A New Approach for Estimating Fuzzy Linear Regression Parameters with Application

Hazim Mansoor Gorgees[1], Mariam Mohammed Hilal[2]

[1, 2]Department of Mathematics, College of Education for Pure Science, Ibn-Al-Haitham, University of Baghdad, Iraq

Abstract: ***The typical causes of pavement deterioration include: traffic loading; environment or climate influences; drainage deficiencies; materials quality problems; construction deficiencies; and external contributors, for this situation, a fuzzy linear regression model was employed and analyzed by using the traditional methods and our proposed method. The total spread error was used as a criterion to compare the performance of the studied methods.***

Keywords: Fuzzy numbers, Fuzzy linear regression, principle component method, linear programming

1. Introduction

As years are passing, statistical-linear-regression had been utilized in mostly every scientific field. The reason of the regression-analysis is to clarify the dependent variable Y variation in terms of the variables explanatory variation X as Y = f(X) in which f(X) is a linear-function. The utilization of linear-regression statistics is restricted through some firm estimation about the taken data, which is the non-observed error-term is identically distributed and mutually independent. Consequently, the model of the statistical-regression could be useful only when the taken data are distributed depending upon a statistical-model & the relationship between y and x is crisp; when obtaining the linguistic-data, the symbolic-numbers are utilized to identify qualitative-terms, e.g., 4 is a number for "excellent", number 3 for "very-good", number 2 for "good" and number 1 is for "fair". For many problems in the real-world, data over-simplification can give significant information about regression-models. Some explanations could be demonstrated in linguistic-terms only (like excellent, good and fair). For that data, fuzzy-set theory gives means about such linguistic-variables modelling using functions of fuzzy-membership. Fuzzy-regression was dealing with fuzzy-data. Regression is founded upon probability-theory in which fuzzy-regression is dependent on fuzzy-set-theory and possibility theory.

The fuzzy-uncertainty was described by Zadeh (1965) through vagueness and ambiguity and introduced the fuzzy theory in order to make such a system that is required to deal with vague and ambiguous information or sentences. The fuzzy uncertainty of dependent-variables was explained by Tanaka et al (1982) with the response functions fuzziness or coefficients of regression in regression-model & initially introduces the fuzzy-regression-model. This model might be classified roughly through dependent and independent variables conditions into 3 categories, as in the following [4]:

(i) Output and input data are both non fuzzy number.
(ii) Output data is fuzzy number but input data is non-fuzzy number.
(iii) Both output and input data are fuzzy-number

2. Multiple Linear Regressions

The most widely used regression model is the multiple linear regression models, Moreover, the ordinary least squares approach is the most popular estimation procedure. The assumptions beyond the ordinary least squares estimation method may be summarized by the independence of error terms with identical distribution, Moreover, no exact linear relationship be exist between two or more independent variables.

The model can be written as:

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \cdots + +\beta_k X_{ik} + \epsilon_i \quad for\ i = 1,2,\ldots,n \tag{1}$$

$$Y = X\beta + \epsilon \tag{2}$$

Where, Y is $n \times 1$ matrix of n observations, β is $(K+1) \times 1$ matrix of the beta coefficients, X is $n \times (K+1)$ matrix containing n observations for K independent variables and ϵ is $n \times 1$ matrix of error terms.

The ordinary least square method is based on the minimization of
$\epsilon' \epsilon = (y - X\beta)^T (y - X\beta)$by differentiating $\epsilon' \epsilon$ with respect to β and setting the resultant matrix equation equal to zero and solving for β.

The ordinary least square estimator is:

$$\hat{\beta} = (X'X)^{-1} X' y \tag{3}$$

3. The Case of Near Multi-Collinearity [5]

The problem of near multi-collinearity occurs when there exist an approximate linear relationship among two or more explanatory variables; the case in which we cannot decide which of these explanatory variables is producing the observed change in the response variable.

The Farrar-Gloabar test based on the chi square statistic may be used to detect near Multi-collinearity. The null hypothesis to be tested is:
H_0: x_j are orthogonal, j=1, 2, …, k.
Against an alternative
H_1: x_j are not orthogonal.
The test statistic is:

$$\chi^2 = -\left[(n-1) - \frac{1}{6}(2k+5)\right] Ln\,|D| \quad (4)$$

Where n is the number of observations, k is the number of explanatory variables, $|D|$ is the determinant of the correlation matrix of explanatory variables.

Comparing the calculated value of χ^2 with theoretical value at $K\,(K-1)/2$ degrees of freedom and specifies level of significant. We reject H_0 if the calculated value is more than the theoretical value which means that the problem of near near-collinearity exist.

Different methods were proposed to handle this problems, one of those popular methods was the principal component method in which y is regressed on the principal components of X matrix. If we use only the larger principal component, the larger variances in the estimated coefficients due to Multi-collinearity are reduced with introducing some bias in the new estimators.

Let G be an orthogonal matrix. The multiple linear regression models can be written as:

$$y = XG\,G'\beta + \epsilon = Z\alpha + \epsilon \quad (5)$$

Where $Z = XG\,, \alpha = G'\beta$.

$$Z'Z = G'X'XG = \Lambda = diag(\lambda_1, \lambda_2, \dots, \lambda_k) \quad (6)$$

Where $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_k > 0$ are the eigenvalue of $X'X$ and the columns of G are the eigenvectors of $X'X$.

The columns of Z are called the principal components and these are orthogonal to each other.

Assuming that the first q principal components are selected then the reduced estimated can be written as:

$$\hat{\alpha}_q = (Z'_q\, Z_q)^{-1} Z'_q\, y = \Lambda_q^{\ -1} G'_q\, X'y \quad (7)$$

Where $Z_q = X\,G_q$, G_q is the matrix of the first q eigenvectors of $X'X$ and Λ_q is the diagonal matrix that contains the first q eigenvalues of $X'X$.

And hence:

$$b_{PC} = G_q \hat{\alpha}_q = G_q \Lambda_q^{\ -1}\, G'_q\, X'y \quad (8)$$

4. Fuzzy Linear Regression

In the complex systems, such as the systems existing in biology, agriculture, engineering and economy we frequently cannot get the exact numerical data for the information of systems because of the complexity of systems themselves, the vagueness in people's thinking and judgment and the influence of various uncertain factors existing in boundary environment around the systems. For this situation, the traditional least squares regression may not be applicable. We need therefore to investigate some soft methods for dealing with these situations. Fuzzy set theory provides suitable tools for regression analysis when the relationship between variables is vaguely defined and the observations are reported as imprecise quantities.

After introducing fuzzy set theory, several approaches to fuzzy regression have been developed by many researchers studies related to fuzzy linear regression may be roughly divided into two approaches, namely, linear programming based methods (possibilistic approach) and fuzzy least squares methods (least squares approach).

5. Tanaka's Model

Fuzzy linear regression developed by Tanaka et.al (1982) seeks to modeling vague and imprecise.

This model use fuzzy linear function to determine the regression of fuzzy phenomena. Deviations generally between the observations and the estimated values in the traditional regression emerging from the observations errors, but here this deviations are depend on not specified the system structure, where this fuzzy deviations are the fuzzy parameters of the system, this fuzzy parameters are triangular membership functions.

To determine the fuzzy coefficients $\tilde{B}_k = (\alpha_k, c_k)$, the linear programming (LP) problem is formulated as follows [2]:

$$Minimize\ J = \sum_{k=1}^{M} c_k \quad (9)$$

Subject to:

$$\sum_{k=0}^{N} \alpha_k x_{ik} + (1-h)\sum_{k=0}^{N} c_k |x_{ik}| \geq y_i \quad (10)$$

$$\sum_{k=0}^{N} \alpha_k x_{ik} - (1-h)\sum_{k=0}^{N} c_k |x_{ik}| \leq y_i \quad (11)$$

$$c_k \geq 0, \alpha_k \in R, \quad x_{i0} = 1, \quad i = 1,2,\dots,M, \quad 0 \leq h \leq 1 \quad (12)$$

Where the h value is belong to $[0, 1]$ which mean it is the threshold level to chosen by the decision maker.

This problem is called as Tanaka method in 1982, which is developed in its objective function, which is written as follows [3]:

$$Minimize\ J = \sum_{k=0}^{N} (c_k \sum_{i=1}^{M} |x_{ik}|) \quad (13)$$

Subject to:

$$\sum_{k=0}^{N} \alpha_k x_{ik} + (1-h)\sum_{k=0}^{N} c_k |x_{ik}| \geq y_i \quad (14)$$

$$\sum_{k=0}^{N} \alpha_k x_{ik} - (1-h)\sum_{k=0}^{N} c_k |x_{ik}| \leq y_i \quad (15)$$

$$c_k \geq 0, \alpha_k \in R, \quad x_{i0} = 1, \quad i = 1,2,\dots,M, \quad 0 \leq h \leq 1 \quad (16)$$

Where the h value is belong to $[0, 1]$ which mean it is the threshold level to chosen by the decision maker.

The number of subjects (conditions) in the problem is $2 \times N$ which always are larger than the number of the variables [3]. Later in 1987 Tanaka is developed this problem because of several of the values of the thresholds becomes equal to zero in solved of the linear problem, so the relationship between the response variable (depended) and the explanatory variables (independent) will be crisp, to avoid this problem he conducted amendment to the objective function which was in minimize the total sum of the spread of the fuzzy parameters while it becomes with minimizing in the total sum of the spread of the prediction value (thresholds value) of the response variable because this value is also fuzzy, then the problem becomes [3]:

$$Minimize\ J = \sum_{i=0}^{N} \sum_{k=1}^{M} c_k\, x_{ik} \quad (17)$$

Subject to:

$$\alpha^t x_i + (1-h)\sum_{k=1}^{M} c_k |x_{ik}| \geq y_i + (1-h)e_i \quad (18)$$

$$-\alpha^t x_i + (1-h)\sum_{k=1}^{M} c_k |x_{ik}| \geq -y_i + (1-h)e_i \quad (19)$$

$$c_k \geq 0, \quad (20)$$

$$\alpha_k \in R, \quad x_i \geq 0, \quad i = 1,2,\dots,N, \quad 0 \leq h \leq 1$$

Where the h value is belong to $[0, 1]$ which mean it is the threshold level to chosen by the decision maker [3]. All the Tanaka models have crisp explanatory variables and fuzzy in parameters and response variable.

6. Savic and Pedrycz method [1]

Savic and Pedrycz formulated the fuzzy regression by combining the ordinary least squares with minimum fuzziness criterion. The method is constructed in two successive steps. The first step employs ordinary least square regression to obtain fuzzy regression parameters. The minimum fuzziness criterion is used in the second step to find the spread of fuzzy regression parameters.

In the first step, the available information about the value of the center of the fuzzy observations is used to fit a regression line to the data.

In fact, the fuzzy data are regress as simplified crisp data and the regression analysis is conducted as it is an ordinary least squares regression. The results of this step are employed as center values of the fuzzy regression parameters.

In the next step, the minimum fuzziness criterion is used to determine fuzzy parameters. Spreads of the fuzzy parameters are obtained by equation (18), (19) as the minimum fuzziness method with the distinction of employing the fuzzy centers of regression parameters resulting from the first step.

7. Proposed Method

Our proposed method is a modification of Savic&Pedrycz method to deal with case of multi-collinearity among the crisp explanatory variables. The method may be summarized as follows:

In the first step the principle component regression is used instead of ordinary least squares regression to determine fuzzy center values of fuzzy regression coefficients. In the second step, the minimum fuzziness criterion is used to find the spread of fuzzy regression coefficients.

8. Practical Study

A numerical example is used in this section to illustrate the proposed method that is summarized in previous sections. Data used in the experiment consist of 54 observations taken from transportation laboratory of the Civil Engineering Department of the University of Baghdad which is illustrated in the following table (1):

Table 1: The Data of Fatigue life

	Y=Ln Nf	$X1$= Ln $\epsilon_0\left(\frac{mm}{mm}\right)$	$X2$=Ln $S_o(MPa)$	$X3$=Ln $\sigma(MPa)$	$X4$=Ln $Av(\%)$
1	0.795855349	-8.111728083	8.538954683	-0.494296322	6.081
2	0.831788925	-8.111728083	8.559869466	-0.494296322	3.822
3	0.70662007	-8.517193191	8.74687532	-0.798507696	5.984
4	0.866475944	-8.111728083	8.724532511	-0.494296322	5.716
5	0.843485463	-8.111728083	8.714239144	-0.494296322	3.95
6	0.102663757	-7.824046011	8.543445563	-0.494296322	2.114
7	0.28092649	-7.824046011	8.350429974	-0.494296322	6.57
8	0.432167891	-7.824046011	8.285765421	-0.494296322	6.873
9	0.563511756	-7.60090246	8.378160983	-0.198450939	4.589
10	0.70662007	-7.824046011	8.330863613	-0.494296322	6.962
11	0.359403715	-7.60090246	8.547722396	-0.198450939	2.68
12	0.320934739	-7.60090246	8.298539545	-0.198450939	6.584
13	0.260305654	-8.517193191	8.753371421	-0.798507696	3.05
14	0.795855349	-8.517193191	8.77971129	-0.798507696	2.404
15	0.217742741	-8.111728083	8.428361978	-0.798507696	5.46
16	0.499994677	-8.111728083	8.541885804	-0.798507696	6.345
17	0.63762373	-8.517193191	8.708639656	-0.798507696	7.7
18	0.758582267	-8.517193191	8.396154863	-0.798507696	6.291
19	0.888949719	-8.517193191	8.697345731	-0.798507696	6.36
20	0.48346438	-8.111728083	8.777709596	-0.494296322	2.615
21	0.623234239	-8.517193191	8.76623838	-0.798507696	2.61
22	0.745842651	-8.111728083	8.402679805	-0.798507696	2.823
23	0.877775964	-8.517193191	8.56674497	-0.798507696	2.825
24	0.679590067	-8.111728083	8.803874764	-0.494296322	2.829
25	0.665796052	-7.824046011	8.377471248	-0.494296322	2.827
26	0.51625616	-8.111728083	8.582044164	-0.798507696	2.829
27	0.608634663	-8.111728083	8.29529886	-0.798507696	2.828
28	0.819953958	-8.517193191	8.730690366	-0.798507696	2.61
29	0.771161626	-8.111728083	8.427487278	-0.798507696	2.829
30	0.578780077	-8.517193191	8.649974303	-0.798507696	2.82
31	0.732938639	-8.111728083	8.362642432	-0.798507696	2.828
32	0.466656233	-8.517193191	8.528528701	-0.798507696	2.82
33	0.340354198	-7.824046011	8.321664807	-0.494296322	2.826
34	0.466656233	-7.824046011	8.426392827	-0.494296322	2.82
35	0.855046772	-8.111728083	8.648396877	-0.494296322	7.26
36	0.651809098	-7.824046011	8.514990768	-0.494296322	6.86
37	0.783584708	-7.418580903	8.610683535	-0.198450939	7.05
38	0.320934739	-7.60090246	8.607216694	-0.198450939	6.67

39	0.807977244	-7.418580903	8.239857411	-0.198450939	7.67
40	0.693196393	-8.111728083	8.701180028	-0.494296322	6.44
41	0.532257434	-7.824046011	8.160232492	-0.494296322	7.87
42	0.831788925	-7.418580903	8.222553638	-0.198450939	6.5
43	0.150295937	-7.60090246	8.640295389	-0.198450939	6.49
44	0.320934739	-7.60090246	8.644002038	-0.198450939	7.45
45	0.026670837	-7.418580903	8.30474227	-0.198450939	7.55
46	0.126763423	-7.824046011	8.58634605	-0.198450939	7.28
47	0.026670837	-7.418580903	8.538563217	-0.198450939	6.9
48	0.239250631	-8.111728083	8.50512061	-0.494296322	6.35
49	0.37809712	-7.60090246	8.494538501	-0.198450939	3.54
50	0.102663757	-7.418580903	8.183118079	-0.198450939	7.495
51	0.150295937	-7.824046011	8.563885919	-0.198450939	7.33
52	0.077968924	-7.418580903	8.335431478	-0.198450939	7.32
53	0.195762075	-7.60090246	8.478452363	-0.198450939	6.485
54	0.026670837	-7.60090246	8.318742253	-0.198450939	7.316

$x_{1j} = \epsilon_0 \left(\frac{mm}{mm}\right)$: Initial tensile strain at 5th repetition of bending beam, (the first independent variable).
$x_{2j} = S_o(\text{MPa})$: Initial flexural stiffness modulus, (the second independent variable).
$x_{3j} = \sigma(\text{MPa})$: stress level, (the third independent variable).
$x_{4j} = Av(\%)$: Percent air void (%), (the fourth independent variable).

Applying the Farrar-Glauber test stated in equation (4) on the data set we found that the calculated value of χ^2 was equal to (55.8970) which is greater than the theoretical value of χ^2 with 6 degrees of freedom and 0.05 level of significant (12.592) and hence, a high degree of multi-collinearity is exist. For Savic and Pedrycz the result is shown in the following table (2): (taken h=0.5)

Table 2: Results of the Savic & Pedrycz method

	Crisp Data		*fuzzy data before using Lp*		*Outputs fuzzy data for Savic&Pedrycz method*	
	Y	*$\hat{y}$ from Least square*	*y_i=centers*	*e_i= spread*	*$\hat{y}_i$=center*	*$\hat{e}_i$=spread*
1	0.795855349	0.541123231	0.66848929	0.00501367	0.66848929	-0.000121904
2	0.831788925	0.544206037	0.687997481	0.005159981	0.687997481	-0.000123521
3	0.70662007	0.709034706	0.707827388	0.005308705	0.707827388	-0.000240924
4	0.866475944	0.545660329	0.706068137	0.005295511	0.706068137	-0.000119136
5	0.843485463	0.547479709	0.695482586	0.005216119	0.695482586	-0.000120843
6	0.102663757	0.48449724	0.293580499	0.002201854	0.293580499	-7.92855E-05
7	0.28092649	0.475052009	0.377989249	0.002834919	0.377989249	-7.86197E-05
8	0.432167891	0.473267211	0.452717551	0.003395382	0.452717551	-7.94314E-05
9	0.563511756	0.355750034	0.459630895	0.003447232	0.459630895	1.17789E-05
10	0.70662007	0.474163761	0.590391916	0.004427939	0.590391916	-7.86042E-05
11	0.359403715	0.361722248	0.360562982	0.002704222	0.360562982	1.2938E-05
12	0.320934739	0.351671905	0.336303322	0.002522275	0.336303322	1.21899E-05
13	0.260305654	0.712580497	0.486443075	0.003648323	0.486443075	-0.000243367
14	0.795855349	0.713913397	0.754884373	0.005661633	0.754884373	-0.000243491
15	0.217742741	0.61614865	0.416945695	0.003127093	0.416945695	-0.000181851
16	0.499994677	0.617639155	0.558816916	0.004191127	0.558816916	-0.000179194
17	0.63762373	0.706197503	0.671910616	0.00503933	0.671910616	-0.000240068
18	0.758582267	0.70090396	0.729743113	0.005473073	0.729743113	-0.000246488
19	0.888949719	0.707500782	0.798225251	0.005986689	0.798225251	-0.000241421
20	0.48346438	0.550434574	0.516949477	0.003877121	0.516949477	-0.000120949
21	0.623234239	0.713375884	0.668305062	0.005012288	0.668305062	-0.000243536
22	0.745842651	0.618636758	0.682239704	0.005116798	0.682239704	-0.000184571
23	0.877775964	0.70870422	0.793240092	0.005949301	0.793240092	-0.000246666
24	0.679590067	0.550766486	0.615178276	0.004613837	0.615178276	-0.000120328
25	0.665796052	0.47999123	0.572893641	0.004296702	0.572893641	-8.1425E-05
26	0.51625616	0.622605965	0.569431062	0.004270733	0.569431062	-0.000181584
27	0.608634663	0.616250531	0.612442597	0.004593319	0.612442597	-0.000186352
28	0.819953958	0.712587853	0.766270906	0.005747032	0.766270906	-0.000244127
29	0.771161626	0.619179735	0.695170681	0.00521378	0.695170681	-0.000184153
30	0.578780077	0.710555051	0.644667564	0.004835007	0.644667564	-0.000245287
31	0.732938639	0.617743409	0.675341024	0.005065058	0.675341024	-0.000185232
32	0.466656233	0.707862835	0.587259534	0.004404447	0.587259534	-0.000247306
33	0.340354198	0.478755268	0.409554733	0.00307166	0.409554733	-8.23537E-05
34	0.466656233	0.481083844	0.473870038	0.003554025	0.473870038	-8.06177E-05
35	0.855046772	0.542182379	0.698614576	0.005239609	0.698614576	-0.000119059
36	0.651809098	0.478363769	0.565086434	0.004238148	0.565086434	-7.56315E-05
37	0.783584708	0.319185857	0.551385282	0.00413539	0.551385282	4.69339E-05

38	0.320934739	0.358414974	0.339674856	0.002547561	0.339674856	1.73968E-05
39	0.807977244	0.310246503	0.559111873	0.004193339	0.559111873	4.13076E-05
40	0.693196393	0.544303218	0.618749805	0.004640624	0.618749805	-0.000118895
41	0.532257434	0.469328429	0.500792931	0.003755947	0.500792931	-8.06515E-05
42	0.831788925	0.311219451	0.571504188	0.004286281	0.571504188	4.00025E-05
43	0.150295937	0.359356963	0.25482645	0.001911198	0.25482645	1.77902E-05
44	0.320934739	0.358326074	0.339630407	0.002547228	0.339630407	1.86866E-05
45	0.026670837	0.311824008	0.169247422	0.001269356	0.169247422	4.22821E-05
46	0.126763423	0.404812442	0.265787932	0.001993409	0.265787932	-1.80953E-05
47	0.026670837	0.317761003	0.17221592	0.001291619	0.17221592	4.56044E-05
48	0.239250631	0.540061306	0.389655968	0.00292242	0.389655968	-0.000122233
49	0.37809712	0.359546148	0.368821634	0.002766162	0.368821634	1.28016E-05
50	0.102663757	0.309191602	0.205927679	0.001544458	0.205927679	4.02121E-05
51	0.150295937	0.404256572	0.277276255	0.002079572	0.277276255	-1.84253E-05
52	0.077968924	0.312770999	0.195369961	0.001465275	0.195369961	4.25923E-05
53	0.195762075	0.355775011	0.275768543	0.002068264	0.275768543	1.5095E-05
54	0.026670837	0.351271053	0.188970945	0.001417282	0.188970945	1.31623E-05

Where the parameters of this method is as follows:

$$y_i = (-1.49500932075108, 0.001121256946788) + (-0.213169442048952, 0.000159877088712)x_{1j} + (0.022168081623166, 0.000016626047275)x_{2j} + (-0.252314818567323, 0.000189236132903)x_{3j} + (-0.001159435826688, 0.000000869575698)x_{4j}$$

The total spread value for the parameters was0.14224008. And for the result of the proposed method is shown in the following table (3): (taken h=0.5)

Table 3: Results of the First Proposed method

	Crisp data		*fuzzy data before using Lp*		*Outputs fuzzy data for first proposed method*	
	Y	$\hat{y}$ *from Principle component*	y_i*=centers*	e_i*= spread*	$\hat{y}_i$*=center*	$\hat{e}_i$*=spread*
1	0.795855349	0.533955501	0.664905425	0.004986791	0.664905425	-0.000313579
2	0.831788925	0.539194706	0.685491816	0.005141189	0.685491816	-0.000320426
3	0.70662007	0.696688391	0.701654231	0.005262407	0.701654231	-0.000535658
4	0.866475944	0.534984229	0.700730087	0.005255476	0.700730087	-0.000312889
5	0.843485463	0.539053396	0.691269429	0.005184521	0.691269429	-0.000318504
6	0.102663757	0.492721863	0.29769281	0.002232696	0.29769281	-0.000298454
7	0.28092649	0.482235488	0.381580989	0.002861857	0.381580989	-0.000286444
8	0.432167891	0.481470893	0.456819392	0.003426145	0.456819392	-0.000286136
9	0.563511756	0.358990964	0.46125136	0.003459385	0.46125136	-0.000091028
10	0.70662007	0.481310401	0.593965236	0.004454739	0.593965236	-0.000285413
11	0.359403715	0.363570315	0.361487015	0.002711153	0.361487015	-0.000095316
12	0.320934739	0.354302892	0.337618816	0.002532141	0.337618816	-0.000085584
13	0.260305654	0.703472427	0.481889041	0.003614168	0.481889041	-0.000544755
14	0.795855349	0.704991027	0.750423188	0.005628174	0.750423188	-0.000546512
15	0.217742741	0.626542831	0.422142786	0.003166071	0.422142786	-0.000501723
16	0.499994677	0.624612005	0.562303341	0.004217275	0.562303341	-0.000497840
17	0.63762373	0.692686196	0.665154963	0.004988662	0.665154963	-0.000530677
18	0.758582267	0.695628501	0.727105384	0.00545329	0.727105384	-0.000538158
19	0.888949719	0.695770302	0.79236001	0.0059427	0.79236001	-0.000534973
20	0.48346438	0.542200716	0.512832548	0.003846244	0.512832548	-0.000322047
21	0.623234239	0.704501694	0.663867967	0.00497901	0.663867967	-0.000546002
22	0.745842651	0.632608619	0.689225635	0.005169192	0.689225635	-0.000510209
23	0.877775964	0.703805551	0.790790757	0.005930931	0.790790757	-0.000547298
24	0.679590067	0.541732542	0.610661304	0.00457996	0.610661304	-0.000321121
25	0.665796052	0.490908859	0.578352455	0.004337643	0.578352455	-0.000297864
26	0.51625616	0.632774123	0.574515142	0.004308864	0.574515142	-0.000508422
27	0.608634663	0.632489688	0.620562175	0.004654216	0.620562175	-0.000511252
28	0.819953958	0.704466146	0.762210052	0.005716575	0.762210052	-0.000546353
29	0.771161626	0.632619566	0.701890596	0.005264179	0.701890596	-0.000509946
30	0.578780077	0.70390033	0.641340204	0.004810052	0.641340204	-0.000546493
31	0.732938639	0.632557031	0.682747835	0.005120609	0.682747835	-0.000510588
32	0.466656233	0.703778885	0.585217559	0.004389132	0.585217559	-0.000547690
33	0.340354198	0.490855363	0.41560478	0.003117036	0.41560478	-0.000298418
34	0.466656233	0.490973951	0.478815092	0.003591113	0.478815092	-0.000297404
35	0.855046772	0.531341454	0.693194113	0.005198956	0.693194113	-0.000308819
36	0.651809098	0.481730148	0.566769623	0.004250772	0.566769623	-0.000283916

37	0.783584708	0.321595839	0.552590274	0.004144427	0.552590274	-0.000063644
38	0.320934739	0.354412909	0.337673824	0.002532554	0.337673824	-0.000082272
39	0.807977244	0.319792813	0.563885029	0.004229138	0.563885029	-0.000065364
40	0.693196393	0.533288437	0.613242415	0.004599318	0.613242415	-0.000310859
41	0.532257434	0.47904229	0.505649862	0.003792374	0.505649862	-0.000284260
42	0.831788925	0.32247821	0.577133567	0.004328502	0.577133567	-0.000069188
43	0.150295937	0.354861788	0.252578863	0.001894341	0.252578863	-0.000082508
44	0.320934739	0.352647895	0.336791317	0.002525935	0.336791317	-0.000079474
45	0.026670837	0.320134898	0.173402868	0.001300522	0.173402868	-0.000065099
46	0.126763423	0.392077689	0.259420556	0.001945654	0.259420556	-0.000101878
47	0.026670837	0.321870219	0.174270528	0.001307029	0.174270528	-0.000064823
48	0.239250631	0.533300277	0.386275454	0.002897066	0.386275454	-0.000313073
49	0.37809712	0.361530531	0.369813826	0.002773604	0.369813826	-0.000093156
50	0.102663757	0.320140324	0.21140204	0.001585515	0.21140204	-0.000066470
51	0.150295937	0.391939729	0.271117833	0.002033384	0.271117833	-0.000101943
52	0.077968924	0.320696887	0.199332906	0.001494997	0.199332906	-0.000065515
53	0.195762075	0.354711495	0.275236785	0.002064276	0.275236785	-0.000084119
54	0.026670837	0.352632175	0.189651506	0.001422386	0.189651506	-0.000083099

9. Conclusions

1) From the results obtained by applying the Savic-Pedrycz method and our proposed method, we conclude that the proposed method has total spread less than the total spread obtained by applying the original Savic-Pedrycz method, which means that it is more accurate and suitable for various real-life situations.
2) The results obtained from applying the proposed method agree with engineering theory beyond the considered problem.

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