An Improved Algorithm to Obtain Initial Basic Feasible Solution for the Transportation Problem

A. Seethalakshmy¹, N. Srinivasan²

¹Research Scholar, Dept. of Mathematics, St.Peter’s University, Chennai, TN
²Assistant Professor, Dept. of Mathematics, St.Peter’s College of Engineering and Technology, Chennai, TN

Abstract: This proposed method constitutes alternate method for initial solution in transportation problem. This method provides the initial solution. A transportation matrix is solved by a difference between maximum and next maximum element for each row and difference between maximum and minimum element for each column. In which the maximum value is marked and allocation is given to the least element. Salient features of this method depict lesser calculation time, easy applicability. Depiction with examples provides easy understanding of this method.

Keywords: Transportation problem, supply, Demand, Vogel method, initial solution

1. Introduction

Transportation problem is most referred and analyzed aspect where there is always efforts to match the demand-supply prerogative or to put them in balance. With each demand there needs to be sufficient supply to maintain balance and create optimal solution for transportation. The efficient way would be to provide a satisfactory logistical solution for the demand and supply with minimum cost incurred.

Considering the decision variable \( X_{ij} \) of model in transportation the \( i^{th} \) supply at source to \( j^{th} \) demand in destination.

The below listed methods are used to deduce feasible solutions with initial transportation problem:
1) North West Corner Method
2) Least Cost Method
3) Vogel’s Approximation Method

The overtly popular method employed on solving this is from MODI and stepping stone method. The former being widely discussed and articulated. This new method can be applied instead of the above three methods to obtain an initial basic feasible solution.

The beginning of assignment problem dates back to 1941 by Hitchcoockin, the subsequent evolution of the methods put forth Koopmans in 1949 and Dantzig in 1951. Though the Simplex method forms the basis of the problem solution, this would however be devoid of being implemented as it would not suit to the environment. With more research on its way in 1954, Charnes and Cooper introduced Stepping Stone Method. This proved to be more efficient, thereby driving the goal of being optimal. This one step further laid foundation towards our grit on further optimizing eventually giving birth to Heuristic method being formulated by Kirca and Stair through the VAM from Goyal;s version in the year 1958.

This article is formed in sections and flows as below:
Mathematical representation of transportation problem is depicted in Section 2, followed by algorithm in Section 3, numerical examples and conclusion follows in Section 4 and 5 respectively.

2. Mathematical form for transportation problem

The LP problem as given below

Minimize \( Z = \sum_{i=1}^{n} \sum_{j=1}^{m} c_{ij} x_{ij} \)

Subject to the constraints

\( \sum_{i=1}^{n} x_{ij} \leq S_i \) For all \( i \)

\( \sum_{j=1}^{m} x_{ij} \geq d_j \) For all \( j \)

\( x_{ij} \geq 0 \)

Transportation problem is considered to be balanced if

\( \sum_{i=1}^{n} S_i = \sum_{j=1}^{m} d_j \)

3. Algorithm

Step 1
Construct the matrix of a transportation problem from given problem. In case if the problem is unbalanced we make it balanced.

Step 2
Find the difference between maximum and next maximum in each row which is called as row penalty and difference between maximum and minimum in each column as column penalty and write it in the side and bottom. From that select the maximum value. From the selected row / column we need to allocate the minimum of supply/demand in the minimum element of the row or column. Eliminate by deleting the columns or rows corresponding to where the supply or demand is satisfied.

Step 3
Thus obtained table is then discussed. The process is continued to the remaining table till \( m + n - 1 \) cells are allocated with satisfaction to its supply and demand.

Volume 6 Issue 4, April 2017

www.ijsr.net
Licensed Under Creative Commons Attribution CC BY
Step 4
If obtained condition in step 2 is contrary, that is if there is tie in maximum value select that value which has least element. If there is tie in the least element then allocate the least element which has minimum supply/demand.

Step 5
Repeating the steps 2 to step 4 until satisfaction of all the supply and demand is met.

Step 6
The total minimum cost is calculated by
\[ \text{Total cost} = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} \]

4. Numerical Example

1. Obtain an initial basic feasible solution for the following problem.

<table>
<thead>
<tr>
<th>Factories</th>
<th>Distribution</th>
<th>D1</th>
<th>D2</th>
<th>D3</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td>6</td>
<td>4</td>
<td>1</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>F2</td>
<td>3</td>
<td>8</td>
<td>7</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>F3</td>
<td>4</td>
<td>4</td>
<td>2</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td>Req.</td>
<td>20</td>
<td>95</td>
<td>35</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Solution

Step 1
Find the difference between the maximum and next maximum element in row-wise and difference between maximum and minimum in column-wise. For that row/column allocate the minimum supply/demand for the least element in that row/column.

<table>
<thead>
<tr>
<th>Factories</th>
<th>D1</th>
<th>D2</th>
<th>D3</th>
<th>Supply</th>
<th>Row Penalty</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td>6</td>
<td>4</td>
<td>35</td>
<td>1</td>
<td>15</td>
</tr>
<tr>
<td>F2</td>
<td>3</td>
<td>8</td>
<td>4</td>
<td>7</td>
<td>40</td>
</tr>
<tr>
<td>F3</td>
<td>4</td>
<td>4</td>
<td>2</td>
<td>60</td>
<td>60</td>
</tr>
<tr>
<td>Req.</td>
<td>20</td>
<td>95</td>
<td>35</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Here the difference is maximum in F2. Hence allocate to the smallest element with minimum supply/demand. Now delete D1.

Step 2
<table>
<thead>
<tr>
<th>Factories</th>
<th>D1</th>
<th>D2</th>
<th>Supply</th>
<th>Row Penalty</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td>6</td>
<td>4</td>
<td>15</td>
<td>(2)</td>
</tr>
<tr>
<td>F2</td>
<td>20</td>
<td>8</td>
<td>40</td>
<td>(5)</td>
</tr>
<tr>
<td>F3</td>
<td>4</td>
<td>4</td>
<td>60</td>
<td>(2)</td>
</tr>
<tr>
<td>Req.</td>
<td>20</td>
<td>95</td>
<td>35</td>
<td></td>
</tr>
</tbody>
</table>

Step 3
<table>
<thead>
<tr>
<th>Factories</th>
<th>D2</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>F2</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>F3</td>
<td>60</td>
<td>60</td>
</tr>
<tr>
<td>Req.</td>
<td>95</td>
<td></td>
</tr>
</tbody>
</table>

Step 4
<table>
<thead>
<tr>
<th>Factories</th>
<th>D1</th>
<th>D2</th>
<th>D3</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td>6</td>
<td>15</td>
<td>35</td>
<td>50</td>
</tr>
<tr>
<td>F2</td>
<td>20</td>
<td>3</td>
<td>8</td>
<td>40</td>
</tr>
<tr>
<td>F3</td>
<td>4</td>
<td>60</td>
<td>4</td>
<td>60</td>
</tr>
<tr>
<td>Req.</td>
<td>20</td>
<td>95</td>
<td>35</td>
<td></td>
</tr>
</tbody>
</table>

Minimum cost = \[ 15 \times 4 + 35 \times 1 + 20 \times 3 + 20 \times 8 + 60 \times 4 \]
Minimum cost = 555

Initial Solution

<table>
<thead>
<tr>
<th>Proposed Method</th>
<th>Vogels Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>555</td>
<td>555</td>
</tr>
</tbody>
</table>

2. Obtain an initial basic feasible solution for the following problem.

<table>
<thead>
<tr>
<th>Factories</th>
<th>Distribution</th>
<th>D1</th>
<th>D2</th>
<th>D3</th>
<th>D4</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td>15</td>
<td>10</td>
<td>17</td>
<td>18</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>F2</td>
<td>16</td>
<td>13</td>
<td>12</td>
<td>13</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td>F3</td>
<td>12</td>
<td>17</td>
<td>20</td>
<td>11</td>
<td>70</td>
<td></td>
</tr>
<tr>
<td>Req.</td>
<td>30</td>
<td>30</td>
<td>40</td>
<td>50</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Solution

Step 1

Find the difference between the maximum and next maximum element in row-wise and difference between maximum and minimum in column-wise. For that row/column allocate the minimum supply/demand for the least element in that row/column.

Here the difference is maximum in \( F_2 \). Hence allocate to the smallest element with minimum supply/demand.

Now delete \( D_4 \).

\[
\begin{array}{c|cccc|c|c}
\hline
\text{Factories} & D_1 & D_2 & D_3 & D_4 & \text{Supply} & \text{Row Penalty} \\
\hline
F_1 & 15 & 10 & 17 & 18 & 20 & (1) \\
F_2 & 16 & 13 & 40 & 12 & 13 & 60 & 20 & (3) \\
F_3 & 12 & 17 & 20 & 11 & 70 & (3) \\
\hline
\text{Req.} & 30 & 30 & 40 & 50 & \\
\hline
\text{Column Penalty} & (4) & (7) & (9) & (7) \\
\hline
\end{array}
\]

Step 2

\[
\begin{array}{c|cccc|c|c}
\hline
\text{Factories} & D_1 & D_2 & D_3 & D_4 & \text{Supply} & \text{Row Penalty} \\
\hline
F_1 & 15 & -20 & 10 & 13 & 20 & (3) \\
F_2 & 16 & 13 & 13 & 20 & (3) \\
F_3 & 12 & 17 & 11 & 70 & (5) \\
\hline
\text{Req.} & 30 & 30 & 40 & 50 & \\
\hline
\text{Column Penalty} & (4) & (7) & (7) \\
\hline
\end{array}
\]

Step 3

\[
\begin{array}{c|cccc|c|c}
\hline
\text{Factories} & D_1 & D_2 & D_3 & D_4 & \text{Supply} & \text{Row Penalty} \\
\hline
F_2 & 16 & 13 & 13 & 20 & (3) \\
F_3 & 12 & 17 & -50 & 11 & 70 & 20 & (5) \\
\hline
\text{Req.} & 30 & 10 & 50 & \\
\hline
\text{Column Penalty} & (4) & (4) & (2) \\
\hline
\end{array}
\]

Here we take the row penalty as \( F_2 (18-15=3) \) and \( F_3 (17-12=5) \) and column penalty as \( D_1 (16-12=4) \) and \( D_3 (17-13=4) \) and \( D_4 (13-11=2) \).

Now delete \( F_2 \).

Minimum cost = \( 20 \times 10 + 10 \times 16 + 10 \times 13 + 40 \times 12 \\
+ 20 \times 12 + 50 \times 11 \)

Minimum cost = 1760

Initial Solution

\[
\begin{array}{c|cccc|c|c}
\hline
\text{Factories} & D_1 & D_2 & D_3 & D_4 & \text{Supply} \\
\hline
F_1 & 15 & 20 & 10 & 17 & 18 & 20 \\
F_2 & 16 & 10 & 40 & 13 & 12 & 13 & 60 \\
F_3 & 20 & 12 & 17 & 50 & 20 & 70 & 70 \\
\hline
\text{Req.} & 30 & 30 & 40 & 50 & \\
\hline
\text{Proposed Method} & 1760 & 1760 \\
\end{array}
\]

3. Obtain an initial basic feasible solution for the following problem.

\[
\begin{array}{c|cccc|c|c}
\hline
\text{Factories} & D_1 & D_2 & D_3 & D_4 & \text{Supply} \\
\hline
F_1 & 4 & 19 & 22 & 11 & 160 \\
F_2 & 1 & 9 & 14 & 14 & 30 \\
F_3 & 6 & 6 & 16 & 14 & 70 \\
\hline
\text{Req.} & 40 & 20 & 60 & 80 & \\
\hline
\text{Distribution} & \\
\end{array}
\]

Solution

Step 1

Find the difference between the maximum and next maximum element in row-wise and difference between maximum and minimum in column-wise. For that row/column allocate the minimum supply/demand for the least element in that row/column.

Volume 6 Issue 4, April 2017

www.ijsr.net

Licensed Under Creative Commons Attribution CC BY

Paper ID: ART20172589

1227
Now delete smallest element with minimum supply/demand.

Here the difference is maximum in \( D_2 \). Hence allocate to the smallest element with minimum supply/demand. Now delete \( D_2 \).

**Step 2**

\[
\begin{array}{c|c|c|c|c|c|c}
\text{Factories} & D_1 & D_2 & D_3 & D_4 & \text{Supply} & \text{Row Penalty} \\
\hline
F_1 & 40 & 9 & 14 & 14 & 30 & (0) \\
F_2 & 6 & 20 & 16 & 14 & 70 & (2) \\
F_3 & 40 & 6 & 60 & 80 & & \\
\hline
\text{Column Penalty} & (5) & (13)^* & (8) & (3) & &
\end{array}
\]

*we take the row penalty as \( F_1 (22-19=3) \) and \( F_2 (14-14=0) \) and \( F_3 (16-14=2) \) and column penalty as \( D_1 (6-1=5) \) and \( D_2 (22-14=8) \) and \( D_3 (14-11=3) \)*

Here the difference is maximum in \( D_2 \). Hence allocate to the smallest element with minimum supply/demand. Now delete \( D_2 \).

**Step 3**

\[
\begin{array}{c|c|c|c|c|c|c}
\text{Factories} & D_1 & D_2 & D_3 & D_4 & \text{Supply} & \text{Row Penalty} \\
\hline
F_1 & 40 & 9 & 14 & 14 & 30 & (0) \\
F_2 & 1 & 14 & 14 & 30 & & (0) \\
F_3 & 6 & 16 & 14 & 50 & & (2) \\
\hline
\text{Column Penalty} & (5) & (8) & (3) & & &
\end{array}
\]

*we take the row penalty as \( F_1 (22-11=10) \) and \( F_2 (14-14=0) \) and \( F_3 (16-14=2) \) and column penalty as \( D_1 (6-1=5) \) and \( D_2 (22-14=8) \) and \( D_3 (14-11=3) \)*

Here the difference is maximum in \( F_1 \). Hence allocate to the smallest element with minimum supply/demand. Now delete \( D_1 \).

**Step 4**

\[
\begin{array}{c|c|c|c|c|c|c}
\text{Factories} & D_1 & D_2 & D_3 & D_4 & \text{Supply} & \text{Row Penalty} \\
\hline
F_1 & 40 & 9 & 14 & 14 & 30 & (0) \\
F_2 & 6 & 20 & 16 & 14 & 70 & (2)^* \\
F_3 & 40 & 6 & 60 & 80 & & \\
\hline
\text{Column Penalty} & (2) & (0) & & & &
\end{array}
\]

**Step 5**

\[
\begin{array}{c|c|c|c|c|c|c}
\text{Factories} & D_1 & D_2 & D_3 & D_4 & \text{Supply} & \text{Row Penalty} \\
\hline
F_1 & 40 & 19 & 22 & 60 & 11 & (10)^* \\
F_2 & 1 & 9 & 30 & 14 & 14 & (30) \\
F_3 & 6 & 20 & 30 & 14 & 14 & (70) \\
\hline
\text{Column Penalty} & & & & & &
\end{array}
\]

**Distribution**

\[
\begin{array}{c|c|c|c|c|c}
\text{Factories} & D_1 & D_2 & D_3 & D_4 & \text{Supply} \\
\hline
F_1 & 40 & 19 & 22 & 60 & 11 \\
F_2 & 1 & 9 & 30 & 14 & 14 \\
F_3 & 6 & 20 & 30 & 14 & 14 \\
\hline
\text{Column Penalty} & & & & &
\end{array}
\]

\[
\text{Minimum cost} = 40 \times 41 + 60 \times 11 + 30 \times 14 + 20 \times 6 + 30 \times 16 + 20 \times 14
\]

\[
\text{Minimum cost} = 2120
\]

**Initial Solution**

- **Proposed Method**: 2120
- **Vogels Method**: 2170

**5. Conclusion**

The above discussed method on initial solution gives another paradigm on solutions with more efficient way. This method further qualifies to be more efficient and initial in solving the problems being a game to further research and development through this. This proposed method gives result exactly or even lesser to VAM method. All necessary qualities of being time efficient, easy applicability etc., forms the core of being implemented successfully.

**References**


[8] Smita Sood and Keerti Jain,'The maximum difference method to find initial basic feasible solution for Transportation Problem',Asian Journal of management Sciences,03[07] 2015:


Author Profile

A.Seethalakshmy received B.Sc degree from Bharadhidhasan women’s College, Pondicherry University and M.Sc degree in Pondicherry University and M.Phil from Pondicherry University. She has 13 years of teaching experience and presently working as Assistant Professor in Department of Mathematics, St. Peter’s College of Engineering and Technology and also a Research Scholar in Dept of Mathematics, St. Peter’s University, Avadi, Chennai , India. She has published 7 international papers.

Dr. N. Srinivasan received M.Sc degree from Vivekanandha College Mylapore Chennai and M.Phil from Madurai Kamaraj University. He received Phd degree in mathematics from HONOLULU University USA. He has 37 years of teaching experience and presently working as Professor and Head. of the department of Mathematics, St. Peter’s University, Avadi, Chennai – 54, India. He has published 12 international papers.