Mathematical Behaviour of a System with Failures due to Machinery Defects and Random Shocks

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Abstract: ***Many engineering systems have been analyzed in the field of reliability theory with the assumption that all the units of the system are of similar type. In most situations, there exists some two unit standby systems in which both the units are dissimilar with different costs and operating conditions. Keeping this view, the present paper analyze a two unit redundant system in which both the units are dissimilar. Here the units can fail due to machinery defects as well as due to random shocks. The reliability characteristics of interest are obtained using regenerative point technique with Markov renewal process.***

Keywords: Transition probabilities, mean sojourn time, mean time to system failure, availability analysis, busy period analysis and expected number of visits by repair facility

1. Introduction

Agarwal, Manju and Kumar, A (1981), Eric Chatelat (2005), Dhillon (1982), Goyel and Agnihotri (1981) and Al-Ali (1990) analyzed many engineering systems in the field of reliability with the assumption that all the units of the system are of similar type. There may exists two unit standby systems in which both the units are dissimilar with different costs and operating conditions. Keeping this view in the present paper, we analyze two unit redundant system in which both the units are dissimilar. The units here can fail due to machinery defects as well as due to random shocks. The reliability characteristics of interest using regenerative point technique with Markov renewal process are obtained such as Transition and Steady state transition probabilities, Mean Sojourn time, MTSF, Point wise and steady state availability of the system, expected Busy period of the repairman, end expected number of visits by repairman in (o, t).

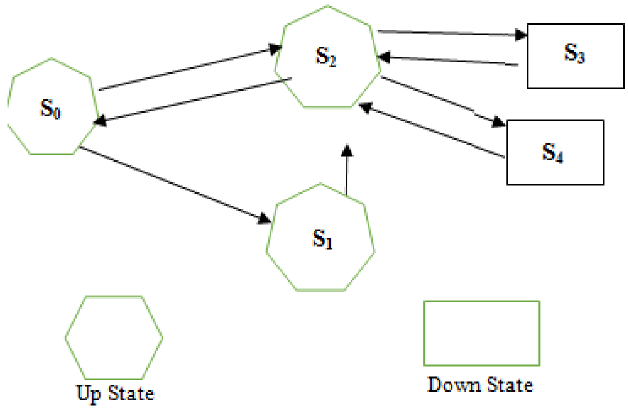
2. Model Description and Assumptions

1) The system consists of only two non-identical units in which first is operative and the second unit is kept as cold standby.
2) First unit can sustain almost two shocks i.e. if the unit does not fail in the first shock then it will definitely fail in the second shock. It can also fail directly due to machinery defects.
3) Second unit can fail due to machinery defects as well as due to random shocks and it cannot sustain more than one random shock.
4) A single repair facility is considered in the system.
5) Second unit is repairable if it is failed due to machinery defects otherwise in case of random shocks send it for replacement.
6) The priority in repair and replacement is given to the second unit over first unit.
7) All the failure time distributions are assumed to be negative exponential while the distribution of repair and replacement are arbitrary.

3. Notation and Symbols

N_0	Normal unit as operative
N_5	Normal unit as cold standby
N_{01}	Normal unit as operative after observing first random shock
F_r	Failed unit under repair
F_{wr}	Failed unit waiting for repair
F_{rep}	Failed unit under replacement
α	Constant rate of occurring first random shock to the first unit
β	Constant rate of occurring second random shock to the first unit
γ	Constant failure rate of first unit failed due to machinery defects
δ	Constant failure rate of second unit failed due to random shock
λ	Constant failure rate of second unit failed due to machinery defects
f(.), F(.)	pdf and cdf of time to repair of first unit
g(.), G(.)	pdf and cdf of time to replacement of second unit failed due to random shock
h(.), H(.)	pdf and cdf of time to repair of second unit failed due to machinery defects

The possible states of the system are here under using the notations and symbols above:
Up States: $S_0 \equiv (N_0, N_s)$ $S_1 \equiv (N_{01}, N_s)$ $S_2 \equiv (F_r, N_0)$
Down States: $S_3 \equiv (F_{wr}, F_{rep})$ $S_4 \equiv (F_{wr}, F_t)$
The transitions between various states are shown below:



4. Transition Probabilities

Let T_0 (= 0), T_1, T_2,………be the epochs at which enters the states S_i ε E. Let X_0 denotes the state entered at epoch T_{n+1} i.e. just after the transition of T_n. Then $\{T_n, X_n\}$ constitutes a Markov-renewal process with state space E and

$Q_{ik}(t) = \Pr[\, X_{n+1} = S_k, T_{n+1}-T_n \le t \mid X_n = S_i]$ (1.1)

is semi Markov over E. The stochastic matrix of embedded Markov chain is

$P = p_{ik} = \lim_{t\to\infty} Q_{ik}(t) = Q(\infty)$ (1.2)

By simple probabilistic consideration, the non-zero elements of $Q_{ik}(t)$ are:

$Q_{01}(t) = \int_0^t \propto e^{-(\alpha+\gamma)u}\, du = \frac{\alpha}{\alpha+\gamma}[1-e^{-(\alpha+\gamma)t}]$

$Q_{02}(t) = \int_0^t \gamma e^{-(\alpha+\gamma)u}\, du = \frac{\gamma}{\alpha+\gamma}[1-e^{-(\alpha+\gamma)t}]$

$Q_{12}(t) = \int_0^t \beta e^{-\beta u}\, du = [1-e^{-\beta t}]$

$$Q_{20}(t) = \int_0^t e^{-(\delta+\lambda)u} f(u)du$$

$Q_{23}(t) = \int_0^t \delta e^{-(\delta+\lambda)u}\, \bar{F}(u)du = \frac{\delta}{\delta+\lambda}[1-e^{-(\delta+\lambda)t}]$ - $\delta\int_0^t e^{-(\delta+\lambda)u}\, F(u)du$

$Q_{24}(t) = \int_0^t \lambda e^{-(\delta+\lambda)u}\, \bar{F}(u)du = \frac{\lambda}{\delta+\lambda}[1-e^{-(\delta+\lambda)t}]$ - $\lambda\int_0^t e^{-(\delta+\lambda)u}\, F(u)d\,F(u)du$

$Q_{32}(t) = \int_0^t g(u)du$ and $Q_{42}(t) = \int_0^t h(u)du$

(1.3.1-7)

The steady state transition p_{ij}can be obtain by taking limit as $t \to \infty$

i.e. $p_{ik} = \lim_{t\to\infty} Q_{ik}(t)$

(1.4)

Thus,

$p_{01} = \frac{\alpha}{\alpha+\gamma}$, $p_{02} = \frac{\gamma}{\alpha+\gamma}$, $p_{12} = 1$, $p_{20} = f^*(\delta+\lambda)$,

$p_{23} = \frac{\delta}{\delta+\lambda}[1-f^*(\delta+\lambda)]$

and $p_{24} = \frac{\lambda}{\delta+\lambda}[1-f^*(\delta+\lambda)]$, $p_{32} = 1$ *and* $p_{42} = 1$

(1.4.1-7)

The above probabilities establish the following relations:

$p_{01} + p_{02} = 1 = p_{12} = p_{13} = p_{42}$

$p_{20} + p_{23} + p_{24} = 1$

(1.5)

5. Mean Sojourn Times

The mean time taken by the system in a particular state S_i before transiting to any other state is known as mean sojourn time and is defined by

$\mu_i = \int_0^\infty P[T > tdt$

(2.1)

where T is time of stay in state S_i by the system.

To calculate mean sojourn time μ_i in state S_i, we assume that so long as the system is in state S_i, it will not transit to any other state. Therefore,

$\mu_0 = \int_0^\infty e^{-(\alpha+\gamma)t}\, dt\ \frac{1}{\alpha+\gamma}$, $\mu_1 = \int_0^\infty e^{-\beta t}\, dt\ \frac{1}{\beta}$, $\mu_2 = \int_0^\infty e^{-(\delta+\lambda)t}\, \bar{F}(t)dt\ \frac{1}{\delta+\lambda}[1-f^*(\delta+\lambda)]$

$\mu_3 = \int_0^\infty \bar{G}(t)dt = \int_0^\infty t.g(t)dt$ and $\mu_4 = \int_0^\infty \bar{H}(t)dt = \int_0^\infty t.h(t)dt$ (2.1.1-5)

For the contribution to mean sojourn time in state $S_i \in E$ and non – regenerative state occurs, before transiting to $S_j \in E$, i.e.

$m_{ij} = -\int t q_{ij}(t)dt = -q_{ij}'^*(0)$ (2.2)

Therefore,

$$m_{01} = \int_0^\infty \alpha.t.e^{-(\alpha+\gamma)t}dt = \frac{\alpha}{(\alpha+\gamma)^2}$$

$$m_{02} = \int_0^\infty \gamma.t.e^{-(\alpha+\gamma)t}dt = \frac{\gamma}{(\alpha+\gamma)^2}$$

$$m_{12} = \int_0^\infty \beta.t.e^{-\beta t}dt = \frac{1}{\beta}$$

$$m_{20} = \int_0^\infty t.e^{(\delta+\lambda)t}f(t)dt$$

$$m_{23} = \delta.\int_0^\infty t.e^{(\delta+\lambda)t}\bar{F}(t)dt$$

$$m_{24} = \lambda.\int_0^\infty t.e^{(\delta+\lambda)t}\bar{F}(t)dt$$

$$m_{32} = \int_0^\infty t.g(t)dt$$

$m_{42} = \int_0^\infty t.h(t)dt$ (2.2.1-8)

Hence,

$m_{01} + m_{02} = \frac{1}{\alpha+\gamma} = \mu_0$, $m_{12} = \frac{1}{\beta} = \mu_1$

$m_{20} + m_{23} + m_{24} = \frac{1}{\delta+\lambda}[1-f^*(\delta+\lambda) = \mu_2$

$m_{32} = \int_0^\infty t.g(t)dt = \mu_3$ and $m_{42} = \int_0^\infty t.h(t)dt = \mu 4$ (2.3.1-5)

6. Mean Time to System Failure (MISF)

The mean time to system failure (MTSF) can be obtained by E (T) given below by using Laplace Stieltjes transform of the relations for the distribution function $\pi_i(t)$of the time to system failure with starting time S_0

$E(T) = \frac{d}{ds}\pi_0(s)|_{s=0} = \frac{D_1'(0)-N_1'(0)}{D_1(0)}$ (3.1)

where $N_1 = \mu_0 + \mu_1 p_{01} + \mu_2$ (3.2)

and $D_1 = 1 - p_{20}$ (3.3)

7. Availability Analysis

System availability is defined as

A_i (t) = P_r [Starting from state S_i the system is available at epoch t without passing through any regenerative state]

M_i(t) = P_r [Starting from up state S_i the system remains up till epoch t without passing through any regenerative state]

Hence, obtaining A_i(t) by using elementary probability argument, we get

$$A_0(t) = M_0(t) + q_{01}© A_1(t) + q_{02}© A_2(t)$$
$$A_1(t) = M_1(t) + q_{12}© A_2(t)$$
$$A_2(t) = M_2(t) + q_{20}© A_0(t) + q_{23}(t)A_3(t) + q_{24}(t)A_4(t)$$
$$A_3(t) = q_{32}(t)©A_2(t)$$
$$A_4(t) = q_{42}(t)©A_2(t)$$

(4.1.1-5)

where $M_0(t) = e^{-(\alpha+\gamma)t}, M_1(t) = e^{-\beta t}$ and $M_2(t) = e^{-(\delta+\lambda)t}.\bar{F}(t)$

Taking Laplace transform of the equations (4.1.1-5) and solving for point wise availability by omitting the arguments „s" for brevity, the steady state functioning availability of the system, when the system starts operation from the state S_i, we get

$$A_0(\infty) = \lim_{t\to\infty} A_0(t) = \lim_{s\to 0} A_0^*(s) = \frac{N_2(0)}{D_0'(0)} = \frac{N_2}{D_2} \quad (4.2)$$

where $N_2 = (\mu_0 + \mu_1 p_{01})p_{20} + \mu_2$ and

$$D_2 = (\mu_0 + \mu_1 p_{01})p_{20} + \mu_2 + \mu_3 p_{23} + \mu_4 p_{42} \quad (4.3)$$

8. Busy Period Analysis

(a) Let $W_i(t)$ be the probability that the system is under repair by repair facility in the state $S_i \in E$at time t without transiting to any regenerative state. Therefore,

$W_2(t) = e^{-(\lambda+\delta)t}\bar{F}(t)$ and $W_4(t) = \bar{H}(t)$ (5.1)

Let $B_i(t)$ be the probability that the system is under repair at time t. We obtain the following recursive relations among $B_i(t)$"s:

$$B_0(t) = q_{01}(t) \copyright B_1(t) + q_{02}(t) \copyright B_2(t)$$
$$B_1(t) = q_{12}(t) \copyright B_2(t)$$
$$B_2(t) = W_2(t) + q_{20}(t) \copyright B_0(t) + q_{23}(t) \copyright B_3(t) + q_{24}(t) \copyright B_4(t)$$
$$B_3(t) = q_{32}(t) \copyright B_{11}(t)$$
$$B_4(t) = W_2(t) + q_{42}(t) \copyright B_2(t) \quad (5.1.1\text{-}5)$$

Taking Laplace transform of the equations (5.1.1-5) and solving the equations by omitting the argument for brevity we get the fraction of time for which the repair facility is busy in repair as

$$B_0(t) = \lim_{t\to\infty} B_0(t) = \lim_{s\to\infty} B_0^*(s) = N_3(0)/D_3'(0) = N_3/D_3 \quad (5.2)$$

where $N_3 = \mu_0 + p_{24}\mu_4$ and D_3 is same as D_2 in (4.3).

(b) In case if the system is under replacement in state $S_i \in E$at time to without transiting to any regenerative state, the fraction of time for which the repair facility is busy in replacement can be obtained as

$$R_0(t) = \lim_{t\to\infty} R_0(t) = \lim_{s\to\infty} R_0^*(s) = N_4(0)/D_4'(0) = N_4/D_4 \quad (5.3)$$

where $N_4 = \mu_3 p_{23}$ and D_4 is same as D_2 in (4.3).

9. Expected Number of Visits by Repair Facility

Let $V_i(t)$ be the expected number of visits by the repair facility in (0, t] given that the system initially started from regenerative state S_i at t = 0. The following recurrence relations among $V_i(t)$"s can be obtained as:

$$V_0(t) = Q_{01}(t)\$V_1(t) + Q_{02}(t)\$[1 + V_2(t)]$$
$$V_1(t) = Q_{12}(t)\$[1 + V_2(t)]$$
$$V_2(t) = Q_{20}(t)\$V_0(t) + Q_{23}(t)\$V_3(t) + Q_{24}(t)\$V_4(t)$$
$$V_3(t) = Q_{32}(t)\$V_2(t)$$
$$V_4(t) = Q_{42}(t)\$V_2(t)$$
(6.1.1-5)

Using Laplace Stieltjes transform of the above equations and omitting the argument „s" for brevity, we can get the number of visits per unit of time when the system starts after entrance into state S_0 as:

$$V_0 = \lim_{t\to\infty}[V_0(t)/t] = \lim_{s\to 0} s\,\tilde{V}_0(s) = N_5/D_5 \quad (6.2)$$

Where $N_5 = p_{20}$ and D_5 is same as in (4.3)

With the help of this study we concluded that the performance of the manufacturing system can be improved by improving the procedures, proper training of employees and proper maintenance of the system. The results derived in this paper are valuable in a study of improving the reliability of the systems and additionally they can be extensively used in many engineering disciplines.

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