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Calculation Quadrupole Moments, Magnetic Dipole Moments and Occupation Numbers for Some Exotic Scandium Isotopes

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***Abstract:** The quadrupole moments, magnetic dipole moments and the occupation numbers on the ground state have been calculated for some exotic scandium nuclei (A= 41, 43, 44, 46, and 47), and for stable nucleus (A=45) based on the shell model using FPBM interaction. ^{40}Ca nucleus is considered as inert core for all isotopes under consideration and the other valence nucleons are moving over the fp-shell model space within $1f_{7/2}$, $2p_{3/2}$, $1f_{5/2}$ and $2p_{1/2}$ orbits. The simple harmonic oscillator potential (HO) is used to generate the single particle matrix elements of Sc isotopes. Calculations with configuration mixing shell model with these limited model spaces, usually underestimate the measured quadrupole moments and magnetic dipole moments. Including the discarded space through the effective charges and effective g-factors, this is called core-polarization effect (CP), make the present result in agreement with the experimental data.*

Keywords: exotic nuclei, quadrupole and magnetic dipole moments.

1. Introduction

The shapes of nuclei far from the stability valley is one of the major topics of interest of modern nuclear physics. Advancements in experimental techniques have probed extreme types of nuclear structures not previously known, termed "exotic nuclei" which are a nuclei with extreme properties, such as an extraordinary ratio of protons and neutrons, large isospin, short -lived isotopes, and loosely binding energy. These exotic nuclei (proton or neutron rich) are weakly bound and some of which have a thin cloud of nucleons orbiting at large distances from the others that forming an inert core [1].

The magnetic moment (μ) and the quadrupole moments (Q) are very sensitive to the single particle orbits occupied by the unpaired nucleons. Magnetic moments and the quadrupole moments provide the information about nuclear structure by the single particle configurations. The *g*-factor reveals which single particle orbits are occupied by the unpaired nucleons, while the quadrupole moment is sensitive to the nuclear deformation and to collective components in the nuclear wave function [2, 3].

Avgoulea *et. al* have been used the laser spectroscopy on ^{42,43,44,44m,45,45m,46}Sc to reveal the nuclear moments and rms charge radii. They concluded from a comparison between theory and experiment more qualitative for the quadrupole moments than for the magnetic moments [4]. Yang *et. al* have been measured the moments and mean-square charge radii of $^{47\text{-}51}$Sc. The measurements will cross the N = 28 shell closure and constitute a prominent test of state-of-the-art theoretical calculations developed in the Ca region [5].

The aim of the present work is to investigate the nuclear structure for exotic $^{41,\ 43,\ 44,\ 46,\ 47}$Sc isotopes and stable ^{45}Sc nucleus through electromagnetic transitions. The quadrupole moments Q and the magnetic dipole moments μ are calculated for each exotic isotopes and stable nucleus using FPBM interaction [6] in *fp*- shell model space, the current results are compared with the measured values. The core-polarization (CP) effects are included through effective charge and effective *g*-factors. Also the occupation numbers of the valence nucleons outside ^{40}Ca core are also calculated for the ground state of $1f_{7/2}$, $2p_{3/2}$, $1f_{5/2}$ and $2p_{1/2}$ orbits to identify the contribution of these orbits for the valence nucleons.

2. Theory

The electric transition operator for a system of *n* nucleon in [7]:

$$\hat{O}_{EJM}(\bar{r}) = \sum_{k=1}^{n} e(k)\, r_k^J Y_{JM}(\Omega_k), \qquad (1)$$

where *e*(*k*) denotes the charge of the nucleon numbered *k*, i.e. *e*(*k*) = 0 for a neutron and *e*(*k*) = *e* for a proton. We will keep the ground expression in terms of *e*(*k*) because we will introduce effective charges for the proton and neutron. Using isospin formalism, with the operator $\hat{\tau}_z$, such that $\hat{\tau}_Z|p\rangle = p$ for (proton) and $\hat{\tau}_Z|n\rangle = -n$ for neutron. Equation (1) can be writer as [7]:

$$\hat{O}_{EJM}(\bar{r}) = \sum_{k=1}^{n}\left[\frac{e_p+e_n}{2} + \frac{e_p-e_n}{2}\hat{\tau}_z\right] r_k^J Y_{JM}(\Omega_k), \qquad (2)$$

and which can wrttien as:

$$\hat{O}_{EJM}(\bar{r}) = \sum_{k=1}^{n}\left[e_{IS} + e_{IV}\,\hat{\tau}_z\right] r_k^J Y_{JM}(\Omega_k), \qquad (3)$$

Where e_{IS} and e_{IV} are the isoscalare and isovector charges, $e_{IS} = \frac{e_p+e_n}{2}$ and $e_{IV} = \frac{e_p-e_n}{2}$, with e_p and e_n are the proton and neutron charge, respectively.

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The reduce matrix element of the electric transition operator $\hat{O}_{JT}$ is expressed as the sum of the product of the elements of the one-body density matrix (*OBDM*) times the single-particle matrix elements, and is gives by:

$$\left\langle J_f T_f \,|||\, \hat{O}_{JT} \,||\, J_i T_i \right\rangle = \sum_{j_f\, j_i} OBDM \quad (j_f, j_i, J_f T_f, J_i T_i)^{\;JT} \times \left\langle j_f\, t \left\| \hat{o}_{J\,T} \right\| j_i\, t \right\rangle \qquad (4)$$

where $j_f t$ and $j_i t$ label single-particle states (isospin is included) for the shell model space. The states $\left| J_f T_f \right\rangle$ and $\left| J_i T_i \right\rangle$ are described by the model space wave functions. The nuclear shell model calculations were performed using the OXBASH Shell model code [8], where the one body density matrix (*OBDM*) elements in spin-isospin formalism are obtained. Using Wickner-Eckart theorem, the single-particle matrix elements reduced in both spin and isospin, are written in terms of the single- particle matrix elements reduced in spin only [7].

$$\left\langle j_f \,|||\, \hat{O}_{JT} \,|||\, j_i \right\rangle = \sqrt{\frac{2T+1}{2}} \sum_{tz} I_T(t_z) \left\langle j_f \left\| \hat{O}_{J t_z} \right\| j_i \right\rangle, \quad (5)$$

$$\text{With } I_T(t_z) = \begin{cases} 1 & for\, T=0 \\ (-1)^{\frac{1}{2}-t_z} & for\, T=1 \end{cases} \qquad (6)$$

where t_z=1/2 for a proton and −1/2 for a neutron. The single-particle matrix element of the electric transition operator reduced in spin is

$$\left\langle j_f \left\| \hat{O}_{J t_z} \right\| j_i \right\rangle = e_{t_z} \left\langle j_f \left\| Y_J \right\| j_i \right\rangle \left\langle n_f l_f \left| r^J \right| n_i l_i \right\rangle, \quad (7)$$

where $\left| nl \right\rangle$ is the single-particle radial wave function. The reduced single particle matrix element of the electric transition operator in spin space is given by [7, 9]:

$$\left\langle j_f T_f \,|||\, \hat{O}_{JT} \,|||\, j_i T_i \right\rangle = e_T \sqrt{2\,(2\,T+1)} \times \left\langle j_f \,\|\, Y_J \,\|\, j_i \right\rangle \left\langle n_f\, l_f \,|\, r^J \,|\, n_i\, l_i \right\rangle, \qquad (8)$$

The Quadrupole moment is define ($J = 2$) as [7]:

$$Q = 2 \begin{pmatrix} J_i & 2 & J_i \\ -J_i & 0 & J_i \end{pmatrix} \sqrt{\frac{4\pi}{5}} \sum_{T=0,1} (-1)^{T_i - T_z} \begin{pmatrix} T_i & T & T_i \\ -T_z & 0 & T_z \end{pmatrix} \times \left\langle J_i T_i \,|||\, \hat{O}_{2T} \,|||\, J_i T_i \right\rangle \qquad (9)$$

Equation (9) can be written as

$$Q = 2 \begin{pmatrix} J_i & 2 & J_i \\ -J_i & 0 & J_i \end{pmatrix} \sqrt{\frac{4\pi}{5}} \sum_{T=0,1} e_T \begin{pmatrix} T_i & T & T_i \\ -T_z & 0 & T_z \end{pmatrix} \tilde{M}_{2T} \qquad (10)$$

where $\tilde{M}_{JT} = \left\langle J_f T_f \,|||\, \tilde{\hat{M}}_{JT} \,|||\, J_i T_i \right\rangle$, which is given in equation (10). The isoscalar (T=0) and isovector (T=1) charges are given by $e_0 = e_{\text{IS}} = \frac{1}{2} e$, $e_1 = e_{\text{IV}} = \frac{1}{2} e$ .The quadrupole moment (Q) can be represented in terms of only the model space matrix elements as [7]:

$$Q = 2 \begin{pmatrix} J_i & 2 & J_i \\ -J_i & 0 & J_i \end{pmatrix} \sqrt{\frac{4\pi}{5}} \sum_{T=0,1} e_T^{\text{eff}} \begin{pmatrix} T_i & T & T_i \\ -T_z & 0 & T_z \end{pmatrix} M_{2T} \quad (11)$$

Then the isoscalar and isovector effective charges are given by:

$$e_T^{\text{eff}} = \frac{M_{JT} + \Delta M_{JT}}{2 M_{JT}}, \quad e = \frac{e_p^{\text{eff}} + (-1)^T e_n^{\text{eff}}}{2} \qquad (12)$$

The proton and neutron effective charges can be obtained as follows:

$e_p^{\text{eff}} = e_0^{\text{eff}} + e_1^{\text{eff}}$, and $e_n^{\text{eff}} = e_0^{\text{eff}} - e_1^{\text{eff}}$

The single particle matrix element in spin- isospin state is given by [7]:

$$\left\langle a \left\| \hat{O}_T(m1) \right\| b \right\rangle = \left\langle n_a l_a \left| r^{J-1} \right| n_b l_b \right\rangle f_T^{m1}(a,b) \qquad (13)$$

where

$$f_T^{m1}(a,b) = (-1)^{\ell_a} (2J+1) \sqrt{\frac{J(2J-1)(2\ell_a+1)(2\ell_b+1)(2j_a+1)(2j_b+1)}{4\pi}}$$
$$\times \begin{pmatrix} \ell_a & J-1 & \ell_b \\ 0 & 0 & 0 \end{pmatrix} \sqrt{2T+1} \left[\frac{g_p^{\ell} + (-1)^T g_n^{\ell}}{J+1} (-1)^{\ell_b + j_b + 1/2} \sqrt{2\ell_b(\ell_b+1)(2\ell_b+1)} \right.$$
$$\left. \times \begin{Bmatrix} \ell_a & \ell_b & J \\ j_b & j_a & 1/2 \end{Bmatrix} \begin{Bmatrix} J-1 & 1 & J \\ \ell_b & \ell_a & \ell_b \end{Bmatrix} + \frac{1}{2} \left\{ g_p^s + (-1)^T g_n^s \right\} \sqrt{3} \begin{Bmatrix} \ell_a & 1/2 & j_a \\ \ell_b & 1/2 & j_b \\ J-1 & 1 & J \end{Bmatrix} \right] \mu_N \qquad (14)$$

where $\mu_N = \dfrac{e\hbar}{2 m_p c} = 0.1051 e.fm$ is the nuclear magneton, with m_p the proton mass. The orbital and spin free nucleon *g*-factors are: g_l (p) = 1, g_s (p) = 5.585 for proton and g_l (n) = 0, g_s (n) = -3.826 for neutron [7]. The bracket $\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$, $\begin{Bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{Bmatrix}$ and $\begin{Bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{Bmatrix}$ are denotes the 3j, 6j and 9j-symbol, respectively. For $J=1$ equation (13) becomes:

$$\left\langle a \left\| \hat{O}_T(m1) \right\| b \right\rangle = f_T^{m1}(a,b) \qquad (15)$$

The reduce matrix element of the magnetic transition operator $\hat{O}_k(m1)$ is given by [7]:

$$\left\langle J_i \left\| \sum_{k=1}^{n} \hat{O}_k(m1) \right\| J_i \right\rangle = \sum_{a,b,T} (-1)^{T_i - T_z} \begin{pmatrix} T_i & T & T_i \\ -T_z & 0 & T_z \end{pmatrix} OBDM \quad (a,b,J=1,T) \times \left\langle a \left\| \hat{O}_T(m1) \right\| b \right\rangle \qquad (16)$$

The triple bare denote that the matrix elements are reduced for both J and T. The magnetic dipole moment (μ) for a state of total angular momentum J is given by [7]:

$$\mu = \sqrt{\frac{4\pi}{3}} \begin{pmatrix} J_i & 1 & J_i \\ -J_i & 0 & J_i \end{pmatrix} \left\langle J_i \left\| \sum_{k=1}^{n} \hat{O}_k(m1) \right\| J_i \right\rangle, \quad (17)$$

The average occupation numbers in each subshell j is given by [10]:

$$occ\#(j,t_z) = OBDM(a,b,t_z,J=0) \sqrt{\frac{2j+1}{2J_i+1}}, \quad (18)$$

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3. Results and Discussion

Calculations of the quadrupole moments (Q) and magnetic dipole moments (μ) are presented for proton rich nuclei (A=41, 43 and 44), for neutron rich nuclei (A=46 and 47) and for stable nucleus (A=45) of Sc isotopes. The electric quadrupole moment Q, representing a deviation from a spherical distribution of the electric charges in a nucleus, is sensitive to the admixture of collective components. The quadrupole moment and magnetic dipole moments is an excellent tool to study the deformation of nuclei. Calculations are presented with model space (MS) only, and with core-polarization (CP) effects. The calculated and experimental values of the magnetic dipole moments and quadrupole moments are tabulated in table (7) and table (8), respectively.

3.1 ^{41}Sc nucleus ($J^{\pi}T = 7/2^{-}$ 1/2, $\tau_{1/2}$ = 596.3(17) ms)

^{41}Sc nucleus (proton rich) is composed of the core ^{40}Ca plus one nucleon surrounding the core and active in the *fp*-shell model space. Calculations are performed with FPBM interaction. The calculated occupation numbers of ^{41}Sc in its ground state are presented in table (1). It is so clear from this table that the largest ratio of the occupation numbers of these protons goes to the $1f_{7/2}$ orbit outside the core, where the percentages of the occupation over $1f_{7/2}$ are 100% and shown in figure (1).

The calculated magnetic dipole moment of this transition for g_s(free) is $\mu = 5.793\, n.m$, which overestimates the experimental value $\mu_{exp.} = 5.431(2)\ \ n.m$ [11]. Using the orbital g-factors for proton and neutron as g_l^p = 0.88, g_l^n = 0.38 the result of μ agrees with that of measured value 5.433 n.m. The calculated quadrupole moments by using the model space only with free effective charge e_p=1.0e and e_n=0.0e is Q= -11.56 e fm^2 (oblate), which is underestimates the measured values of Ref. [11]. Including the core-polarization effect by using the standard effective charges 1.3e, 0.5e, for the proton and neutron, respectively, the Q moments is -15.03 e fm^2 which agree with the measured values.

Table 1: The calculated occupation numbers as predicted by FPBM interaction for ^{41}Sc nucleus.

State 1 and state2	Proton and neutron occupation numbers
$1s_{1/2}$	4.0
$1p_{3/2}$	8.0
$1p_{1/2}$	4.0
$1d_{5/2}$	12.0
$1d_{3/2}$	8.0
$2s_{1/2}$	4.0
$1f_{7/2}$	1.0

Table 2: The calculated occupation numbers as predicted by FPBM interaction for ^{43}Sc nucleus.

State 1 and state2	Proton and neutron occupation numbers
$1s_{1/2}$	4.0
$1p_{3/2}$	8.0
$1p_{1/2}$	4.0
$1d_{5/2}$	12.0
$1d_{3/2}$	8.0
$2s_{1/2}$	4.0
$1f_{7/2}$	2.5693
$2p_{3/2}$	0.2536
$1f_{5/2}$	0.1325
$2p_{1/2}$	0.0446

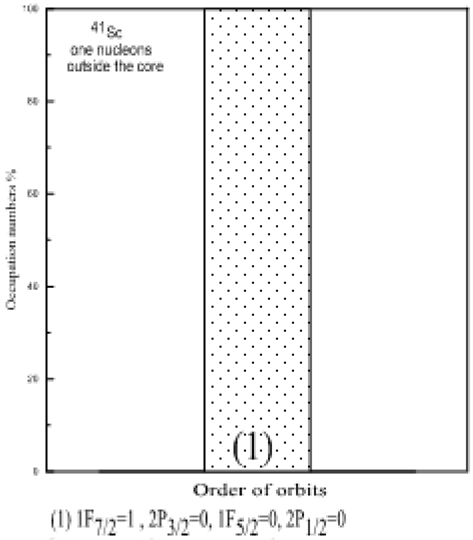

Figure 1: The percent occupation numbers for the ground states of $1f_{7/2}$, $2p_{3/2}$, $1f_{5/2}$ and $2p_{1/2}$ orbits outside ^{40}Ca core of the considered ^{41}Sc nucleus.

3.2 ^{43}Sc nucleus ($J^{\pi}T = 7/2^{-}$ 1/2, $\tau_{1/2}$ =3.891(12) h)

The ground state of unstable nucleus ^{43}Sc (exotic proton-rich system) is specified by $J^{\pi}T = 7/2^{-}$ 1/2 with half-life time ($\tau_{1/2}$=3.891(13) h). This isotope is consisting of ^{40}Ca core nucleus plus three active nucleons distributed over *fp*-shell. Table (2) containing the occupation numbers of each orbit, including the core orbits where the $1f_{7/2}$ orbit is occupied by the large ratio of occupation numbers. The percentages of the occupation numbers of $1f_{7/2}$, $2p_{3/2}$, $1f_{5/2}$ and $2p_{1/2}$ orbits by the active three nucleons are shown in figure (2).The calculated magnetic dipole moment of this transition for g_s(free) is $\mu = 4.678\, n.m$, which admissible with the experimental values $\mu_{exp.} = 4.62\ (4), 4.528(10)\quad n.m$ [11,4]. Using the orbital g-factors for proton and neutron as g_l^p = 0.9, g_l^n = 0.21 the result of μ agrees with that of measured values 5.624n.m. The calculated quadrupole moments by using the model space only with free effective charge e_p=1.0e and e_n=0.0e is Q = $-8.882\, e$ fm^2 (oblate), which underestimates the measured values $Q_{exp.=}-26.0(6), -27.0(5)\ e$ fm^2 [11,4]. Including the core-polarization (Cp) effect by using the standard effective charges 1.3e, 0.5e, for the proton and neutron, respectively,

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the Q moments is $-18.35e$ fm^2 which acceptable with the measured values.

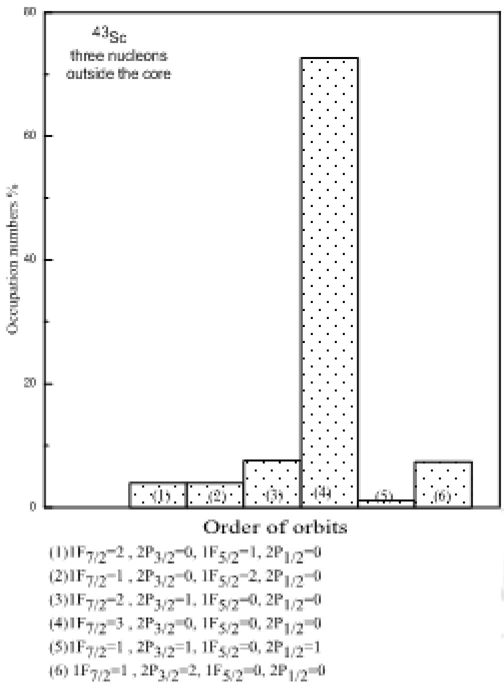

Figure 2: The percent occupation numbers for the ground states of $1f_{7/2}$, $2p_{3/2}$, $1f_{5/2}$ and $2p_{1/2}$ orbits outside ^{40}Ca core of the considered ^{43}Sc nucleus

3.3 ^{44}Sc nucleus ($J^{\pi}T = 2^{+}$ 1, $\tau_{1/2}$= 3.97(4) h)

The ground state of unstable nucleus ^{44}Sc (exotic proton-rich system) is specified by $J^{\pi}T= 2^{+}$ 1 with half-life time ($\tau_{1/2}$=3.97(4) h). Using FPBM interaction with ^{40}Ca core nucleus plus four active nucleons distributed over *fp*-shell. Such interaction and model space give occupation numbers viewed in table (3). The percentages of the occupation numbers for the four active nucleons are shown in figure (3).

Table 3: The calculated occupation numbers as predicted by FPBM interaction for ^{44}Sc nucleus.

State 1 and state2	Proton and neutron occupation numbers
$1s_{1/2}$	4.0
$1p_{3/2}$	8.0
$1p_{1/2}$	4.0
$1d_{5/2}$	12.0
$1d_{3/2}$	8.0
$2s_{1/2}$	4.0
$1f_{7/2}$	3.3928
$2p_{3/2}$	0.3893
$1f_{5/2}$	0.1529
$2p_{1/2}$	0.0650

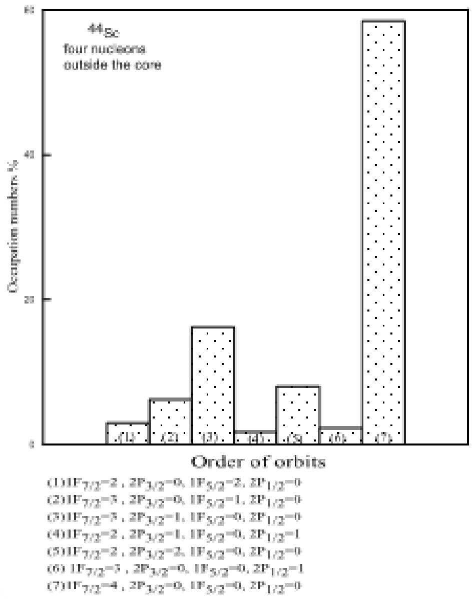

Figure 3: The percent occupation numbers for the ground states of $1f_{7/2}$, $2p_{3/2}$, $1f_{5/2}$ and $2p_{1/2}$ orbits outside ^{40}Ca core of the considered ^{44}Sc nucleus.

The calculated magnetic dipole moment of this transition for g_s(free) is $\mu = 0.2.641\ n.m$, which overestimates both experimental valus $\mu_{exp.} = 2.56\,(3),\ 2.499\ (5)\ n.m$ [11,4].

Using the orbital *g*-factors for proton g_l^p =0.92 the result of μ agrees with that of experimental value 2.56 n.m. The calculated Q moment is 2.748 *e* fm^2 (prolate) which is in underestimates with the measured values $Q_{exp.=}$10.0(5),16.0(4) *e* fm^2 [11,4]. Using the standard effective charges 1.3e, 0.5e, for the proton and neutron, respectively, the Q moment is 6.323 *e* fm^2 which is also underestimates with the measured values.

3.4 ^{45}Sc nucleus ($J^{\pi}T = 7/2^{-}$ 3/2, $\tau_{1/2}$ =stable)

According to the many – particle shell model, the stable nucleus ^{45}Sc is considered as a core of ^{40}Ca plus five nucleons distributed over the $1f_{7/2}$, $2p_{3/2}$, $1f_{5/2}$ and $2p_{1/2}$ orbits. The calculated occupation numbers of ^{45}Sc in its ground state are presented in table (4). It is so clear from this table that the largest ratio of the occupation numbers of these nucleons goes to the $1f_{7/2}$ orbit outside the core. The percentage occupation numbers are shown in figure (4).

Table 4: The calculated occupation numbers as predicted by FPBM interaction for ^{45}Sc nucleus.

State 1 and state2	Proton and neutron occupation numbers
$1s_{1/2}$	4.0
$1p_{3/2}$	8.0
$1p_{1/2}$	4.0
$1d_{5/2}$	12.0
$1d_{3/2}$	8.0
$2s_{1/2}$	4.0
$1f_{7/2}$	4.3382
$2p_{3/2}$	0.3738
$1f_{5/2}$	0.2205
$2p_{1/2}$	0.0675

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The calculated magnetic dipole moment of this transition for g_s(free) is $\mu = 4.713\ n.m$, which is a good agreement with the measured value $\mu_{\exp.} = 4.756487\ (2)\ n.m$ [11]. Using the orbital *g*-factors for neutron as $g_l^n = 0.07$ the result of μ an excellent agreement with the experimental value 4.7562 n.m as shown in table (7). The calculated Q moment is -9.341 *e* fm^2 (oblate) which is in underestimates the measured values of Refs. [11]. Using the standard effective charges 1.3e, 0.5e, for the proton and neutron, respectively, the Q moment is -21.06 *e* fm^2 which is a good agreement with the measured values.

Table 5: The calculated occupation numbers as predicted by FPBM interaction for ^{46}Sc nucleus.

State 1 and state2	Proton and neutron occupation numbers
$1s_{1/2}$	4.0
$1p_{3/2}$	8.0
$1p_{1/2}$	4.0
$1d_{5/2}$	12.0
$1d_{3/2}$	8.0
$2s_{1/2}$	4.0
$1f_{7/2}$	5.3270
$2p_{3/2}$	0.4191
$1f_{5/2}$	0.1915
$2p_{1/2}$	0.0624

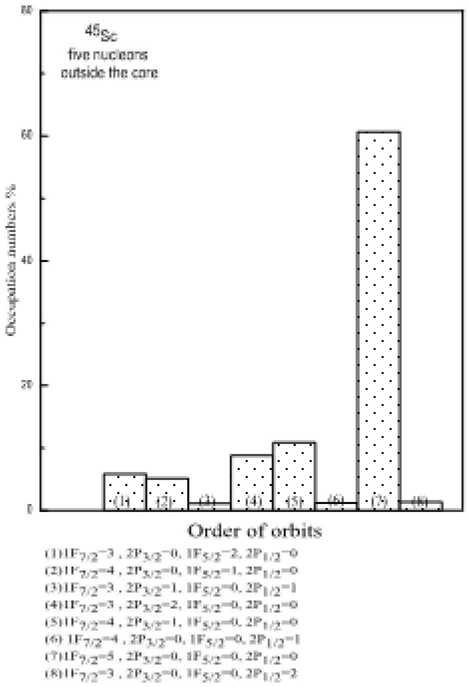

Figure 4: The percent occupation numbers for the ground states of $1f_{7/2}$, $2p_{3/2}$, $1f_{5/2}$ and $2p_{1/2}$ orbits outside ^{40}Ca core of the considered ^{45}Sc nucleus.

3.5 ^{46}Sc nucleus ($J^{\pi}T = 4^+\ 2$, $\tau_{1/2}$=83.79(4) d)

According to the many– particle shell model, the exotic ^{46}Sc nucleus (exotic neutron-rich system) is considered as a core of ^{40}Ca plus six nucleons distributed over the $1f_{7/2}$, $2p_{3/2}$, $1f_{5/2}$ and $2p_{1/2}$ orbits. Such interaction and model space give occupation numbers viewed in table (5). The percentages of the occupation numbers for the six active nucleons are shown in figure (5). The calculated magnetic dipole moment of this transition for g_s(free) is $\mu = 3.022\ n.m$, which is a good agreement with the measured value $\mu_{\exp.} = 3.03(2)\ , 3.042(8)\ n.m$ [11, 4]. Using the orbital g-factors for neutron as $g_l^n = 0.1$ the result of μ agrees with that of experimental values 3.036n.m. The calculated Q moment is 1.672 (prolate) *e* fm^2 which is in underestimates with the measured values $Q_{\exp.=}$ 11.9(6), 12.0(2) *e* fm^2 [11, 4]. Using the standard effective charges 1.3e, 0.5e, for the proton and neutron, respectively, the Q moment is 0.433 *e* fm^2 which is still remaining underestimates the measured value.

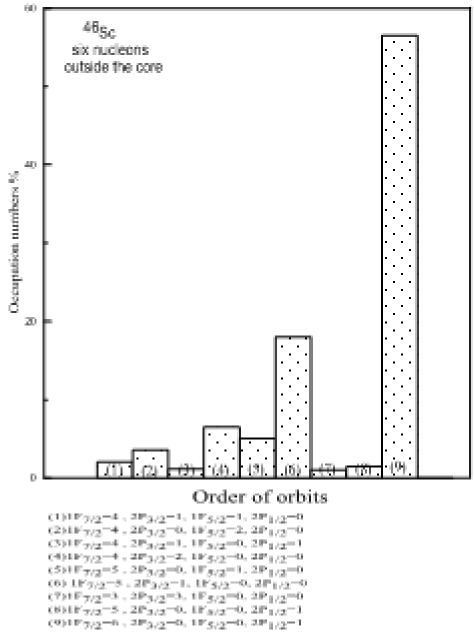

Figure 5: The percent occupation numbers for the ground states of $1f_{7/2}$, $2p_{3/2}$, $1f_{5/2}$ and $2p_{1/2}$ orbits outside ^{40}Ca core of the considered ^{46}Sc nucleus.

3.6 ^{47}Sc nucleus ($J^{\pi}T = 7/2^-\ 5/2$, $\tau_{1/2}$ =3.3492(6) d)

^{47}Sc nucleus (neutron rich) is composed of the core ^{40}Ca plus seven nucleons surrounding the core and active in the *fp*-shell model space. Calculations are performed with FPBM interaction. The calculated occupation numbers of ^{47}Sc in its ground state are presented in table (6). It is so clear from this table that the largest ratio of the occupation number of these neutrons goes to the $1f_{7/2}$ orbit outside the core. Such interaction and model space give occupation numbers viewed in table (6).

Table 6: The calculated occupation numbers as predicted by FPBM interaction for ^{47}Sc nucleus.

State 1 and state2	Proton and neutron occupation numbers
$1s_{1/2}$	4.0
$1p_{3/2}$	8.0
$1p_{1/2}$	4.0
$1d_{5/2}$	12.0
$1d_{3/2}$	8.0
$2s_{1/2}$	4.0
$1f_{7/2}$	6.2129
$2p_{3/2}$	0.4127
$1f_{5/2}$	0.2884
$2p_{1/2}$	0.0860

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The calculated magnetic dipole moment of this transition for g_s(free) is $\mu = 4.49 n.m$, which is underestimates the measured value $\mu_{exp.} = 5.34(2)$ $n.m$ [11]. Using the orbital g-factors for proton and neutron as g_l^p = 1.15, g_l^n = 0.05 the result of μ an excellent agreement with the experimental value 5.342 n.m. The calculated Q moment is -9.737 (oblate) e fm^2 which is in underestimates with the measured values $Q_{exp.=}$ $-22.0(3)$ e fm^2 [11]. Using the standard effective charges 1.3e, 0.5e, for the proton and neutron, respectively, the Q moment is -21.54 e fm^2 which a good agreement with the measured values.

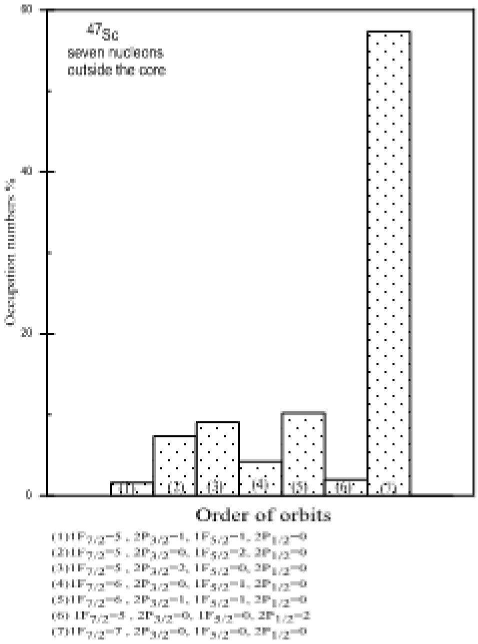

Figure (6): The percent occupation numbers for the ground states of $1f_{7/2}$, $2p_{3/2}$, $1f_{5/2}$ and $2p_{1/2}$ orbits outside ^{40}Ca core of the considered ^{47}Sc nucleus.

Table 7: The calculated magnetic dipole moments (μ) of ^{41,43,44,45,46,47}Sc isotopes are compared with experimental results

A	$J^{\pi}T$	$\mu_{Calc.}$(n. m)				$\mu_{exp.}$ (n. m) [11]	$\mu_{exp.}$ (n. m) [4]
		g(free)	g_l^p	g_l^n	$\mu_{Calc.}$(n. m)		
41	7/2- 1/2	5.793	0.88	0.38	5.433	5.431(2)	------
43	7/2- 1/2	4.678	0.9	0.21	4.624	4.62(4)	4.528(10)
44	2+ 1	2.641	0.92	0.0	2.56	2.56(3)	2.499(5)
45	7/2- 3/2	4.713	1.0	0.07	4.7562	4.75648 (2)	------
46	4+ 2	3.022	1.0	0.01	3.036	3.03(2)	3.042(8)
47	7/2- 5/2	4.49	1.15	0.05	5.342	5.34(2)	-------

Table 8: The calculated quadrupole moments (Q) of $^{41, 43, 44, 45}$Sc isotopes are compared with experimental results.

A	$J^{\pi}T$	$Q_{Calc.}$(e. fm^2)		Q_{exp} (e. fm^2) [11]	$Q_{exp.}$ (e. fm^2) [4]
		$Q_{calc.}$ e_p=1.0 e_n=0.0	$Q_{calc.}$ e_p=1.3 e_n=0.5		
41	7/2- 1/2	-11.56	-15.03	-15.6(3)	------
43	7/2- 1/2	-8.882	-18.35	-26.0(6)	−27.0(5)
44	2+ 1	2.748	6.323	10.0(5)	16.0(4)
45	7/2- 3/2	-9.341	-21.06	-15.6(3)	-----
46	4+ 2	1.672	0.433	11.9(6)	12.0(2)
47	7/2- 5/2	-9.737	-21.54	-22.0(3)	-----

4. Conclusions

In the present work, one can summarize the following conclusions:

1) The quadrupole moments, magnetic dipole moments and the occupation numbers of the Sc nuclei depend clearly on assigned configurations and their experimental data will be useful to determine the deformations of the ground states of nuclei near the drip line.
2) The shell model is less successful in describing the quadrupole and magnetic moments unless taking the core-polarization effects into account. The core polarization is necessary, beneficial and plays a major role for describing nuclear properties
3) Inclusion the standard effective charges used for stable nuclei gives good results for the quadrupole moments comparison with the exotic nuclei (neutron rich).
4) Calculations of the persent occupation number of Sc isotopes show a strong contribution of $1f_{7/2}$ orbit and a clear exotic behavior for the valence nucleons outside ^{40}Ca core.

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