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The Triangular Libration Point L_4 in the the Restricted Three Body Problem when the Bigger Primary is a Triaxial Rigid Body and Source of Radiation, Perturbation Effects Act in Coriolis and Centrifugal Forces

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Abstract: ***This paper deals with the stationary solutions of the triangular libration point L_4 of the planar restricted three body problem when the bigger primary is a triaxial rigid body and source of radiation, perturbation effects act in coriolis and centrifugal forces with one of the axes as the axis of symmetry and its equatorial plane coinciding with the plane of motion. It is seen that there are five libration points two triangular and three collinear. It is further observed that the collinear points are unstable, while the triangular points are stable for the mass parameter $0 \leq \mu < \mu_{crit}$ (the critical mass parameter). It is further seen that the triangular points have long or short periodic elliptical orbits in the same range of μ.***

Keywords: Restricted Three Body Problem: Libration Points; Rigid Body; Source of Radiation; Coriolis force; Centrifugal force

1. Introduction

It is well known that the classical planar restricted three body problem possesses five libration points, two triangular and three collinear. The collinear libration points L_1, L_2, L_3 are unstable, while the two equilateral libration points L_4, L_5 are stable for $\mu < \mu_{crit} = 0.0385208965 \ldots\ldots\ldots Szebehely$. Winter showed that the stability of the two equilateral points is due to the existence of coriolis terms in the equations of motion written in a synodic co-ordinate system.

In recent times many perturbing forces i.e., oblateness and radiation forces of the primaries, coriolis and centrifugal forces, variation of the masses of the primaries and of the infinitesimal mass etc., have been included in the study of the restricted three body problem. In the case of restricted three body problem where both the primaries are oblate spheroids whose equatorial plane coincides with the plane of motion, the location of libration points and their stability in the Liapunov sense has been studied by Vidyakin. For the case, where the bigger primary is an oblate spheroid whose equatorial plane coincides with the plane of motion, Subba Rao and Sharma have studied the stability of libration points. A similar problems has been studied by El-Shaboury. Khanna and Bhatnagar have studied the problem when the smaller primary is a triaxial rigid body.

In this paper, we consider the bigger primary as a triaxial rigid body and a source of radiation, perturbation effects act in coriolis and centrifugal forces with one of the axes as the axis of symmetry and its equatorial plane coinciding with the plane of motion. Further we assume that both the primaries are moving without rotation about their centre of mass in circular orbits. An attempt is made to study the existence and stability of libration points.

Garain, Mandal and Ahikary (2007) found triangular equilibrium points and also examined the stability of restricted problem of three bodies when bigger primary is a triaxial rigid body and perturbation effects act in coriolis and centrifugal forces.

D.N.Garain and B.K.Mandal (2015) found first order normalization of the triangular libration point L_4

Mandal (2017) found triangular libration point L_4 in 2+2 body problem when perturbation effects act in coriolis and centrifugal forces, small primary is a radiating body and bigger primary is a triaxial rigid body.

2. Equations of Motion

We shall adopt the notation and terminology of Szebehely. As a consequence, the distance between the primaries does not change and is taken equal to one; the sum of the masses of the primaries is also taken as one. The unit of time is so chosen as to make the gravitational constant unity. Using dimensionless variables, the equations of motion of the infinitesimal mas m_3 in a synodic co-ordinate system (x, y) are

$$\ddot{x} - 2n\,\dot{y} = \frac{\partial\Omega}{\partial x}$$

and

$$\ddot{y} - 2n\,\dot{x} = \frac{\partial\Omega}{\partial x}, \qquad (1)$$

$$where\ \Omega = \frac{1}{2}n^2[(1-\mu)r_1^2 + \mu r_2^2] + (1-p)\left[\frac{1-\mu}{r_1} + \frac{1-\mu}{2m_1\, r_1^3}(I_1 + I_2 + I_3 - 3I)\right] + \frac{\mu}{r_2}, (2)$$

McCusky

$$r_1^2 = (x-\mu)^2 + y^2,$$
$$r_2^2 = (x+1-\mu)^2 + y^2 \qquad (3)$$
$$and\ p = \frac{Radiation\ pressure\ due\ to\ bigger\ primary}{Gravitation\ force\ due\ to\ bigger\ primary} < < 1.$$

Here, we have applied effect of small perturbation in coriolis and centrifugal forces with the help of perturbations α and β the unperturbed value of both being unity

$\alpha = 1 + \epsilon, |\epsilon| \ll 1$

$\beta = 1 + \epsilon', |\epsilon'| \ll 1$

Hence Ω change to Ω' the equilibrium points are obtained from equations

$\Omega' x = \Omega' y = 0.$

Here μ is the ratio of the mass of the smaller primary to the total mass of the primaries and $0 < \mu \leq \frac{1}{2}$. That is , $\mu = \frac{m_2}{m_1+m_2} \leq \frac{1}{2}$ with $m_1 \geq m_2$ being the masses of the primaries. l_1, l_2, l_3 are the principal moments of inertia of the triaxial rigid body of mass $m_1$at its centre of mass, with a, b, c as lengths of its semi-axes. I is the moment of inertia about a line joining the centre of the rigid body of mass m_1 and the infinitesimal body of mass m_3 and is given by

$$I = I_1 l_1^2 + I_2 m_1^2 + I_3 n_1^2\ ,$$

where l_1, m_1, n_1 are the direction cosines of the line with respect to its principal axes.

Here, we have also assumed that the principal axes of m_1 and m_2 are parallel to the synodic axes $O(xyz)$.

The axes $O(xyz)$ have been defined by Szebehely.

The mean motion, n, is given by

$$n^2 = 1 + \frac{3}{2}(2A_1 - A_2 - A_3). \qquad (4)$$

$$where \qquad A_1 = \frac{a^2}{5R^2}, A_2 = \frac{b^2}{5R^2}, A_3 = \frac{c^2}{5R^2},$$

and R is the distance between the primaries.

Here we are neglecting the perturbation in the potential between m_1 and m_2 due to radiation pressure because m_2 is supposed to be sufficiently large.

Ω in the eq. (2) can also be written as

$$\Omega = \frac{1}{2}n^2[(1-\mu)r_1^2 + \mu r_2^2] + \frac{1-\mu}{r_1} + \frac{1-\mu}{2r_1^3}(2\sigma_1 - \sigma_2) - \frac{3(1-\mu)}{2r_1^5}(\sigma_1 - \sigma_2)y^2 - p\frac{1-\mu}{r_1} + \frac{\mu}{r_2},$$

$where\ \ \sigma_1 = A_1 - A_3$ and $\sigma_2 = A_2 - A_3$.

We assume that σ_1 and $\sigma_2 \ll 1$.

The mean motion gives in the eq. (4), becomes

$n^2 = 1 + \frac{3}{2}(2\sigma_1 - \sigma_2).$

It may be noted that the mean motion, n, is independent of the solar radiation pressure p.

3. Location of Libration Point L4

Equations (1) permits an integral analogous to Jacobi integral

$\dot{x}^2 + \dot{y}^2 - 2\Omega + C = 0$

The libration points are the singularities of the manifold

$F(x, y, \dot{x}, \dot{y}) = \dot{x}^2 + \dot{y}^2 - 2\Omega + C = 0$

Therefore, these points are the solutions of the equations

$\Omega_x = 0, \Omega_y = 0$

$$\begin{aligned}
L_4 = x^4 \Big[& -\frac{74}{256} + \sigma_1\left(\frac{585}{512} - \frac{2425}{512}\mu + \frac{25}{32\mu}\right) \\
& + \sigma_2\left(-\frac{1555}{512} + \frac{3225}{512}\mu - \frac{25}{32\mu}\right) - \frac{95}{128}\epsilon' \\
& + \frac{15}{32}\mu\epsilon'\Big] \\
+y^4 \Big[& \frac{3}{128} + \sigma_1\left(\frac{1785}{512} - \frac{3705}{512}\mu + \frac{45}{32\mu}\right) \\
& + \sigma_2\left(-\frac{1395}{512} + \frac{3705}{512}\mu - \frac{45}{32\mu}\right) \\
& - \frac{75}{128}\epsilon'\Big] \\
+x^3 y \Big[& \frac{105\sqrt{3}}{192} - \frac{300\sqrt{3}}{192}\mu \\
& + \sigma_1\left(\frac{2555\sqrt{3}}{384} - \frac{1060\sqrt{3}}{192}\mu + \frac{320\sqrt{3}}{192\mu}\right) \\
& + \sigma_2\left(-\frac{2565\sqrt{3}}{384} - \frac{530\sqrt{3}}{192}\mu - \frac{320\sqrt{3}}{192\mu}\right) \\
& + \frac{215\sqrt{3}}{288}\epsilon' - \frac{215}{144}\mu\epsilon'\Big] \\
+xy^3 \Big[& -\frac{90\sqrt{3}}{64} + \frac{45\sqrt{3}}{16}\mu \\
& + \sigma_1\left(-\frac{1505\sqrt{3}}{128} + \frac{2285\sqrt{3}}{128}\mu - \frac{5\sqrt{3}}{4\mu}\right) \\
& + \sigma_2\left(+\frac{615\sqrt{3}}{128} - \frac{865\sqrt{3}}{128}\mu + \frac{5\sqrt{3}}{4\mu}\right) \\
& - \frac{185\sqrt{3}}{96}\epsilon' + \frac{185}{48}\mu\epsilon'\Big] \\
+x^2 y^2 \Big[& \frac{246}{128} + \frac{345}{65}\epsilon' + \sigma_1\left(-\frac{1755}{256} + \frac{8835}{256}\mu - \frac{840}{128\mu}\right) \\
& + \sigma_2\left(\frac{6225}{256} - \frac{12915}{256}\mu + \frac{840}{128\mu}\right)\Big]
\end{aligned}$$

4. Stability of Triangular Libration Points

Now, we write the variational equations by putting $x = a + \xi$ and $y = b + \eta$ in the equations of motion (1), where (a,b) are the co-ordinate of L_4 (or L_5)

The characteristic equation is

$$\lambda^4 + (4n^2 - \Omega' x^\circ x - \Omega' y^\circ y)\,\lambda^2 + \Omega' x^\circ x \Omega' y^\circ y) - (\Omega' x^\circ y)^2 = 0 \qquad (5)$$

where

$$\begin{aligned}
\Omega' x^\circ y = \frac{3\sqrt{3}}{2}[& \mu - \frac{1}{2} + \frac{11}{18}\epsilon'(2\mu - 1) \\
& + \frac{\sigma_1}{24\mu}(8 - 47\mu + 89\mu^2) \\
& + \frac{\sigma_2}{24\mu}(-8 + 9\mu - 37\mu^2)
\end{aligned}$$

$$\Omega' x^\circ x = \frac{3}{4} + \frac{5}{4}\epsilon' + \frac{3}{16\mu}\sigma_1(15\mu^2 - 8 + 19\mu) + \frac{3}{16\mu}\sigma_2(-31\mu^2 + 8 - \mu) > 0$$

$$\Omega' y^\circ y = \frac{9}{4} + \frac{7}{4}\epsilon' + \frac{3}{16\mu}\sigma_1(8 + 29\mu - 15\mu^2) + \frac{3}{16\mu}\sigma_2(-8 - 7\mu + 15\mu^2) < 0$$

Replacing λ^2 by $\wedge$ in the equation (5), we get

$$\wedge^2 + P\wedge + Q = 0 \qquad (6)$$

Now we have the roots

$\lambda_1 = +\wedge_1^{\frac{1}{2}}, \lambda_2 = -\wedge_1^{\frac{1}{2}}, \lambda_3 = +\wedge_2^{\frac{1}{2}}$ *and* $\lambda_4 = -\wedge_2^{\frac{1}{2}}$
depend in a simple manner, on the value of the mass parameter μ, P, σ_1 *and* σ_2

If P, q_1, σ_1 *and* σ_2 are equal to zero then $\mu = \mu_0$ is a root of the given equation

where $\mu_0 = 0.0385208965 \ldots \ldots \ldots \ldots \ldots Szebehely$

when P, q_1, σ_1 *and* σ_2 are not equal to zero. We suppose

$$\mu_{crit} = \mu_0 + r_1 P + r_2\sigma_1 + r_3\sigma_2 + q_1\epsilon'$$

$$\mu_{crit} = 0.0385208965 \ldots \ldots . . -0.0089174706P + 0.8126474\sigma_1 - 1.09626653\sigma_2 - 0.14267953\epsilon'$$

Now we shall treat the three regions of the values of μ separately.

$0 \le \mu < \mu_{crit}$

we have $-\frac{1}{2}P < \wedge_1 \le 0$ and $-\frac{1}{2}P > \wedge_2 \ge -P$

But $P > 0$, therefore $\wedge_1$ and $\wedge_2$ are negative. In this case the four roots of the characteristic equation are written as

$$\lambda_{1,2} = \pm i(-\wedge_1)^{\frac{1}{2}} = \pm i\, s_1$$

$$and\ \lambda_{3,4} = \pm i(-\wedge_2)^{\frac{1}{2}} = \pm i\, s_2$$

This show that the triangular libration points are stable in linear case.

5. Conclusion

In the restricted three body problems, when the bigger primary is a triaxial rigid body and a source of radiation, perturbation effects act in coriolis and centrifugal forces. There are five libration points, three collnear and two triangular. The collinear points are unstable for all values of the mass parameter μ. Such that $0 < \mu \le \frac{1}{2}$ and the triangular points are stable for $\mu < \mu_{crit}$ and $0 \le \mu < \mu_{crit}$.

$$where\ \mu_{crit} = 0.0385208965 \ldots \ldots . . -0.0089174706P + 0.8126474\sigma_1 - 1.09626653\sigma_2 - 0.14267953\epsilon'$$

Hence L_4 is stable.

For $\mu_{crit} < \mu < \frac{1}{2}$, L_4 is unstable and

$\mu = \mu_{crit}$, L_4 is unstable

when $P = \sigma_1 = \sigma_2 = 0$ and also $q_1 = 0$

The results obtained are in conformity with the classical problem Szebehely.

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