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T-Pure Fuzzy Submodules

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Abstract: ***The main aim of this paper is to extend and study the notion of (ordinary) T-pure submodule into T-pure fuzzy submodule and T-pure ideal into T-pure fuzzy ideal. This lead us to introduced and study other notions such as T-pure fuzzy submodule and T-pure fuzzy ideal***

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1. Introduction

Let X be a fuzzy module of an R-module M, we denoted by X-F(M), it is well known that A is fuzzy submodules of X denoted by F-S(X) is called T-pure fuzzy submodule of X. if for each fuzzy ideal **I** of R such that $I^2X \cap A = I^2A$. And an fuzzy ideal I of a ring R denoted by F- I(R) is called T-pure fuzzy ideal of R if for each fuzzy ideal J^2 of R $J^2 \cap I = J^2 I$

In this paper, we fuzzify these concepts T-pure fuzzy submodule and T-pure fuzzy ideal , moreover we generalize many properties of T-pure fuzzy submodule and T-pure fuzzy ideal

This paper consists of two part. In part one, various basis properties about T-pure fuzzy submodule are discussed . part two included T-pure fuzzy ideal and basic properties about this concept

1.1 T-pure Fuzzy submodule

In this section we inerdue the concept of T-pure fuzzy submodule by of provided some properties of this concepts.

Definition (1.1): Let X-F(M) .let A be a F-S(X). A is called a T-pure fuzzy submodule if for each fuzzy ideal **K** of R, $KX \cap A = KA$. [4]

Proposition (1.2):
Let X-F(M) ,and let A be fuzzy sub modules of X. Then A is a T-pure F-S(X) $\Leftrightarrow$ A_t is a T-pure submodules of X_t. $\forall$ t $\in$ (0,1]

Definition (1.3):
Let X-F(M) and let A be a F-S(X). A is called T-pure F-S(X) . if for each fuzzy ideal **I** of R such that $I^2X \cap A = I^2A$.

Proposition (1.4):
Let X-F(M) and let A be fuzzy sub modules of X. Then A is T-pure fuzzy submodule of X if and only if A_t is T-pure submodules of X_t. $\forall$ t $\in$ (0,1]

Proof:
Let J be an ideal of ring R

Define $I^2: R \to [0,1]$ by $I^2(x) = \begin{cases} t & if\ x \in J \\ o & otherwise \end{cases}$ $\forall$ t∈(0,1]

And let N ≪ M

Define A: M⟶[0,1] by $A(x) = \begin{cases} t & if\ x \in N \\ o & otherwise \end{cases}$ $\forall$ t∈(0,1]

It is clear that I^2 is F-I(R) and A is F-S(X).

Now, $A_t = N$, $I_t^2 = J$, $X_t = M$

(⟹) Let A is T-pure fuzzy submodule of X .To prove A_t is T-pure submodules of X_t.
$\forall$ t∈(0,1].

To show that $I_t^2 X_t \cap A_t = A_t I_t^2$

$I_t^2 X_t \cap A_t = (I^2X)_t \cap A_t$ by[6]
$= (I^2X \cap A)_t$ by[1]
$=(I^2A)_t$ since A is T-pure
$=A_t I_t^2$

Thus A_t is T-pure submodules of X_t. $\forall$ t∈(0,1].

Conversely Let I^2 be F-I(R) and A be a F-S(X).

T.p A is T-pure fuzzy submodule of X

$(I^2X \cap A)_t = (I^2X)_t \cap A_t$ $\forall$ t∈(0,1].
$= I_t^2 X_t \cap A_t$

but A_t is T-pure submodules of X_t.

Then $I_t^2 X_t \cap A_t = I_t^2 A_t$
$= (I^2A)_t$

Hence $(I^2X \cap A)_t = (I^2A)_t$

$I^2X \cap A = I^2A$

Therefore A is T-pure fuzzy submodule of X .

Remarks and Examples (1.5):

1- Let X-F(M) and let A be a pure F-S(X) , Then A is T-pure F-S(X).

Proof:
It is clear

The converse not true by

Example: Let M=Z_4 as Z-module and N=2Z_4

Define X: M⟶[0,1] by $X(x) = \begin{cases} 1\ if\ x \in M \\ o\ otherwise \end{cases}$

Define A: M⟶[0,1] by $A(x) = \begin{cases} t\ if\ x \in N \\ o\ otherwise \end{cases}$ $\forall$ t∈(0,1]

It is clear that X is F(M) , A is F-S(X) and X_t=M ,A_t=N

A_t is T-pure submodules of X_t. by[7]

Thus A is T-pure fuzzy submodule of X by (Proposition1.4)

But A is not pure fuzzy submodule of X since if I_t=2Z where I:R⟶[0,1]

Such that I(x)=t if x∈2Z and I(x)=0 if x∉2Z

Now 2Z.4Z∩ $2Z_4$={$\overline{0},\overline{2}$} but 2Z.$2Z_4$=2{$\overline{0},\overline{2}$}={$\overline{0}$}

Thus A_t is not pure submodules of X_t

Therefore A is not pure fuzzy submodule of X. [4]

2- Let X-F(M) . It is clear that the fuzzy singleton $\{o_t\}$ and X are always T-pure fuzzy submodule of X. ∀ t∈(0,1]

3-In the fuzzy module Z as Z-module. The only T-pure fuzzy submodule are fuzzy singleton $\{o_t\}$ and X

Proof:

Let X: Z⟶[0,1] by $X(x) = \begin{cases} 1 \text{ if } x \in Z \\ o \text{ otherwise} \end{cases}$

Define O_t: X⟶[0,1] by $O_t(x) = \begin{cases} t \text{ if } x = 0 \\ o \text{ otherwise} \end{cases}$ ∀ t∈(0,1]

X_t=Z and O_t=0

By(2) clear that X and $O_t$are T-pure fuzzy submodule

If there exists a fuzzy submodule

A:nZ⟶[0,1] by $A(x) = \begin{cases} t \text{ if } x \in nZ \\ o \text{ otherwise} \end{cases}$ ∀ t∈(0,1]

A_t=nZ

Let I(x):$\langle n\rangle^2$ ⟶[0,1] by $I(x) = \begin{cases} t \text{ if } x \in \langle n\rangle^2 \\ o \text{ otherwise} \end{cases}$ ∀ t∈(0,1]

It is clear that I_t=$\langle n\rangle^2$ and I is a fuzzy ideal

n^2=n^2. $1\epsilon\langle n\rangle^2$Z∩nZ= $\langle n\rangle^2$.nZ=n^3Z

But $n^2 \notin n^3$Z Thus A_t is not T- pure by [7]

X_t=Z , O_t=0 only two T-pure submodule of Z-module ∀ t∈(0,1] by (Proposition1.4)

4-Let X be a fuzzy module of an Z-module Q. and let A be a non-empty FC-S(X) . then A is not T-pure F-S(X).

Proof:

Define X: Q⟶[0,1] by $X(x) = \begin{cases} 1 \text{ if } x \in Q \\ o \text{ otherwise} \end{cases}$

Define A: Q⟶[0,1] by $A(x) = \begin{cases} 1 \text{ if } x \in N \\ o \text{ otherwise} \end{cases}$ ∀ t∈(0,1]

where N is submodule of Q, X_t=Q and A_t=N

N is not T- pure fuzzy submodule of Q by [7]

Then A is not T-pure F-S(X) by (Proposition(1.4))

5- Let X-F(M) . let A be a T-pure F-S(X) such that A≅B where B is F-S(X) , then B is not T- pure F-S(X) for example.

Example: Let M=Z

Let X: M⟶[0,1] by $X(x) = \begin{cases} 1 & \text{if } x \in M \\ o & \text{otherwise} \end{cases}$

Let A: Z⟶[0,1] by $A(x) = \begin{cases} t & \text{if } x \in Z \\ o & \text{otherwise} \end{cases}$ ∀ t∈(0,1]

Let B: Z⟶[0,1] by $B(x) = \begin{cases} t & \text{if } x \in 2Z \\ o & \text{otherwise} \end{cases}$ ∀ t∈(0,1]

It is clear that A and B are F-S(X), Now A_t=Z and B_t=2Z and Z≅2Z but 2Z is not T- pure submodules [7]

B is not T-pure fuzzy submodule by (Proposition(1.4))

Proposition (1.6):

Let X-F(M), and let A and B are two F-S(X) . if A is T-pure F-S(X), B ⊆ A. and B is T-pure F-S(A) , then B is T-pure fuzzy submodule of X.

Proof:

Since A be a T-pure F-S(X) then $I^2X \cap A = I^2A$…..(1)

where I^2 is F-I(R) and since B be a T-pure F-S(A) then

$I^2A \cap B = I^2B$….(2)

Now, we get $I^2B = I^2A \cap B$….(2)

= $(I^2X \cap A)$∩B…..(1)

= $I^2X \cap (A \cap B)$ since B ⊆ A

Therefore B is T-pure fuzzy submodule of X .

Proposition(1.7):

Let X-F(M) . and let C be a T-pure F-S(X). If B is a F-S(X) contating A, then A is T-pure F-S(B)

Proof :

Let I^2 be a F-I(R) and let C be a T-pure F-S(X)

Hence $I^2X \cap C = I^2C$

Now, $I^2B \cap C = (I^2B \cap I^2X) \cap C$ since C ⊆B⊆X

$=I^2B \cap (I^2X \cap C)$

$=I^2B \cap I^2C$

$=I^2C$

Thus C is T-pure fuzzy submodule of a fuzzy submodule B .

“Definition (1.8):

Let X ,Y -F(M_1, M_2)respectively. Define X⨁Y: M_1⨁M_2 ⟶ [0,1] by (X⨁Y)(a, b) = min{X(a), Y(b) for all (a, b) ∈ M_1⨁M_2} X⨁Y is called a fuzzy external direct sum of X and Y. [9]”

Lemma(1.9):

Let N_1 and N_2 be two S(M_1) and S (M_2) if $N_1 \oplus N_2$ is T-pure submodule of $M_1 \oplus M_2$ then N_1 and N_2 are T-pure submodule in M_1 and M_2.

Proof:

T.p $I^2 M_1 \cap N_1 = I^2N_1$ and $I^2 M_2 \cap N_2 = I^2N_2$ for each ideal I^2 of R.

Since $N_1 \oplus N_2$ is T-pure in $M_1 \oplus M_2$ we get:

$I^2(M_1 \oplus M_2) \cap (N_1 \oplus N_2) = I^2(N_1 \oplus N_2)$

$(I^2 M_1 \oplus I^2 M_2) \cap (N_1 \oplus N_2) = I^2N_1 \oplus I^2N_2$

Hence $I^2 M_1 \cap N_1 = I^2N_1$ and $I^2M_2 \cap N_2$=I^2N_2

Thus N_1 and N_2 are T-pure.

Proposition(1.10):

Let X_1-F(M_1) and X_2-F(M_2), If A , B be are two F-S(X_1) and F-S(X_2). respectiveely then A and B are T-pure fuzzy submodule of X_1 andX_2 if and only if A⨁B is T-pure fuzzy submodule of $X_1 \oplus X_2$.

Proof:

(⟹) $A_t \oplus B_t$=$(A \oplus B)_t$ and $({X_1}_t \oplus {X_2}_t)$=$(X_1 \oplus X_2)_t$. $\forall t \in (0,1]$ by [4]

Therefore $(A \oplus B)_t$ is T-pure of $(X_1 \oplus X_2)_t$ by [7]

Thus A⨁B is T-pure fuzzy submodule of $X_1 \oplus X_2$. (Proposition(1.4))

⟸ let $A \oplus B$ is T-pure fuzzy submodule of $X_1 \oplus X_2$.

To show that A and B are T-pure fuzzy submodule of X_1, X_2 respectively.

By [4, lemma (2.2.4)] and (Proposition1.4) we get :-

$(A \oplus B)_t$=$A_t \oplus B_t$ is T-pure in module $(X_1)_t \oplus (X_2)_t$

Thus A_t and $B_t$are T-pure submodule of $(X_1)_t$ **and** $(X_2)_t$ by (lemma (1.9))

Therefore A and B are two T-pure fuzzy submodules of a fuzzy modules X_1 ***and*** X_2 by (Proposition(1.4)).

Proposition(1.11):

Let H be a direct summand of a fuzzy module X. then H is T-pure fuzzy submodule of X.

Proof:

Let X $=H \oplus C$, where C is a F-S(X) and H is a direct summand of X.

Such that X=H+C and $H \cap C = 0$ by[def of fuzzy direct summand]

To prove H is T-pure (i.e $I^2X \cap H = I^2H$ for each I^2 is F-I(R))

$I^2X \cap H = I^2(H \oplus C) \cap H$

$=(I^2H \oplus I^2C) \cap (H \oplus 0)$

$=(I^2H \cap H) \oplus (I^2C \cap 0)$ by[4]

$=(I^2H \cap H) \oplus 0$

$= I^2H \cap H$

$=I^2H$ since $I^2H \subseteq H$

Therefore H is T-pure fuzzy submodule of X.

Proposition(1.12):

let f :X $\longrightarrow$ Y be epimorphism of X_1-F(M_1) and X_2-F(M_2) respectively, let B be a fuzzy submodule of X and X is f-invariant , if B is a T-pure F-S(X) ,then f(B) is T-pure F-S(Y).

Proof:

To prove $I^2Y \cap f(B) = I^2f(B)$ for each fuzzy ideal I of R.

$I^2Y \cap f(B) = I^2f(X) \cap f(B)$ since f is epimorphism

$=f(I^2X) \cap f(B)$ by[4]

$= f(I^2X \cap B)$ by[3]

$= f(I^2B)$ since B is T-pure

$=I^2f(B)$ by[4]

Thus f(B) is T-pure fuzzy submodule of Y.

Proposition(1.13):

let f :X $\longrightarrow$ Y be epimorphism of X_1-F(M_1)and X_2-F(M_2) respectively, such that every submodule of X is f-invariant , if C is T-pure F-S(Y), then f^{-1} (C) is T-pure fuzzy submodule of X.

Proof:

To prove f^{-1} (C) is T-pure (i.e $I^2X \cap f^{-1}(C) = I^2 f^{-1}(C)$ for each I^2 is F-I(R)).

$f(I^2X \cap f^{-1}(C)) = f(I^2X) \cap f(f^{-1}(C))$ by[3]

$=I^2f(X) \cap C$ by[4]

$=I^2Y \cap C$ since f is epimorphism

$= I^2C$ since A is T-pure

Therefore $f(I^2X \cap f^{-1}(C)) = I^2C$ so $f^{-1}[f(I^2X \cap f\text{-}1C) = f\text{-}1(I2C)$

But $f^{-1}[f(I^2X \cap f^{-1}(A))] = I^2X \cap f^{-1}(C)$ since by hyposse and

$f^{-1}(I^2C) = I^2 f^{-1}(C)$ by[4]

Thus $I^2X \cap f^{-1}(C) = I^2 f^{-1}(C)$

Lemma(1.14):

Let$\{I_i, i \in N\}$ be an ascending chain of T-pure fuzzy submodules of a F(X) and let A be a F-I(R), then $A^2[\cup_{i\in N} I_i] = \cup_{i\in c}[A^2I_i]$.

Proof:

Since $A^2I_i \subseteq A^2[\cup_{i\in N} I_i], \forall i \in N$

implies that $\cup_{i\in c}[A^2I_i] \subseteq A^2[\cup_{i\in N} I_i]$……(1)

and by $A^2[\cup_{i\in N} I_i] \subseteq A^2$ and $A^2[\cup_{i\in N} I_i] \subseteq \cup_{i\in N} I_i$

since $\cup_{i\in N} I_i$ is fuzzy submodule of X

then $A^2[\cup_{i\in N} I_i] \subseteq A^2X \cap [\cup_{i\in N} I_i]$

$=\cup_{i\in c} [A^2X \cap I_i]$ by[4]

$=\cup_{i\in c} [A^2I_i]$ since I_i T-pure fuzzy submodule

Thus $A^2[\cup_{i\in N} I_i] \subseteq \cup_{i\in c} [A^2I_i]$

Therefore $A^2[\cup_{i\in N} I_i] = \cup_{i\in c}[A^2I_i]$

Proposition(1.15):

If $\{I_i, i \in N\}$ be an ascending chain of T-pure fuzzy submodule of fuzzy module X , then $\cup_{i\in N} I_i$ is T-pure fuzzy submodule of X .

Proof:

We must prove that $\cup_{i\in N} I_i$ is T-pure i.e $C^2X \cap [\cup_{i\in N} I_i] = C^2[\cup_{i\in N} I_i]$ for each fuzzy ideal C^2 of R

$C^2X \cap [\cup_{i\in N} I_i] = \cup_{i\in c} [C^2X \cap I_i]$ by [4]

$=\cup_{i\in c} [C^2I_i]$ since I_i T-pure F-I(R)

$= C^2[\cup_{i\in N} I_i]$ by(lemma1.14)

Thus $\cup_{i\in N} I_i$ is T-pure submodule of X .

2. T-pure fuzzy ideal

Definition (2.1):

An fuzzy ideal I of a ring R is called T-pure F-I(R) if for each fuzzy ideal $J^2$of R $J^2 \cap I = J^2I$

Definition (2.2):

If every F-I(R) is T-pure fuzzy ideal ,then we say R is T-regular fuzzy ring.

Proposition (2.3):

Let I be a fuzzy ideal of R then I is T-pure if and only if I_t is a T-pure ideal of R. $\forall$ t$\in$(0,1]

Proof:

($\Longrightarrow$) Let I is T-pure F-I(R) T.p I_t is a T-pure ideal of R $\forall$ t$\in$(0,1]

Let J^2 be an ideal of R

Define K^2: $J^2 \longrightarrow [0,1]$ by $K^2(x) = \begin{cases} t & if\ x \in J^2 \\ o & otherwise \end{cases}$ $\forall$ t$\in$(0,1]

It is clear that K^2 is F-I(R) and $K_t^2 = J^2$

T.P $J^2 \cap I_t = J^2I_t$

$J^2 \cap I_t = K_t^2 \cap I_t$

$=(K^2 \cap I)_t$

$=(K^2I)_t$ since I is T-pure ideal

$=K_t^2I_t$

$=J^2I_t$ $\forall$ t$\in$(0,1]

$\Longleftarrow$ I_t is a T-pure ideal T.p I is T-pure fuzzy ideal

Let K^2 is fuzzy ideal of R T.p $K^2 \cap I = K^2I$

$(K^2 \cap I)_t = K_t^2 \cap I_t$

$=K_t^2I_t$

$=(K^2I)_t$ sinceI_t is T-pure ideal

Thus $(K^2 \cap I)_t = (K^2I)_t$

Hence $K^2 \cap I = K^2I$

I is T-pure fuzzy ideal of R.

Remarks and Examples(2.4):

1-Let X-F(M), . R is regular fuzzy ring then R is T-regular fuzzy ring.

Proof:

It is clear that

The converse not true by

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Example: Let M=Z_4 as Z-module and N={$\bar{0},\bar{2}$}

Define X: M⟶[0,1] by $X(x)=\begin{cases} 1 \text{ if } x\in M \\ o \text{ otherwise}\end{cases}$

Define I: R⟶[0,1] by $A(x)=\begin{cases} t \text{ if } x\in N \\ o \text{ otherwise}\end{cases}$ ∀ t∈(0,1]

It is clear that I is fuzzy ideal of R and X_t=M ,I_t=N

Then X_t is T-regular ring by [7]

Hence X is T- regular fuzzy ring since every ideal of Z_4 is T-pure by (Proposition1.4) every ideal is fuzzy T-pure.

But X_t is not regular ring since ideal {$\bar{0},\bar{2}$} is not pure [7]

Thus I not pure fuzzy ideal by [4]

Then X is not regular fuzzy ring .

2- Let X-F(M), if R is a ring then the fuzzy singleton {o_t} and a ring R are always T-pure F-I(R)

3- Let X-F(M) . if R is a field , then X is T- regular ring.

Proof:

Since every field has only one submodule 0 then by(2), X is T- regular ring.

The converse of (3) is true if we gives the condition, R is a fuzzy integral domain

Proposition (2.5):

Let R_1 , R_2 be two rings and Let g any epimorphism function from R_1 to R_2. If C be a T-pure F-I(R_1) , then g(C) is a T-pure F-I(R_2).

Proof:

Let I^2 be a F-I(R_2). To prove g(C) ∩ I^2=I^2g(C).

g(C) ∩ I^2= g(C) ∩ g^{-1}(g (I^2)). [5]

= g(C ∩ g(I^2)) [3]

But g^{-1}(I^2) is F-I(R_1) by [5]

And C is T-pure F-I(R_1) so that

g(C ∩ g^{-1}(I^2))= g(A. g^{-1}(I^2))

= g(C). g (g^{-1}(I^2)) by [2]

= g(C). I^2 by[5]

Proposition (2.6):

Let R_1 , R_2 be two rings and Let f any epimorphism function from R_1 to R_2 and every fuzzy ideal of R_1 is f-invariant ,then if C is T-pure fuzzy ideal of R_2. Then $f^{-1}(C)$ is T-pure F-I(R_1) .

Proof:

Let C be a F-I(R_2). then $f^{-1}(C)$ is F- I(R_1) .see [5]

Let J^2 be a F-I(R_2). To. Prove $f^{-1}(C)\cap J^2$= $f^{-1}(C)J^2$

And by $f(f^{-1}(C)\cap J^2)$= $f(f^{-1}(C)\cap f(J^2)$ see[3]

=C ∩ $f(J^2)$ see[5]

=C. $f(J^2)$ since C is T-pure

= $f(f^{-1}(C)f(J^2)$ by [5]

= $f(f^{-1}(C)\,J^2)$ by [2]

Hence

f^{-1} [$f(f^{-1}(C)J^2)$] =f^{-1} [$f(f^{-1}(C)\cap J^2)$]

and by hyppose. We get

$f^{-1}(C)\cap J^2$ = $(f^{-1}(C)\,J^2$

Therefore $f^{-1}(C)$ is T-pure F-I(R_1).

Proposition (2.7):

Let K be a is F-I(R_1) and let J be a F-I(R_2), then K⊕J is T-pure fuzzy ideal of $R_1\oplus R_2$ if and only if K and J are T-pure fuzzy ideal in R_1 *and* R_2 respectively.

Proof:

(⟹) Let K ⊕J is T-pure fuzzy ideal To. Prove K and J are T-pure fuzzy ideal

Let A^2 and B^2 be two fuzzy ideal of R_1 *and* R_2 respectively. Then A^2⊕ B^2 is T-pure fuzzy ideal of $R_1\oplus R_2$ see[4]

Hence (K ⊕J) ∩ (A^2⊕ B^2)= (K ⊕J) (A^2⊕ B^2) since (K ⊕J) is T-pure fuzzy ideal

And by (K ⊕J) ∩ (A^2⊕ B^2)= (K ∩ A^2) ⊕(J ∩ B^2) see[4]

(K ⊕J) (A^2⊕ B^2)= (KA^2) ⊕(J B^2) see[4]

There for (K ∩ A^2) = KA^2 and J ∩ B^2=J B^2 see[4]

Thus K and J are T-pure fuzzy ideals of R_1 *and* R_2.

(⟸) let K and J are T-pure fuzzy ideal of R_1 *and* R_2.

Let A^2 and B^2 be two fuzzy ideal of R_1 *and* R_2

Hence A^2⊕ B^2 is fuzzy ideal in $R_1\oplus R_2$ see[8]

T.p (K ⊕J) ∩ (A^2⊕ B^2)= (K ⊕J) (A^2⊕ B^2)

(K ⊕J) ∩ (A^2⊕ B^2)= (K ∩ A^2) ⊕(J ∩ B^2) see [4]

=(KA^2) ⊕(J B^2) since K *and* J are T-pure

=(K ⊕J) (A^2⊕ B^2) see[4]

Hence (K ⊕J) ∩ (A^2⊕ B^2)= (K ⊕J) (A^2⊕ B^2)

Thus K ⊕J is T-pure fuzzy ideal in $R_1\oplus R_2$

Proposition2.8:

Let I and J are T-pure F-I(R) , then I ∩ J is T-pure F-I(R).

Proof:

T.p I ∩ J is T-pure F-I(R).

To show that for each fuzzy ideal K^2 of R (I ∩ J) ∩ K^2= (I ∩ J) K^2

Now, ∀ t ∈ (0,1] ((I ∩ J)K^2) $_t$ = $(I\cap J)_t K_t^2$ by[10]

=$(I_t\cap J_t)K_t^2$ by[1]

=$(I_t J_t)K_t^2$ since J_t is T-pure (by level)

=I_t ($J_t K_t^2$)

=$I_t(JK^2)_t$ by [10]

=$I_t(J\cap K^2)_t$ since J is T-pure

= $I_t\cap(J\cap K^2)_t$ since I_t is T-pure

=$I_t\cap(J_t\cap K_t^2)$ by[1]

=$(I_t\cap J_t)\cap K_t^2$

= $((I\cap J)_t\cap K_t^2)$ by [1]

=$[(I\cap J)\cap K^2]_t$ by [1]

Therefore (I ∩ J) ∩ K^2=(I ∩ J)K^2

Thus I ∩ J is T-pure fuzzy ideal of R

Lemma2.9:

let $\{J_i, i\in N\}$ be an ascending chain of T-pure F-I(R) . let C be a fuzzy ideal of R,

then $C^2[\cup_{i\in N} J_i]=\cup_{i\in c}[C^2 J_i]$.

proof:

$C^2 J_i\subseteq C^2[\cup_{i\in N} J_i], \forall\, i\in N$

implies that $\cup_{i\in c}[\,C^2 J_i]\subseteq C^2[\cup_{i\in N} J_i]$……(1)

but $C^2[\cup_{i\in N} J_i]\subseteq C^2$ and $C^2[\cup_{i\in N} J_i]\subseteq \cup_{i\in N} J_i$

since $\cup_{i\in N} J_i$ is fuzzy ideal

on other side $C^2[\cup_{i\in N} J_i]\subseteq C^2\cap[\cup_{i\in N} J_i]$

$=\cup_{i\in c}\ [C^2\cap J_i]$ by [4]

$=\cup_{i\in c}\ [C^2 J_i]$ since J_i T-pure fuzzy ideal of R

Thus $C^2[\cup_{i\in N} J_i]\subseteq\cup_{i\in c}\ [C^2 J_i]$

Therefore $C^2[\cup_{i\in N} J_i]=\cup_{i\in c}[C^2 J_i]$

Proposition2.10:
If $\{J_i, i \in N\}$ be an ascending chain of T-pure F-I(R), then $\cup_{i\in N} J_i$ is T-pure F-I(R).

Proof:
We must show that for each fuzzy ideal C^2 of R

$C^2 \cap [\cup_{i\in N} J_i] = C^2[\cup_{i\in N} J_i]$

$C^2 \cap [\cup_{i\in N} J_i] = \cup_{i\in c} [C^2 \cap J_i]$ by [4]

$= \cup_{i\in c} [C^2 J_i]$ since J_i T-pure F-I(R).

$= C^2[\cup_{i\in N} J_i]$ by lemma(2.9)

Hence $\cup_{i\in N} J_i$ is T-pure F-I(R).

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