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Analytical Study of Temperature Distribution of a Plane Wall Subjected to a Constant Heat Generation with Variable Thermal Conductivity Using Different Materials

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Abstract: ***Ananalytical study had been accomplished to find the effects of the variation of thermal conductivity on the temperature distribution of a plane wall using different materials. Plane wall exposed to a constant heat generation inside the material under study. A wide range of temperatures has been taken into consideration. Starting from 0C°to a high temperatureapproximately equal to the melting point temperature of the material under study. The patterns of the temperature distribution of plane wall have been computed using analytical solution by programming with MATLAB. The effect of heat generation was taken into consideration. The comparison between results was made for each system when using different types of materials to observe the pattern of temperature distribution.***

Keywords: Thermal conductivity, Temperature Distribution, Heat Generation, Temperature Coefficient of Conductivity

1. Introduction

The study and analyzing of temperature distribution is very important. The importance comes from that it is necessary to understanding the behavior of temperature and its variation with respect to the dimensions and properties of the specimen under study. Many engineering applications design depend upon the temperature distribution in its structure to explain the behavior of thermal properties with temperature variation. The Nuclear reactor is a direct and very important example for such study, because it is undergoing to a huge variety in temperature values that can effects on the value of thermal conductivity for the materials under study, in addition to that the existence of a source of heat generation with a constant value also play an important factor that can control the temperature distribution. Choi [1] studied the variation of thermal conductivity for a solid thin film, he used the two methods of non-equilibrium molecular dynamics, and solid argon had been selected as a specimen material. Thermal conductivity calculated from MD simulation and compared with an experimental data taken from the bulk status. The study proved that the temperature of the system and internal stresses affect thermal conductivity of the material under study. The results showed good agreement with experimental data was found. Rabin [2] investigated the effect of thermal conductivity of the temperature variation of the pure water ice. One dimensional temperature distribution presented for frozen water in this research. The results showthat when temperature increases, the thermal conductivity will decrease with a temperature overestimated by up to 38K. The results showed also that when the temperature dependency of the thermal conductivity is neglected, the heat fluxes through frozen boundaries are underestimated by a factor of 2.

Rahman et al. [3]analyzed and investigated the effects of temperature dependent thermal conductivity along a vertical flat plate with Magneto-hydrodynamic (MHD) natural convection. The boundary conditions applied for governing equations which are converted to the dimensionless form and solved using implicit finite difference and Keller-box scheme. The effects of many parameters are taken into consideration like velocity, skin friction coefficient, and surface temperature. Mottaghy et al.[4]investigated temperature distribution on both thermal diffusivity and conductivity. The study had been accomplished to the crystalline rock samples and the data had been taken from the Kola Peninsula and Eastern Alps. The study results from a general relationship for thermal diffusivity with respect to temperature up to 300 C°.For thermal conductivity the temperature taken as an ambient temperature. The comparison with the other references had been accomplished and showed a good agreement when the conditions taken at ambient temperature. Gonnet [5]measured in his thesis the thermal behavior of SWNT-reinforced composites, thermal conductivity had been measured by using a comparative method and by using constantan foil as a reference. The range of temperature that used in this work started from 115K to room temperature. He used a thermo-mechanical analyzer to find the coefficient of thermal expansion. STAICU [6] studied the effects of the principle of variation in thermal conductivity through a flat plate with a state of quasi- stationary regime of thermal conductivity. The study had been treated with a different regime during a quasi-stationary of heating. The results of this study presented relationships of a flat plate under quasi-stationary heating. Khan el al. [7] studied the temperature distributions of a hollow sphere, the material under study with variable thermal conductivity and under heat generation, analytical formulation are developed for a steady state condition using Homotopy Perturbation Method (HPM). The study showed that the temperature distribution is strongly dependent on thermal conductivity. The results had been compared with the Kirchhoff Transformation results.

2. Objective of Research

An analytical study has been accomplished to investigate deeply the effects of thermal conductivity of different materials when these materials exposed to a change in temperature. Copper, Aluminum, iron have been used as material samples with a distinguished model which is a plane wall. Different boundary conditions have been applied in this study to explain their effect on the temperature distribution. The temperature ranges begin from 0C$^\circ$ to a high-level temperature approximately the melting point temperature for each material that used in this research work.

3. Governing Equations

The energy equation for this research is a most important equation that taken to begin the procedures for analyzing the behavior of thermal conductivity with temperature for selected materials. Thermal conductivity assumed that there is no change in its value except with temperature difference. Heat generation in the plane wall also added to studied its effect on the pattern of temperature distribution and different materials that were taken into consideration.

$$q_{in} + q_{gen} = q_{out} + q_{st} \qquad (1)$$

Where [6]:

q_{in} *is heat enter the right side of the plane wall*
q_{gen} *is heat generated in the plane wall*
q_{out} *is heat leaving from left side of the plane wall*
q_{st} *is heat stored in the plane wall*

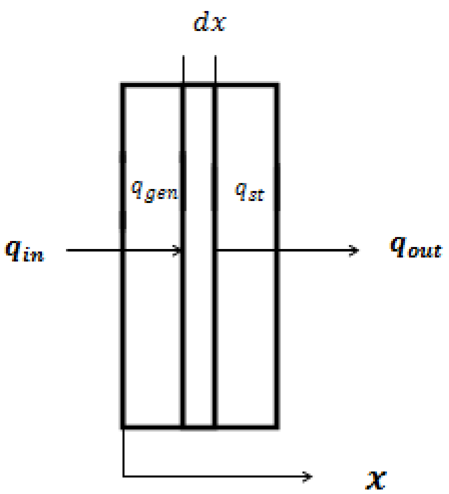

Figure 1: Energy balance of a plane wall

The heat conducting to the plane wall can be represented as:

$$q_{in} = -KA\frac{dT}{dx} \qquad (2)$$

The heat leaving the plane wall is:

$$q_{out} = q_{x+dx} = -\left[KA\frac{dT}{dx} + \frac{\partial}{\partial x}\left(KA\frac{dT}{dx}\right)dx\right] \qquad (3)$$

The heat generation in the plane wall is:

$$q_{gen} = \dot{q}Adx \qquad (4)$$

Last but not least the heat stored in the plane wall is:

$$q_{st} = \rho cA\frac{\partial T}{\partial t}dt \qquad (5)$$

In this research, the heat stored assumed equal to zero, which means there is no change in temperature with respect to time (steady state condition).

Where K in the above equations represented the thermal conductivity of selected material. A is a cross-sectional area for the plane wall.

The final form of the heat energy balance of the plane wall is:

$$\frac{\partial}{\partial x}\left(K\frac{dT}{dx}\right) + \dot{q} = 0 \qquad (6)$$

In order to solve equation (6), a number of assumptions should be used to solve equation (6). The assumptions that concerned to this research are:

- Steady state heat transfer conduction.
- Thermal conductivity is taken ones constant and another one is variable for all models under study.

The general solution for equation (6) is controlled by boundary conditions. The boundary conditions for plane wall had been taken for two types.

1) In the case of constant thermal conductivity:

a) For a plane wall havin**g the same temperature surface at both sides as shown in figure (2).**

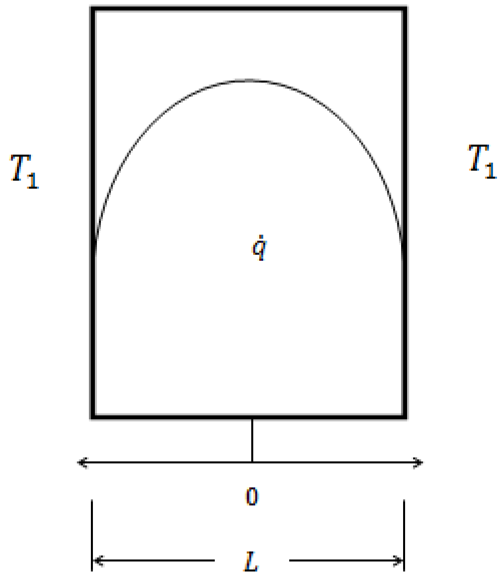

Figure 2: Temperature distribution of a plane wall with same temperatures surface at both sides

Boundary conditions for the figure (2) are:

$AT\ x = 0, \quad \frac{dT}{dx} = 0$

$at\ x = \pm{}^{L}/_{2}, T = T_1$

The general solution for equation (6) is:

$$T(x) = T_1 + \frac{\dot{q}Lx}{2k}\left(1 - \frac{x}{L}\right) \qquad (7)$$

Where Lrepresented thickness of the plane wall

b) For plane wall having different temperature values at both sides as shown in figure (3).

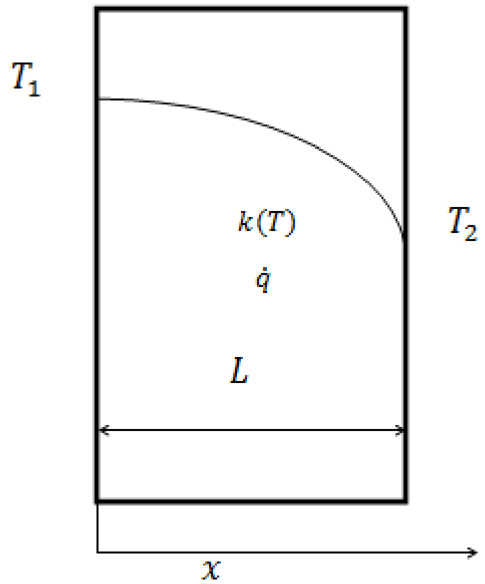

Figure 3: Temperature distribution of a plane wall with different temperatures surface at both sides

Boundary conditions for the figure (3) are:
$At\ x = 0, T = T_1$, $At\ x = L, T = T_2$

The general solution of equation (6) is:

$$T(x) = T_1 - \frac{\dot{q}x^2}{2k} + \left(\frac{T_1 - T_2}{L} + \frac{\dot{q}L}{2k}\right)x \quad (8)$$

Back to the equation (6) and by changing the boundary conditions to satisfy the second part of research, which is the thermal conductivity varies with temperature for the materials that is under study, thermal conductivity varies with temperature depending on physical structure of material, so thermal conductivity can be evaluated from Fourier's law according to[9,10]:

$$k_x = -\frac{q_x''}{\left(\partial T/\partial x\right)} \quad (9)$$

Where q_x'' represents heat per unit area transferred through the material.

Equation (9) is especiallyto compute the thermal conductivity along x axis, in the same, way the value of $k_y$and k_z can be computed in case of an anisotropic material. Accepted approximation for thermal conductivity which is based on that the thermal conductivity of each material varies with a linear formula depending on the temperature variation, and according to the following expression:

$$K(T) = k_0(1 + \beta T) \quad (10)$$

Where$K(T)$is thermal conductivity at a certain temperature. k_0 is thermal conductivity at a reference temperature, and β represented temperature coefficient of thermal conductivity where each material has a specific value. The value of thermal conductivity from equation (10) can be used for general equation of temperature distributionof a plane wall.

$$\frac{\partial}{\partial x}\left(k_0(1 + \beta T)\frac{dT}{dx}\right) + \dot{q} = 0 \quad (11)$$

The temperature distribution in this case depends on the boundary conditions for both sides of a plane wall, taking the first boundary conditions similar to that in figure (2). The general solution of equation (11) is:

$$T(x) = -\frac{1}{\beta} \pm \sqrt{\frac{1}{\beta^2} + \frac{\dot{q}xL}{k_0\beta}\left(1 - \frac{x}{L}\right) + T_1^2 + \frac{2}{\beta}T_1} \quad (12)$$

The second boundary conditions are taken similar to the boundary conditions in figure (3). The general solution of equation (14) in this case is:

$$T(x) = -\frac{1}{\beta} \pm \sqrt{\frac{1}{\beta^2} + \frac{\dot{q}xL}{k_0\beta}\left(1 - \frac{x}{L}\right) + \frac{x}{L}(T_2^2 - T_1^2) + \frac{2x}{\beta L}(T_2 - T_1) + T_1^2 + \frac{2}{\beta}T_1} \quad (13)$$

The derivation of equations (12) and (13) are shown in appendix-A.

4. Results and Discussion

The effects of variation of thermal conductivity with temperature distribution on a plane wall show in the figures below, a wide range of temperature had been taken into considerations to find a suitable behavior for a different materials, the reference temperature is taken as $0C^\circ$ and all thermal conductivity reference had been taken at this temperature. In the table (1), all the information that had been used in this research, which is thermal conductivity, the temperature coefficient of thermal conductivity and the first assumptions of the temperature of aplane wall.

Table 1: Information about models under study

Material	k_0 W/m.C°	β	Plane wall /Temperature C°	
			Symmetric	asymmetric
Copper	385	$-1.93 * 10^{-4}$	60	40 to 60
Aluminum	210	$6.752 * 10^{-4}$		
Iron	80	$-3.59 * 10^{-4}$		

The result figures show the temperature distribution in the case of a constant thermal conductivity and variable thermal conductivity to show the difference between each other and to clarification the effects of change in thermal conductivity to the temperature distribution in a selected material.

Beginning with plane wall that having the same temperature for both sidesas in figure (2), the pattern of the selected materials is strongly identical and shows the same behavior, which is the maximum temperature is at the mid-point of the wall owing to the values of heat generation in the wall, figures (5) and (6) show that explanation which is for constant and variable thermal conductivity respectively, the difference between two figure is very small which is coming from that the effects of variation of thermal conductivity to the temperature distribution becomean evident in a high surface temperature. Stay in figures (5) and (6), because of low thermal conductivity related with iron in comparison with copper and aluminum, the behavior of temperature distribution curvature of this material is more than that the two other materials, this gives the iron a maximum temperature in the center in comparison with two other materials, which is 91.25C° and 92.1273C° for constant and variable thermal conductivity respectively.

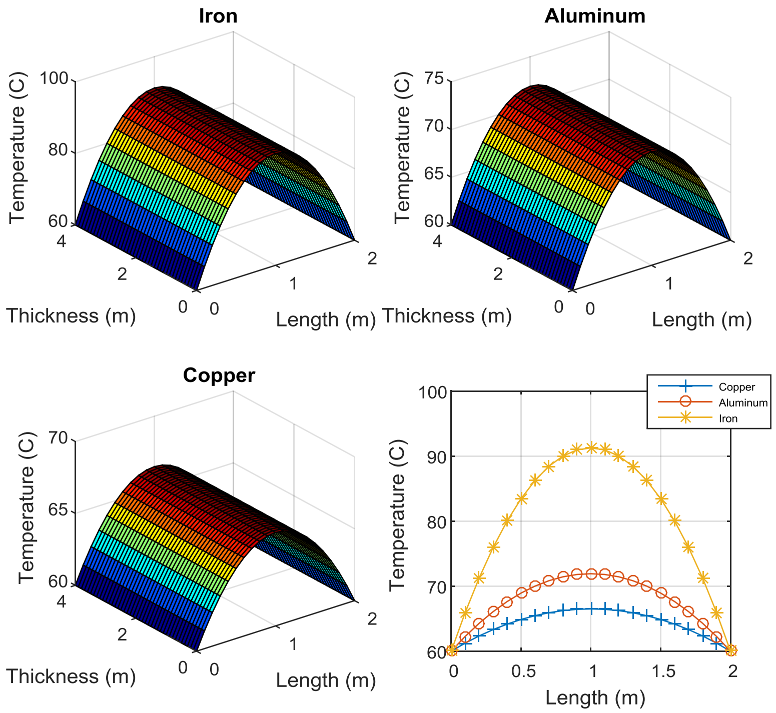

Figure 5: Temperature distribution of a plane wall with constant thermal conductivity (equal temperatures on both sides of plane wall)

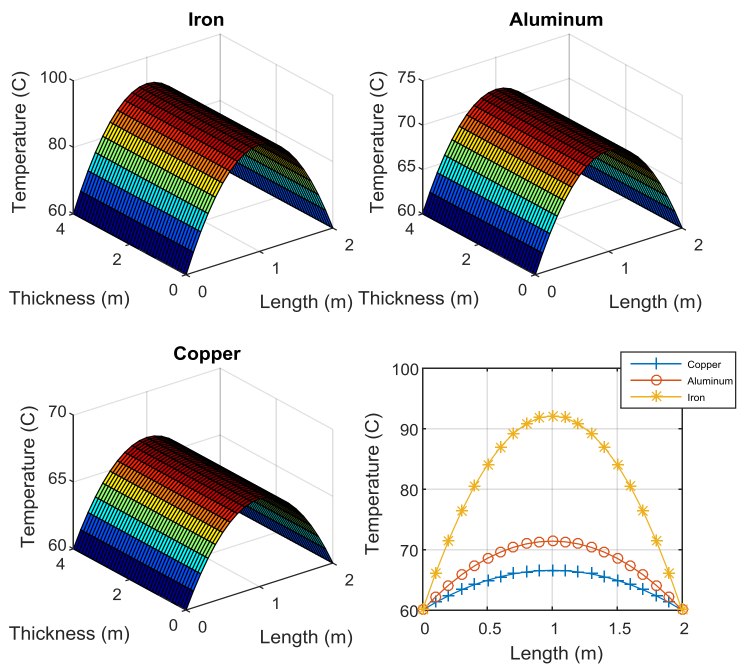

Figure 6: Temperature distribution of a plane wall with variable thermal conductivity (equal temperatures on both sides of plane wall)

Figure (7) review a comparison between temperature distribution of constant and variable thermal conductivity for all three materials that had been selected for this study, the difference as shown un figure is very small, this difference shows in Iron more than other materials due to small thermal conductivity compared with Aluminum and Copper, which is cause to heat transfers slow than that in case of transferring heat in Aluminum and Copper.This difference becomes importance in high-temperature surface for the plane wall, this difference shows in figure (8)as an explanation in case the surface temperature of the plane wall is 200C˚.

From figures (9) and (10) which are explanation about figure (3) in the case of the temperature on both sides of the plane wall are not equal, the behavior of temperature distribution in Copper is very smooth compared to Aluminum and Iron, this is the resultant of high conductivity of Copper which is makes the heat transfers through the material faster than that in other materials. For Iron due to slightly low conductivity, the temperature through the plane wall greater than of the other material's walls. Like the temperature distribution in a plane wall having the same temperatures on both sides, the difference between temperature distribution with constantconductivity and variable conductivity did not appear Significantly, this difference is clear for a high temperature. Figure (11) shows the difference between constant and variable thermal conductivity in case that the plane wall having different surface temperatures on both sides.

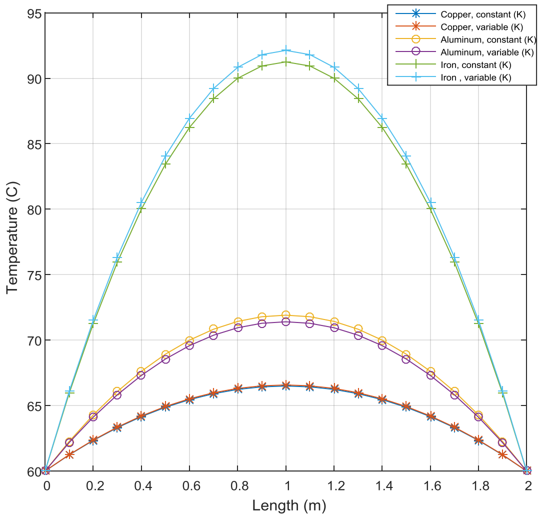

Figure 7: Comparison between temperature distribution for constant and variable thermal conductivity with different materials for plane wall having the same temperatures surfaces

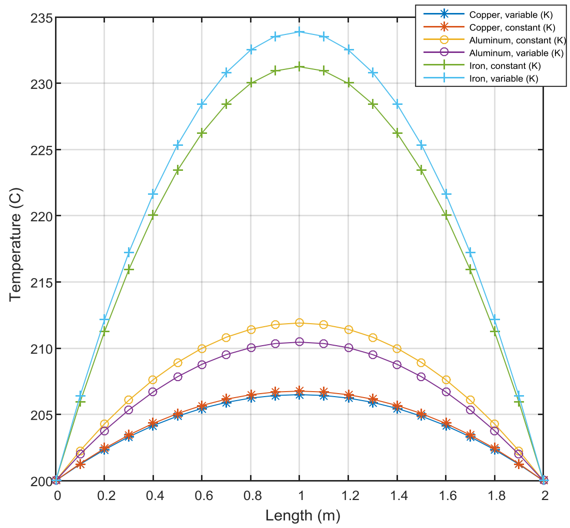

Figure 8: Effects of higher temperature to the temperature distribution for constant and variable thermal conductivity with different materials

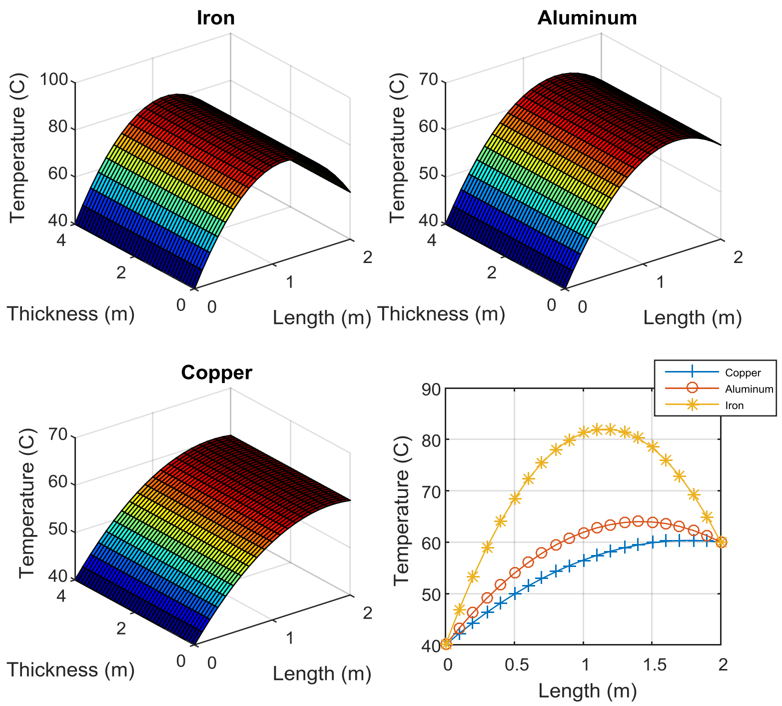

Figure 9: Temperature distribution of a plane wall with constant thermal conductivity (not equal temperatures on both sides of plane wall)

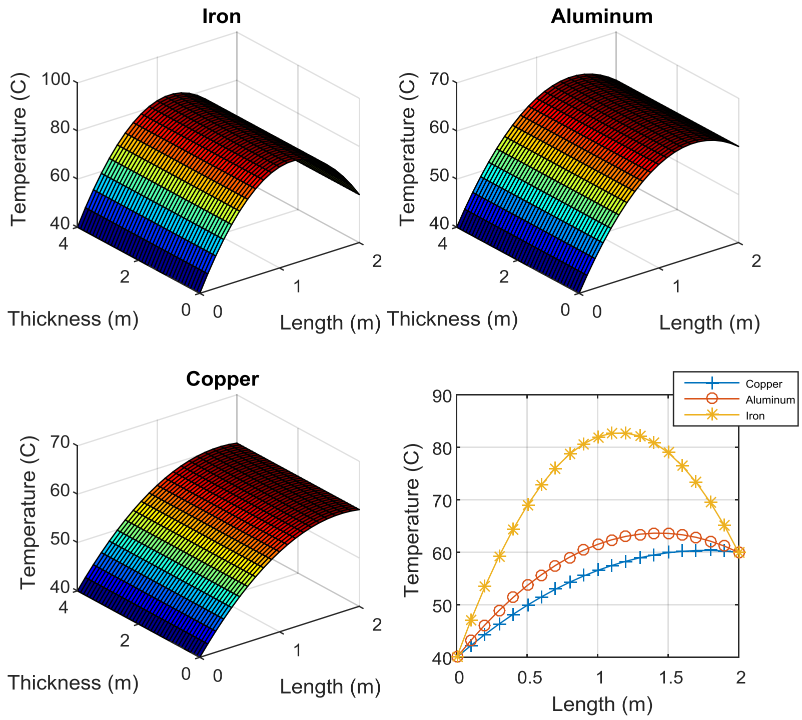

Figure 10: Temperature distribution of a plane wall with variable thermal conductivity (not equal temperatures on both sides of plane wall)

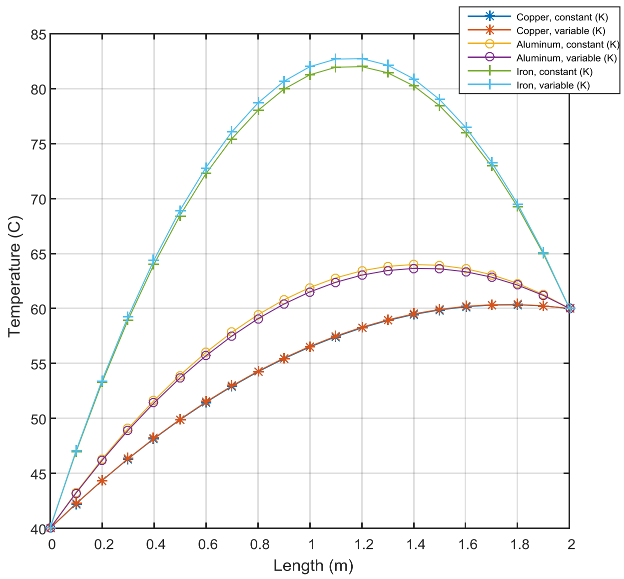

Figure 11: Comparison between temperature distribution for constant and variable thermal conductivity with different materials of a plane wall having different temperature surfaces

5. Conclusion

This study for difference materials leads to several conclusions that can be mentioned

1) The effects of variation of thermal conductivity for the materials did not appear clearly unless in the high temperature that deals with it.
2) The pattern of temperature distribution in the case of variable thermal conductivity is the same pattern as the temperature distribution in the case of constant thermal conductivity, the only difference between two types is the values of temperature.
3) For any shape under study, Iron having the highest value of temperature inside in case of heat generation inside the shape, this is due to the minimum thermal conductivity of Iron compared with the other materials under study.
4) For a plane wall, temperature distribution having a linear pattern if that the thickness of the plane wall is small (less than 0.4m), begin to take the curvatures pattern above this thickness.

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Appendix –A

a- the derivation of the equation of temperature distribution for a plane wall with a variable thermal conductivity and heat generation:

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\dot{q}=0 \;\rightarrow \frac{\partial}{\partial r}\left(k_0(1+\beta T)\frac{\partial T}{\partial x}\right)=-\dot{q}$$

$$k_0(1+\beta T)\frac{\partial T}{\partial x}=-\dot{q}x+C_1 \rightarrow \; k_0\partial T+k_0\beta T\partial T$$
$$=-\dot{q}x\partial x+C_1\partial x$$

$$k_0T+\frac{k_0\beta}{2}T^2=-\frac{\dot{q}x^2}{2}+C_1\mathrm{x}+C_2 \qquad (a1)$$

Boundary Conditions:

$Atx=0, \quad T=T_1(a2) \quad At\;x=L, \quad T=T_1 \quad (a3)$

$$\therefore C_1=\frac{\dot{q}L}{2}$$

$$C_2=\frac{k_0\beta}{2}\left(T_1^2+\frac{2}{\beta}T_1\right)$$

$$\therefore\; T^2+\frac{2}{\beta}T=\frac{2}{k_0\beta}\left(-\frac{\dot{q}x^2}{2}+C_1x+C_2\right) \qquad (a4)$$

By solving equation (a4) and substituting the values of C_1 *and* C_2. The equation of temperature distribution is:

$$T(x)=-\frac{1}{\beta}\pm\sqrt{\frac{1}{\beta^2}+\frac{2}{\beta}T_1+T_1^2+\frac{\dot{q}xL}{k_0\beta}\left(1-\frac{x}{L}\right)} \qquad (a5)$$

This is temperature distribution for a plane wall having the same value of temperature at both sides, with a material of variable thermal conductivity and with heat generation.

In the case of there is a different temperature on both sides of the plane wall, the boundary conditions will change, the temperature distribution is the same as equation $(a1)$.

$$k_0T+\frac{k_0\beta}{2}T^2=-\frac{\dot{q}x^2}{2}+C_1\mathrm{x}+C_2 \qquad (a1)$$

Boundary conditions:

$At\;x=0\,,\;\; T=T_1(a6)\,, \qquad At\;x=L, T=T_2 \quad (a7)$

The solution of equation $(a1)$ will be:

$$T(x)=-\frac{1}{\beta}\pm\frac{1}{2}\sqrt{\frac{4}{\beta^2}-4\left(\frac{\dot{q}x^2-2C_1-2C_2}{k_0\beta}\right)} \quad (a8)$$

Substitute $(a6)$ in $(a8)$

$$C_2=\frac{k_0\beta}{2}\left(T_1^2+\frac{2}{\beta}T_1\right) \qquad (a9)$$

Substitute $(a7) and\;(a9)$ in $(a8)$:

$$C_1=\frac{k_0\beta}{2L}(T_2^2-T_1^2)+\frac{k_0}{L}(T_2-T_1)+\frac{\dot{q}L}{2} \qquad (a10)$$

$$\therefore T(x)$$
$$=-\frac{1}{\beta}$$
$$\pm\sqrt{\frac{1}{\beta^2}+\frac{\dot{q}xL}{k_0\beta}\left(1-\frac{x}{L}\right)+\frac{x}{L}(T_2^2-T_1^2)+\frac{2x}{\beta L}(T_2-T_1)+T_1^2+\frac{2}{\beta}T_1}$$
$$(a11)$$

Equation (a11) is temperature distribution with heat generation and thermal conductivity is variable of a plane wall with different temperature at both sides