0) = \cos s, \text{ with } y_0(s,0) = 0. \quad (47)$$

and the general iterative problem

$$\mathrm{L}(y_{n+1}) + \mathrm{N}(y_n) + \mathrm{h}(s,t) = 0, y_{n+1}(s,0) = 0. \quad (48)$$

By solving the primary problem, we get

$y_0(s,t) = t\cos s.$

The first iteration can be done through and given as

$$(y_1)_t = (y_0)_{ss} + \cos s \text{ with } y_1(s,0) = 0. \quad (49)$$

Thus, the solution of equation (49) is

$$y_1(s,t) = \left(t - \frac{t^2}{2!}\right)\cos s.$$

The second iteration is

$$(y_2)_t = (y_1)_{ss} + \cos s \text{ with } y_2(s,0) = 0. \quad (50)$$

Then, the solution of equation (50) is

$$y_2(s,t) = \left(t - \frac{t^2}{2!} + \frac{t^3}{3!}\right)\cos s.$$

Similarly, we get,

$$y_3(s,t) = \left(t - \frac{t^2}{2!} + \frac{t^3}{3!} - \frac{t^4}{4!}\right)\cos s,$$

$$y_4(s,t) = \left(t - \frac{t^2}{2!} + \frac{t^3}{3!} - \frac{t^4}{4!} + \frac{t^5}{5!}\right)\cos s.$$

This has the closed form:

$$y(s,t) = (1 - e^{-t})\cos s. \quad (51)$$

which is the exact solution to the problem [6], [8], [18].

4.3 Nonlinear heat problem

Consider the following nonlinear heat equation

$$y_t = y y_{ss} + s e^t \text{ , } 0 < s < 1 \text{ , } t > 0, \quad (52)$$

with initial condition $y(s,0) = 0$,

and boundary conditions $y(0,t) = 0, y(1,t) = t$.

The TAM will be applied as follows

$$\mathrm{L}(y) = y_t \text{ , } \mathrm{N}(y) = y y_{ss} \text{ and } \mathrm{h}(s,t) = s e^t, \quad (53)$$

Thus the primary problem is

$$\mathrm{L}(y_0) = 0, \text{ with } y_0(s,0) = 0. \quad (54)$$

A general problem of a relation can generate an iterative problem

$$\mathrm{L}(y_{n+1}) + \mathrm{N}(y_n) + \mathrm{k}(s,t) = 0, y_{n+1}(s,0) = 0. \quad (55)$$

By solving the primary problem, we obtain

$y_0(s,t) = s e^t.$

The first iteration can be done through and given as

$$(y_1)_t = y_0(y_0)_{ss} + s e^t \text{ with } y_1(s,0) = 0. \quad (56)$$

Then, the solution of equation (56) is

$y_1(s,t) = s e^t.$

The second iteration is

$$(y_2)_t = y_1(y_1)_{ss} + s e^t \text{ with } y_2(s,0) = 0. \quad (57)$$

Then, the solution of equation (57) is

$y_2(s,t) = s e^t.$

Similarly, we get,

$y_3(s,t) = s e^t,$

$y_4(s,t) = s e^t,$

The solution is

$$y(s,t) = s e^t. \quad (58)$$

which is the exact solution to the problem.

4.4. Linear heat-like problem

Consider the following linear heat-like equation [5]

$$y_t = \frac{s^2}{2} y_{ss} \text{ , } 0 < s < 1 \text{ , } t > 0, \quad (59)$$

with initial condition $y(s,0) = s^2$,

and boundary conditions $y(0,t) = 0 \text{ , } y(1,t) = e^t$.

The TAM will be applied as follows

$$\mathrm{L}(y) = y_t \text{ , } \mathrm{N}(y) = \frac{s^2}{2} y_{ss} \text{ and } \mathrm{h}(s,t) = 0. \quad (60)$$

Thus, the primary problem is

$$\mathrm{L}(y_0) = 0, \text{ with } y_0(s,0) = s^2. \quad (61)$$

A general iterative problem can be given as

$$\mathrm{L}(y_{n+1}) + \mathrm{N}(y_n) + \mathrm{k}(s,t) = 0, y_{n+1}(s,0) = s^2. \quad (62)$$

By solving the primary problem, we achieve

$y_0(s,t) = s^2.$

The first iteration can be done through and given as

$$(y_1)_t = \frac{s^2}{2}(y_0)_{ss} \text{ with } y_1(s,0) = s^2. \quad (63)$$

Then, the solution of equation (63) is

$y_1(s,t) = (1+t)s^2.$

The second iteration is

$$(y_2)_t = \frac{s^2}{2}(y_1)_{ss} \text{ with } y_2(s,0) = s^2. \quad (64)$$

Then, the solution of equation (64) is

$$y_2(s,t) = (1 + t + \frac{t^2}{2!})s^2.$$

Similarly, we get,

$$y_3(s,t) = (1 + t + \frac{t^2}{2!} + \frac{t^3}{3!})s^2,$$

$$y_4(s,t) = (1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \frac{t^4}{4!})s^2.$$

This has the closed form:

$$y(s,t) = s^2 e^t. \quad (65)$$

which is the exact solution to the problem [5].

4.5. Nonlinear heat-like problem

Consider the following nonlinear heat-like equation

$$y_t = x y y_{ss} + s + 1 \text{ , } 0 < s < 1 \text{ , } t > 0, \quad (66)$$

with initial condition $y(s,0) = 0$,

and boundary conditions $y(0,t) = t \text{ , } y(1,t) = 2t$.

The TAM will be applied as follows

$$\mathrm{L}(y) = y_t \text{ , } \mathrm{N}(y) = s y y_{ss} \text{ and } \mathrm{h}(s,t) = s + 1. \quad (67)$$

Thus the primary problem is

$$\mathrm{L}(y_0) = s + 1, \text{ with } y_0(x,0) = 0. \quad (68)$$

A general iterative problem can be written as

$$\mathrm{L}(y_{n+1}) + \mathrm{N}(y_n) + \mathrm{k}(s,t) = 0, y_{n+1}(s,0) = 0. \quad (69)$$

By solving the primary problem, we obtain

$y_0(s,t) = t(s+1).$

The first iteration can be done through and given as

$$(y_1)_t = s y_0(y_0)_{ss} + s + 1 \text{ with } y_1(s,0) = 0. \quad (70)$$

Then, the solution of equation (70) is

$y_1(s,t) = t(1+s).$

The second iteration is

$$(y_2)_t = s y_1(y_1)_{ss} + s + 1 \text{ with } y_2(s,0) = 0. \quad (71)$$

Thus, the solution of equation (71) is
$y_2(s,t) = t(1+s)$.
Similarly, we get,
$y_3(s,t) = t(1+s)$,
$y_4(s,t) = t(1+s)$.
The solution is

$$y(s,t) = t(1+s). \quad (72)$$

which is the exact solution to the problem.

5. Conclusion

In this paper, a semi-analytical iterative method namely (TAM) has been successfully implemented to obtain the exact solutions for linear and nonlinear wave, wave-like heat and heat-like equations. The exact solution obtained from the application of TAM identical to the results obtained with these methods available in the literature, such as ADM [1], [2], DGJ [5], [6], HPM [3], [4], VIM [7], [8]. It is seems that the TAM appears to be accurate to employ with reliable results and does not required any restricted assumption to deal with nonlinear terms.

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