On Rate of Convergence of Jungck-Picard-S. Iterative Scheme

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Abstract: ***This paper concerns with the convergence, rate of convergence, of Jungck-Picard-S.iterative scheme. We show that the previous iteration converges to a unique common fixed point when applied to a pair of Jungck-contraction mappings under certain condition. Also, we compare the speed of various Jungck-iterative schemes with Jungck-Picard-S for a pair of Jungck-contraction mappings using certain condition.***

Keywords: Jungck iterative schremes, Rate of convergence sequences, Convergent sequences, Jungck contraction mapping

1. Introduction and Preliminaries:

In 1976, Jungck[1] generalized Banach's contraction principle using the concept of commuting mappings which was given by Pfeffer[2] but Jungck has introduced it in more general context.

Proposition (1.1) [1]:
Let S be a mapping on a set X into itself. Thus S has a fixed point if and only if there is a constant mapping $T: X \to X$ which commutes with

Hence Jungck[1] has used this proposition and produced his theorem of common fixed point.

Theorem (1.2) [1]:
Let S be a continuous mapping of a complete metric space (X, d) into itself. Then S has a fixed point in X if and only if there exists $\delta \in (0,1)$ and a mapping $T: X \to X$ which commutes with Swhich satisfies $T(X) \subset S(X)$ and $d(Tx, Ty) \leq \delta d(Sx, Sy)$
And in 1986, Jungck[3], introduced more generalized commuting mappings, called compatible mappings which are useful for obtaining common fixed points of mappings.

Definition (1.3) [3]:
Let (X, d) be a metric space, $T, S: X \to X$ are said to be compatible if $\lim_{n\to\infty} d\big(TS(x_n), ST(x_n)\big) = 0$, where $\{x_n\}_{n=0}^{\infty}$ is a sequence such that $\lim_{n\to\infty} Tx_n = \lim_{n\to\infty} Sx_n = t$ for some $t \in X$.

Thus in 1996 Jungcket. al. [4] introduced the concept of coincidence point and depending on it , in 1998, Jungck and Rhoades [5] defined the notion of weakly compatible and showed that compatible mappings are weakly compatible but the converse is not true.

Definition (1.4) [5]:
Let B be a Banach space and, $T, S: B \to B$. A point $u^* \in B$ is called a coincidence point of a pair of self mappings T, S if there exists a point z (called a point of coincidence) in B such that $z = Su^* = Tu^*$. Two self mappings S and T are weakly compatible if they commute at there coincidence points, that is if $Su^* = Tu^*$ for some $u^* \in B$then $STu^* = TSu^*$. And the point $u^* \in B$ is called common fixed point of S and Tif $u^* = Su^* = Tu^*$. We abbreviate the set of coincidence points of S and $T$by $C(S,T)$.

In 2005, Singh et. al. [6] significantly improved on the result of Jungck[1] when he proved the following result which is now called Jungck-contraction principle.

Theorem (1.5) [6]:
Let (X, d) be a metric space. Let $T, S: X \to X$satisfying $d(Tx, Ty) \leq \delta d(Sx, Sy)$, $0 \leq \delta < 1$, for all $x, y \in X$. $T(X) \subseteq S(X)$and$S(X)$ or $T(X)$ is a complete subspace of X, then S and T have a coincidence. Indeed, for any $x_1 \in X$, there exists a sequence $\{x_n\}_{n=1}^{\infty}$ in X such that
1. $Sx_{n+1} = Tx_n, n = 1,2, \cdots$
2. $\{Sx_n\}_{n=1}^{\infty}$converges to Su^* for some u^* in X, and $Su^* = Tu^*$ that is S and T have a coincidence at u^*.

Furthermore, if S, T commute (just) at u^* then S and T have a unique common fixed point.

Remark (1.6):
If $S = id$ (identity mapping), then the Jungck-contraction mappingis the same as thewell known contraction mapping.
Olatinwo[7] and Chughet. al. [8] built on that work to introduce Jungck-SP iterative schemes and proved their convergences of the coincidence points of some pairs of certain mappings with the assumption that one of each of the pairs of mappings is injective. Their iterative schemes are defined as follows:

Definition (1.7) [7]:
Let B be a Banach space and C be a nonempty subset of B. Let $T, S: C \to C$ be two self mappings such that $T(C) \subseteq S(C)$. For $u_1 \in C$, the Jungck-Noor iterative scheme is the sequence $\{Su_n\}_{n=1}^{\infty}$ defined by

$$Su_{n+1} = (1 - \alpha_n)Su_n + \alpha_n Tv_n$$
$$Sv_n = (1 - \beta_n)Su_n + \beta_n Tw_n$$
$$Sw_n = (1 - \gamma_n)Su_n + \gamma_n Tu_n n \in \mathbb{N}$$

where $\{\alpha_n\}_{n=1}^{\infty}$, $\{\beta_n\}_{n=1}^{\infty}$ and $\{\gamma_n\}_{n=1}^{\infty}$ are real sequences in $[0,1)$ such that $\sum_{n=1}^{\infty} \alpha_n = \infty$.

Definition (1.8) [8]:
Let B be a Banach space and C be a nonempty subset of B. Let $T, S: C \to C$ be two self mappings such that $T(C) \subseteq S(C)$. For $p_1 \in C$, the Jungck-SP iterative scheme is the sequence $\{Sp_n\}_{n=1}^{\infty}$ defined by

$$Sp_{n+1} = (1-\alpha_n)Sq_n + \alpha_n Tq_n$$
$$Sq_n = (1-\beta_n)Sr_n + \beta_n Tr_n$$
$$Sr_n = (1-\gamma_n)Sp_n + \gamma_n Tp_n \quad n \in \mathbb{N}$$

where $\{\alpha_n\}_{n=1}^{\infty}$, $\{\beta_n\}_{n=1}^{\infty}$ and $\{\gamma_n\}_{n=1}^{\infty}$ are real sequences in $[0,1)$ such that $\sum_{n=1}^{\infty}\alpha_n = \infty$.

Hussain et. al. [9] introduced the Jungck-CR iterative scheme and proved its convergence to a unique common fixed point of a pair of certain mappings without assuming the injectivity of any of the mappings but rather they proved their results for a pair of weakly compatible mappings S,T.

Definition (1.9) [9]:

Let B be a Banach space and C be a nonempty subset of B. Let $T,S: C \to C$ be two self mappings such that $T(C) \subseteq S(C)$. For $a_0 \in C$, the Jungck-CR iterative scheme is the sequence $\{Sa_n\}_{n=1}^{\infty}$ defined by

$$Sa_{n+1} = (1-\alpha_n)Sb_n + \alpha_n Tb_n$$
$$Sb_n = (1-\beta_n)Ta_n + \beta_n Tc_n$$
$$Sc_n = (1-\gamma_n)Sa_n + \gamma_n Ta_n \quad n \in \mathbb{N}$$

where $\{\alpha_n\}_{n=1}^{\infty}$, $\{\beta_n\}_{n=1}^{\infty}$ and $\{\gamma_n\}_{n=1}^{\infty}$ are real sequences in $[0,1)$ such that $\sum_{n=1}^{\infty}\alpha_n = \infty$.

Recently, Badri[10] defined the following Jungck-Picard-S iterative scheme. In this section, we will prove its convergent to a unique common fixed point for a pair of Jungck-contraction mappings $S$and T.

Definition (1.10) [10]:

Let B be a Banach space and C be a nonempty subset of B. Let $T,S: C \to C$ be two self mappings such that $T(C) \subseteq S(C)$. For $x_1 \in C$, the Jungck-Picard-S iterative scheme is the sequence $\{Sx_n\}_{n=1}^{\infty}$ defined by

$$Sx_{n+1} = Ty_n$$
$$Sy_n = (1-\beta_n)Tx_n + \beta_n Tz_n$$
$$Sz_n = (1-\gamma_n)Sx_n + \gamma_n Tx_n \quad n \in \mathbb{N}$$

where$\{\beta_n\}_{n=1}^{\infty}$ and $\{\gamma_n\}_{n=1}^{\infty}$ are real sequences in $[0,1)$ such that $\sum_{n=1}^{\infty}\beta_n\gamma_n = \infty$.

2. Convergence of Jungck-Picard-S Iterative Scheme

In this section we will prove the convergence of Jungck – Picard – S. iteration

Theorem (2.1):

Let B be a Banach space and C be a nonempty closed convex subset of B, $S,T: C \to C$ be self-mappings satisfying Jungck-contraction condition (1.6), such that $T(C) \subseteq S(C)$ and S,T are weakly compatible. Suppose that there exists a $z \in C(S,T)$ such that $Sz = Tz = u^*$ and $\{Sx_n\}_{n=1}^{\infty}$ generated by (1.10) be the Jungck-Picard-S iterative scheme, where $\{\beta_n\}_{n=1}^{\infty}$ and $\{\gamma_n\}_{n=1}^{\infty}$ are real sequences in $[0,1)$satisfying $\sum_{n=1}^{\infty}\beta_n\gamma_n = \infty$. Then the Jungck-Picard-S iterative scheme $\{Sx_n\}_{n=1}^{\infty}$ converges to u^*. Moreover u^* is the unique common fixed point of S,T.

Proof:

First we prove $\{Sx_n\}_{n=1}^{\infty}$ converges to u^*. It follows from (1.6) and (1.10) that

$$\|Sx_{n+1} - u^*\| = \|Ty_n - u^*\|$$
$$\leq \delta\|Sy_n - u^*\|$$
$$\leq \delta(1-\beta_n)\|Tx_n - u^*\| + \beta_n\delta\|Tz_n - u^*\|$$
$$\leq (1-\beta_n)\delta^2\|Sx_n - u^*\| + \beta_n\delta^2\|Sz_n - u^*\|$$
$$\leq \delta^2[1-\beta_n\gamma_n(1-\delta)]\|Sx_n - u^*\|$$

Repeating this process n times, we obtain:

$$\|Sx_{n+1} - u^*\| \leq \delta^{2(n+1)}\prod_{k=1}^{n}[1-\beta_k\gamma_k(1-\delta)]\|Sx_1 - u^*\| \tag{2.1}$$

$$\leq \delta^{2(n+1)}\|Sx_1 - u^*\|e^{-(1-\delta)\sum_{k=1}^{n}\beta_k\gamma_k}$$

Since $\delta \in [0,1)$ and$\sum_{k=1}^{\infty}\beta_k\gamma_k = \infty$, $e^{-(1-\delta)\sum_{k=1}^{n}\beta_k\gamma_k} \to 0$ as n→∞, which implies$\lim_{n\to\infty}\|Sx_n - u^*\| = 0$. Thus $\{Sx_n\}_{n=1}^{\infty}$ converges to u^*.

Now, we will prove u^* is the unique common fixed point of S,T.Suppose that there exist two points of coincidence z_1, $z_2 \in C(S,T)$ such that $Sz_1 = Tz_1 = u_1^*$ and $Sz_2 = Tz_2 = u_2^*$.

Using condition (1.6), we have

$$0 \leq \|u_1^* - u_2^*\| = \|Tz_1 - Tz_2\| \leq \delta\|Sz_1 - Sz_2\|$$
$$= \delta\|u_1^* - u_2^*\|$$

Thus $u_1^* = u_2^*$.

Now, since S,T are weakly compatible and $u^* = Tz = Sz$, then $Tu^* = Su^*$, but since the point of coincidence is unique, so $u^* = Tu^*$. Thus $Tu^* = Su^* = u^*$. Therefore u^* is the unique common fixed point of S,T.

3. Rate of Convergence of Jungck-Picard-S Iterative Scheme

In this section we compare the speed of Jungck-Picard-S iterative scheme (1.10) and the speed of Jungck-Noor (1.7), Jungck-SP (1.8) and Jungck-CR (1.9) iterative schemes by the following theorem.

Theorem (3.1):

Let C be a nonempty closed convex subset of a Banach space B, $S,T: C \to C$ be self-mappings satisfying Jungck-contraction condition (1.6) assume $T(C) \subseteq S(C)$, let $\{Sx_n\}_{n=1}^{\infty}$, $\{Su_n\}_{n=1}^{\infty}$, $\{Sp_n\}_{n=1}^{\infty}$ and $\{Sa_n\}_{n=1}^{\infty}$ be the Jungck-Picard-S (1.10), Jungck-Noor (1.7), Jungck-SP (1.8) and Jungck-CR (1.9) iterative schemes respectively satisfying $\lim_{n\to\infty}\alpha_n = \lim_{n\to\infty}\beta_n = \lim_{n\to\infty}\gamma_n =$ lim$n\to\infty an\beta n\gamma n$=lim$n\to\infty\beta n\gamma n$=0. Then $Sxnn=1\infty$ converges to u^* faster than $\{Su_n\}_{n=1}^{\infty}$, $\{Sp_n\}_{n=1}^{\infty}$ and $\{Sa_n\}_{n=1}^{\infty}$ do.

Proof:

From inequality (2.1), we have

$$\|Sx_{n+1} - u^*\| \leq \delta^{2(n+1)}\prod_{k=1}^{n}[1-\beta_k\gamma_k(1-\delta)]\|Sx_1 - u^*\| \tag{3.1}$$

From Jungck-Noor iteration (1.7) and Junck-contraction condition (1.6), we get:

$$\|Su_{n+1} - u^*\| = \|(1-\alpha_n)Su_n + \alpha_n Tv_n - u^*\|$$
$$\leq (1-\alpha_n)\|Su_n - u^*\| + \alpha_n\|Tv_n - u^*\|$$
$$\leq (1-\alpha_n)\|Su_n - u^*\| + \alpha_n\delta\|Sv_n - u^*\| \tag{3.2}$$

By the same argument we get

$$\|Sv_n - u^*\| \leq (1-\beta_n)\|Su_n - u^*\| + \beta_n\delta\|Sw_n - u^*\| \tag{3.3}$$

And

$$\|Sw_n - u^*\| \le (1-\gamma_n)\|Su_n - u^*\| + \gamma_n\delta\|Su_n - u^*\| \quad (3.4)$$

Combining (3.2), (3.3) and (3.4), and since $\delta \in [0,1)$and $\alpha_n, \beta_n \in [0,1]$, $n \in \mathbb{N}$, we get:

$$\|Su_{n+1} - u^*\| \le [1 - \alpha_n\beta_n\gamma_n(1-\delta)]\|Su_n - u^*\|$$

Repeating this process n times, we get:

$$\|Su_{n+1} - u^*\| \le \prod_{k=1}^{n}[1-\alpha_k\beta_k\gamma_k(1-\delta)]\,\|Su_1 - u^*\| \quad (3.5)$$

Using (3.1) and (3.5), we obtain:

$$\frac{\|Sx_{n+1} - u^*\|}{\|Su_{n+1} - u^*\|} \le \frac{\delta^{2(n+1)}\prod_{k=1}^{n}[1-\beta_k\gamma_k(1-\delta)]\|Sx_1 - u^*\|}{\prod_{k=1}^{n}[1-\alpha_k\beta_k\gamma_k(1-\delta)]\,\|Su_1 - u^*\|}$$

Define $\theta_n = \delta^{2(n+1)}\prod_{k=1}^{n}\frac{[1-\beta_k\gamma_k(1-\delta)]}{[1-\alpha_k\beta_k\gamma_k(1-\delta)]}$

By the assumption

$$\lim_{n\to\infty}\frac{\theta_{n+1}}{\theta_n} = \lim_{n\to\infty}\frac{\delta^2[1-\beta_{n+1}\gamma_{n+1}(1-\delta)]}{[1-\alpha_{n+1}\beta_{n+1}\gamma_{n+1}(1-\delta)]} = \delta^2 < 1$$

Thus it follows from ratio test that $\sum_{n=1}^{\infty}\theta_n < \infty$. Hence, we have $\lim_{n\to\infty}\theta_n = 0$ which implies that the iterative sequence defined by Jungck-Picard-S (1.10) converges to u^* faster than the iterative sequence defined by Jungck-Noor iteration method (1.7).

From Jungck-SP iteration (1.8) and Jungck-contraction condition (1.6), we have:

$$\|Sp_{n+1} - u^*\| \le (1-\alpha_n)\|Sq_n - u^*\| + \alpha_n\|Tq_n - u^*\|$$
$$\le [1-\alpha_n(1-\delta)]\|Sq_n - u^*\| \quad (3.6)$$

By the same argument we have

$$\|Sq_n - u^*\| \le \|Sr_n - u^*\| \quad (3.7)$$

Therefore

$$\|Sr_n - u^*\| \le [1-\gamma_n(1-\delta)]\|Sp_n - u^*\| \quad (3.8)$$

Combining (3.6), (3.7) and (3.8), we have:

$$\|Sp_{n+1} - u^*\| \le [1-\gamma_n(1-\delta)]\|Sp_n - u^*\|$$

And so on, we get:

$$\|Sp_{n+1} - u^*\| \le \prod_{k=1}^{n}[1-\alpha_k(1-\delta)][1-\beta_k(1-\delta)]\,[1-\gamma_k(1-\delta)]\,\|Sp_1 - u^*\| \quad (3.9)$$

Using (3.1) and (3.9), we obtain:

$$\frac{\|Sx_{n+1} - u^*\|}{\|Sp_{n+1} - u^*\|} \le \frac{\delta^{2(n+1)}\prod_{k=1}^{n}[1-\beta_k\gamma_k(1-\delta)]\|Sx_1 - u^*\|}{\prod_{k=1}^{n}[1-\alpha_k(1-\delta)][1-\beta_k(1-\delta)]\,[1-\gamma_k(1-\delta)]\|Sp_1 - u^*\|}$$

Define $\theta_n = \delta^{2(n+1)}\prod_{k=1}^{n}\frac{[1-\beta_k\gamma_k(1-\delta)]}{[1-\alpha_k(1-\delta)][1-\beta_k(1-\delta)][1-\gamma_k(1-\delta)]}$

By the assumption

$$\lim_{n\to\infty}\frac{\theta_{n+1}}{\theta_n} = \lim_{n\to\infty}\frac{\delta^2[1-\beta_{n+1}\gamma_{n+1}(1-\delta)]}{[1-\alpha_{n+1}(1-\delta)][1-\beta_{n+1}(1-\delta)][1-\gamma_{n+1}(1-\delta)]} = \delta^2 < 1$$

Thus it follows from ratio test that$\lim_{n\to\infty}\theta_n = 0$ which implies that the iterative sequence defined by Jungck-Picard-S (1.10) converges to u^* faster than the iterative sequence defined by Jungck-SP iteration method (1.8).

From Jungck-CR iteration (1.9) and Jungck-contraction condition (1.6), we get:

$$\|Sa_{n+1} - u^*\| \le [1-\alpha_n(1-\delta)]\|Sb_n - u^*\| \quad (3.10)$$

Also

$$\|Sb_n - u^*\| \le (1-\beta_n)\delta\|Sa_n - u^*\| + \beta_n\delta\|Sc_n - u^*\| \quad (3.11)$$

And

$$\|Sc_n - u^*\| \le [1-\gamma_n(1-\delta)]\|Sa_n - u^*\| \quad (3.12)$$

Combining (3.10), (3.11) and (3.12), we have:

$$\|Sa_{n+1} - u^*\| \le [1-\alpha_n(1-\delta)]\delta(1-\beta_n)\|Sa_n - u^*\|$$
$$+[1-\alpha_n(1-\delta)]\beta_n\delta[1-\gamma_n(1-\delta)]\|Sa_n - u^*\|$$
$$= \delta[1-\alpha_n(1-\delta)][1-\beta_n\gamma_n(1-\delta)]\|Sa_n - u^*\|$$

Repeating this process n times, we get:

$\|Sa_{n+1} - u^*\| \le \delta^{(n+1)}\prod_{k=1}^{n}[1-\alpha_k(1-\delta)][1-$
$\beta k\gamma k1-\delta Sa1-u*$(3.13)

Using (3.1) and (3.13), we obtain:

$$\frac{\|Sx_{n+1} - u^*\|}{\|Sa_{n+1} - u^*\|} \le \delta^{(n+1)}\frac{\|Sx_1 - u^*\|}{\prod_{k=1}^{n}[1-\alpha_k(1-\delta)]\,\|Sa_1 - u^*\|}$$

Define $\theta_n = \frac{\delta^{(n+1)}}{\prod_{k=1}^{n}[1-\alpha_k(1-\delta)]}$

By the assumption

$$\lim_{n\to\infty}\frac{\theta_{n+1}}{\theta_n} = \lim_{n\to\infty}\frac{\delta}{[1-\alpha_{n+1}(1-\delta)]} = \delta < 1$$

Thus it follows from ratio test that $\sum_{n=1}^{\infty}\theta_n < \infty$. Hence, we have $\lim_{n\to\infty}\theta_n = 0$ which implies that the iterative sequence defined by Jungck-Picard-S (1.10) converges to u^* faster than the iterative sequence defined by Jungck-CR iteration method (1.9).

Now to support our result in the above theorem, with the help of computer programs in java, we give an example for comparing the speed of Jungck-Picard-S iterative scheme (1.10) and the speed of Jungck-Noor (1.7), Jungck-SP (1.8) and Jungck-CR (1.9) iterative schemes.

Example (3.2): Let $B = \mathbb{R}$, $C = [1,4]$, $S, T: C \to C$ are mappings defined as $Sx = x^2$ and $Tx = \frac{1+x}{2}$ for all $x \in C$. It is easily seen that the mappings S and T satisfying Jungck-contraction condition (1.6) with the unique common fixed point 1 take $\alpha_n = \beta_n = \gamma_n = 0.1$or all $n = 1, \cdots, 253$ with initial value 0.6. The comparison of the rate of convergence of the speed Jungck-Picard-S iterative scheme (1.10) and the speed of Jungck-Noor (1.7), Jungck-SP (1.8) and Jungck-CR (1.9) iterative schemes to a common fixed point of S and T is shown in the following tables.

Jungck-Picard-S Iterative Scheme			
N	Sx_{n+1}	Tx_n	x_{n+1}
1	0.99073574014397030	0.96300226355683090	0.99535709177358570
2	0.99942366303823990	0.99767854588679280	0.99971178998661400
3	0.99996424268967280	0.99985589499330700	0.99998212118501040
4	0.99999778190468420	0.99999106059250520	0.99999889095172710
5	0.99999986240868020	0.99999944547586360	0.99999993120433770
6	0.99999999146503820	0.99999996560216890	0.99999999573251910
7	0.99999999947056570	0.99999999786625950	0.99999999973528290
8	0.99999999996715850	0.99999999986764140	0.99999999998357920
9	0.99999999999796270	0.99999999999178970	0.99999999999898140
10	0.99999999999987370	0.99999999999949060	0.99999999999993680
11	0.99999999999999220	0.99999999999996850	0.99999999999999610
12	0.99999999999999960	0.99999999999999800	0.99999999999999980
13	1.00000000000000000	0.99999999999999990	1.00000000000000000
14	1.00000000000000000	1.00000000000000000	1.00000000000000000

Jungck-CR Iterative Scheme			
N	Sx_{n+1}	Tx_n	x_{n+1}
1	0.88007642291401620	0.86939502714649830	0.93812388463039150
2	0.97159511324417220	0.96906194231519580	0.98569524359417100
3	0.99343356653019540	0.99284762179708550	0.99671137574033710
4	0.99849041316792920	0.99835568787016850	0.99924492151220320
5	0.99965339508567460	0.99962246075610160	0.99982668252336350
6	0.99992044184664640	0.99991334126168170	0.99996022013210430
7	0.99998173979670350	0.99998011006605210	0.99999086985667200
8	0.99999580897885280	0.99999543492833600	0.99999790448723080
9	0.99999903809415210	0.99999895224361550	0.99999951904696040
	⋮	⋮	⋮
22	0.99999999999999530	0.99999999999999490	0.99999999999999770
23	0.99999999999999890	0.99999999999999890	0.99999999999999940
24	0.99999999999999980	0.99999999999999980	0.99999999999999990
25	1.00000000000000000	1.00000000000000000	1.00000000000000000

Jungck-Noor Iterative Scheme			
N	Sx_{n+1}	Tx_n	x_{n+1}
1	0.12369127957207945	0.62486725945469360	0.35169771050161736
2	0.18268045524212984	0.67584885525080860	0.42741134196711467
3	0.23884042408530287	0.71370567098355730	0.48871302835641990
4	0.29196813683769040	0.74435651417820990	0.54034075992626210
5	0.34200815810894114	0.77017037996313100	0.58481463568291550
⋮	⋮	⋮	⋮
252	0.99999999817978070	0.99999999950702460	0.99999999908989030
253	0.99999999831979540	0.99999999954494510	0.99999999915989770

	Jungck-SP Iterative Scheme		
N	Sx_{n+1}	Tx_n	x_{n+1}
1	0.31654656900855230	0.70115601429551420	0.56262471418215560
2	0.44805414213558790	0.78131235709107780	0.66936846514874590
3	0.55679207518572800	0.83468423257437300	0.74618501404526220
4	0.64544521756179670	0.87309250702263110	0.80339605274223050
5	0.71711162317314380	0.90169802637111520	0.84682443468120580
⋮	⋮	⋮	⋮
252	0.99999999999999930	0.99999999999999980	0.99999999999999970
253	0.99999999999999930	0.99999999999999980	0.99999999999999970

From the above tables, we observe the decreasing order of convergence of Jungck iterative schemes as follows: Jungck-Picard-S, Jungck-CR, Jungck-SP and Jungck-Noor, iterative schemes.

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