New Position Vectors of Space Curves in Euclidean Three-Dimensional Space $\boldsymbol{E^3}$ According to Type-2 Bishop Frame with Constant Curvatures

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Abstract: ***In this paper we consider a unit speed curve α and we denoted by $\{T, N_1, N_2\}$ to the type-two Bishop Frame then study the position vectors of space curves in Euclidean 3- Space E^3 according to type-2 Bishop Frame with constant curvatures***

Keywords: Euclidean three-dimensional space; type-two Bishop Frame; position vectors

1. Introduction

The Bishop Frame has many properties that make it ideal for many research papers in mathematical have been treated in Minkowski space as in [8,9]. This information along with the initial position and orientation of the Bishop Frame provide all of the information necessary to define the curve [6]. The Bishop frame may have applications in the area of computer graphics, the biology areas and it may be so possible to compute information about the shape of sequences of DNA. Ali was accommodate position vector of an arbitrary curve in Galilean 3-space G3 [1]. Author was obtained and sketched some special case of position vectors in Galilean space [2]. In [3] Fathi studied the Position Vectors of Space Curves with constant curvatures in an arbitrary space curves in term of type-1 Bishop Frame in Euclidean 3- SpaceE^3.The Position Vectors in Euclidean 4-space E^4 was studied by the Authors he take the case that which curvatures was constant[4]. In [8] S. Yilmaz was make a system of differential equationinE_1^3 and then he discusssolution that which is give the components of the position vectors of some special space-like curves. In [5] the authors investigated the position vectors of a spacelike curve in the Minkowski 4-space R_1^4 , and he give some characterizations for spacelike curves which lie on some subspaces of R_1^4. In [6] the authors investigated a curve in Euclidean 3-space E^3. And he written the position vector as linear combination of its Bishop frame. Then he shows that its position vector satisfies the parametric equation. In Minkowski 3-spacesome characterization of space like inclined curves according to the type-2 Bishop frame was given by Yassinin [9], and also he show that the position vectors of rectifying curvesal ways lie on the orthogonal complement of the vector field.

The problem of the determination of the position vector of an arbitrary space curve according to type -2 Bishop Frame is still open in the Euclidean 3-space.

2. Preliminaries

In view to what is required in the next section, the following are the basic elements, of the theory of curves in the Euclidean space. Although these elements are briefly presented, the generalizations which give their overall idea are illustrated, with an inner product,

$$\langle x, y\rangle = \|x\|\|y\|cos\theta \qquad (2.1)$$

And a cross product,

$$x \times y = \|x\|\|y\|sin\theta\hat{n} \qquad (2.2)$$

If $x, y \in E^3$, and the angle between x and y is$\theta \in [0, \pi]$ we can define the orthogonality and parallelism conditions as:

$$x \perp y \;\Leftrightarrow\; \langle x, y\rangle = 0 \; and x \parallel y \;\Leftrightarrow\; x \times y = 0 \; (2.3)$$

Since $\hat{n}$ denoted to the positively oriented unit vector which is perpendicular to the plane spanned by x and y. The flat metric In Euclidean 3-space E^3 can defined as:

$$\langle\, ,\rangle = dx_1^2 + dx_2^2 + dx_3^2 \qquad (2.4)$$

The norm of a vector is $\|y\| = \sqrt{\langle y, y\rangle}$

The arbitrary curve $\alpha: I \subset R \longrightarrow E^3$ is said to be of unit speed if $\|\acute{\alpha}(s)\| = 1$.

Let $\{T, N, B\}$ be the Serret- Frenet frame of the curve α which have curvature $\kappa$and torsion τthen:

$$\begin{pmatrix} T \\ N \\ B \end{pmatrix}' = \begin{pmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & -\tau & 0 \end{pmatrix}\begin{pmatrix} T \\ N \\ B \end{pmatrix} \qquad (2.5)$$

Let $\{T, N_1, N_2\}$ Denote to the type-2 Bishop moving frame along the unit Speed curveα. Where the vectors T,N_1 and N_2 are mutually orthogonal vectors satisfying$\langle T, T\rangle =$ $\langle N_1, N_1\rangle = \langle N_2, N_2\rangle = 1$. And k_1 , k_2 the Bishop curvatures Then the type-2 Bishop formulas. And satisfying α are:

$$\begin{pmatrix} T \\ N_1 \\ N_2 \end{pmatrix}' = \begin{pmatrix} 0 & 0 & -k_1 \\ 0 & 0 & -k_2 \\ k_1 & k_2 & 0 \end{pmatrix}\begin{pmatrix} T \\ N_1 \\ N_2 \end{pmatrix} \qquad (2.6)$$

The relation between type-2 Bishop frame and Serret-Frenet frame, is written as:

$$N_2 = -\tau N = k_1 T + k_2 N_1 \qquad (2.7)$$

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By taking the norm to equation (2.7) , we get:

$$\tau(s) = \sqrt{{k_1}^2 + {k_2}^2}\,,\ \kappa(s) = \frac{d\theta(s)}{ds}$$

And also we can write $k_1 = -\tau\cos\theta(s)\, and k_2 = -\tau\sin\theta(s)\, whichleadto$

$$\theta(s) = \arctan\left(\frac{k_2}{k_1}\right) where k_1 \neq 0$$

We can write $= \int \kappa(s)ds$. if we take k_1 and k_2 as Rectangular coordinate form which that correspond to system for the polar coordinates.

I. The Position Vectors of Space Curves with constant Curvatures According to Type-2 Bishop Frame in Euclidean 3- Space E^3

In this section we will determinate of position vector in the Euclidean three space according to the type two Bishop frame. Let $\alpha(s)$ be an arbitrary space curve with constant curvatures in E^3 with respect to the type-2 Bishop frame $\{T, N_1, N_2\}$ then we can conclude that

$$\propto(s) = \lambda(s)T(s) + \mu(s)N_1(s) + \nu(s)N_2(s) \ (3.1)$$

Where $\lambda(s), \mu(s) and \nu(s)$ are differentiable functions of $s \in I \subset R$. The differentiation of the equation (3.1) with respect to s, can be written as:

$$\propto' = \lambda' T + \lambda T' + \mu' N_1 + \mu N_1' + \nu' N_2 + \nu N_2' \ (3.2)$$

Where $\propto' =$ T . And by using the type-2 Bishop equations, we get the following:

$$\begin{cases} \lambda' + \nu k_1 = 1 \\ \mu' + \nu k_2 = 0 \\ \nu' - \lambda k_1 - \mu k_2 = 0 \end{cases} \qquad (3.3)$$

The first equation of (3.3) leads to:

$$\dot{\lambda} = \frac{1}{k_1} - \nu \qquad (3.4)$$

By change of variables as in [3] and write $\theta = \int \kappa(s)ds$.
The second equation of (3.3) can be written as:

$$\dot{\mu} = \frac{-k_2\nu}{k_1} = \frac{-\nu}{f(\theta)} \ (3.5)$$

Since $\frac{k_1}{k_2} = f(\theta)$ and the derivative with respect to θdenoted by the dot

After we differentiate the three equations (3.4), (3.5) and (3.3) with respect to θ we find the differential equation

$$\ddot{\nu} + 2k_1\dot{\nu} = 1 \ (3.6)$$

From the solution of (3.8) we find

$$\nu(\theta) = \frac{1}{2k_1} + c_1\cos\sqrt{2k_1}s + c_2\sin\sqrt{2k_1}s \ (3.7)$$

Fromm the last equations (3.6) and (3.7), we find that $\lambda(\theta)$ is given by

$$\lambda(\theta) = \frac{1}{k_1}s + +c_1\sqrt{2k_1}\sin\sqrt{2k_1}s - c_2\sqrt{2k_1}\cos\sqrt{2k_1}\,s \ (3.8)$$

By substituted equations(3.7) in to equation (3.5) and integrate the both side we find $\mu(\theta)$ as:

$$\mu(\theta) = \frac{-1}{m}\left[\frac{1}{k_1}s + c_1\sqrt{2k_1}\sin\sqrt{2k_1}s - c_2\sqrt{2k_1}\cos\sqrt{2k_1}\,s\right] \ (3.9)$$

If we substitute equations (3.7), (3.8) and (3.9) in to (3.1) we find that the position vector of an arbitrary curve in the type-2 Bishop frame can be written as:

$$\begin{aligned} \propto(s) = {} & \left[\frac{1}{k_1}s + c_1\sqrt{2k_1}\sin\sqrt{2k_1}s - c_2\sqrt{2k_1}\cos\sqrt{2k_1}\,s\right]T(s) \\ & - \frac{1}{m}\left[\frac{1}{k_1}s + c_1\sqrt{2k_1}\sin\sqrt{2k_1}s - c_2\sqrt{2k_1}\cos\sqrt{2k_1}\,s\right]N_1(s) \\ & + \left[\frac{1}{2k_1} + c_1\cos\sqrt{2k_1}s + c_2\sin\sqrt{2k_1}s\right]N_2(s) \ (3.10) \end{aligned}$$

Thus we have the following theorem

Theorem 3.1: In Euclidean three space E^3 with respect to the type two Bishop frame if we take $\alpha(s)$ as space curve with constant curvatures. Then the position vector of $\alpha(s)$can be given by

$$\begin{aligned} \propto(s) = {} & \left[\frac{1}{k_1}s + c_1\sqrt{2k_1}\sin\sqrt{2k_1}s - c_2\sqrt{2k_1}\cos\sqrt{2k_1}\,s\right]T(s) \\ & - \frac{1}{m}\left[\frac{1}{k_1}s + c_1\sqrt{2k_1}\sin\sqrt{2k_1}s - c_2\sqrt{2k_1}\cos\sqrt{2k_1}\,s\right]N_1(s) \\ & + \left[\frac{1}{2k_1} + c_1\cos\sqrt{2k_1}s + c_2\sin\sqrt{2k_1}s\right]N_2(s) \end{aligned}$$

Where c_1, c_2 are arbitrary constants and $m = f(\theta)$.

3. Conclusion

This paper presented the problem of the determination of position vector of an arbitrary space curve according to the type-2 Bishop frame in the Euclidean 3- space. Our main problem we established a system of differential equations whose solution gives the components of the position vector of a curve on the type-2 Bishop frame.

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