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Estimation of Mean Time to Recruitment for a Two Graded Manpower System Involving Independent and Non-Identically Distributed Random Variables with Thresholds having SCBZ Property

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Abstract: ***In this paper, an organization subjected to a random exit of personnel due to policy decisions taken by the organization is considered; there is an associated loss of manpower if a person quits the organization. As the exit of personnel is unpredictable, a recruitment policy involving two thresholds, optional and mandatory is suggested to enable the organization to plan its decision on recruitment. Based on shock model approach, a mathematical model is constructed using an appropriate univariate policy of recruitment. The analytical expression for the mean time to recruitment is obtained when i) the loss of manpower forms a sequence of independent and non-identically distributed continuous random variables ii) inter-decision times are independent and non-identically distributed continuous random variables and iii) the optional and mandatory thresholds having SCBZ property.***

Keywords: Manpower planning, Shock models, Univariate recruitment policy, SCBZ property, Hypo-Exponential distribution

1. Introduction

Exits of personnel which in other words known as wastage is an important aspect in the Manpower planning. Many models have been discussed using different types of distributions. Such models could be seen in [1] [2] [3] and [4].In [5] expected time to recruitment is obtained when inter-decision times are independent and identically distributed random variables. The expected time to distribution has SCBZ property [8].The results of [7] are extended in [6] when the inter-decision times are recruitment is obtained in [7] when the threshold exchangeable constantly correlated exponential random variables. In [9] the mean time to recruitment is obtained when the survival time process is geometric and the threshold distribution has SCBZ property. In all the above cited works, the problem of time to recruitment in a single graded organization involves only one threshold value. Since the number of exits in a policy decision making epoch is unpredictable and the time at which the cumulative loss of man hours crossing a single threshold is probabilistic, the organization has left with no choice except making recruitment immediately upon the threshold crossing. In [10] this limitation is removed by considering the following new recruitment policy involving two thresholds optional and mandatory. If the cumulative loss of manpower crosses the optional threshold, the organization may or may not go for recruitment. However, recruitment is necessary whenever the cumulative loss of manpower crosses the mandatory threshold. In view of this policy, the organization can plan its decision upon the time for recruitment. In [11] the mean time to recruitment is obtained when the optional and mandatory thresholds distribution has may not be permitted. Most of these authors have used univariate CUM policy of recruitment by which recruitment is done whenever the cumulative loss of manpower crosses a threshold. In [12] the author has obtained the performance measures namely mean and variance of the time to recruitment for a two graded system when the loss of manpower, the threshold for the loss of manpower in each grade, the inter-decision times are independent and identically distributed exponential random variables forming the same renewal process for both grades and the threshold for the organization is the max (min) of the thresholds for the two grades using univariate CUM policy. In [13] the authors have studied the maximum model discussed in [12] when both thresholds have SCBZ property. In [14],[15],[16] and [17] the authors have extended the results in [10] for a two graded system according as the thresholds are exponential random variables or extended exponential random variables or SCBZ property possessing random variables or geometric random variables. In [18] the authors have extended the results in [10] for a two grade system according as the optional thresholds are exponential random variable and the distributions of the mandatory thresholds have SCBZ property. In [14] the authors have also studied this work in [19] by considering optional and mandatory thresholds for the loss of manpower in the two grades. In [20] the authors have extended the work of [15] and [18] when the loss of manpower in the organization is the maximum of the loss of manpower in the two grades by assuming exponential, extended exponential and SCBZ property. The objective of the present paper is to study the problem of time to recruitment for a two graded manpower system and to obtain mean time for recruitment using CUM univariate recruitment policy for different cases of the threshold distributions by assuming that when the loss of manpower and inter-decision times form separately a sequence of independent and non-identically distributed exponential random variables with thresholds optional and mandatory having SCBZ property.

2. Model Description and Assumptions

1) An organization having two grades in which decisions are taken at random epochs in $(0, \infty)$ and at every decision making epoch a random number of persons quit the organization. There is an associated loss of man hours to the organization if a person quits.
2) It is assumed that the loss of man hours is linear and cumulative.
3) The loss of manpower at any decision epoch forms a sequence of independent and non- identically distributed random variables.
4) The inter-decision times are independent and non-identically distributed random variables.
5) The loss of manpower process and the process of inter-decision times are statistically independent.
6) There is an optional and mandatory threshold for the loss of manpower in the organization.
7) The loss of manpower at any decision epoch, the optional and mandatory threshold levels are statistically independent.
8) The optional threshold level is less than the mandatory threshold level.
9) **Uivariate CUM recruitment policy**: Recruitment is done whenever the cumulative loss of manpower crosses the mandatory threshold. The organization may or may not go for recruitment if the cumulative loss of manpower crosses the optional threshold.

3. Notations

X_i : The loss of manpowers due to the i^{th} decision epoch i=1,2,3……forming a sequence of independent and non-identically distributed exponential random variables with parameters α_i (α_i>0).

$G_i(.)$: The Distribution function of X_i

$g_i(.)$: The probability density function of X_i with mean $\frac{1}{\alpha_i}$ (α_i >0)

S_k : Cumulative loss of manpower in the first k-decisions (k=1,2,3….)

$$S_k = \sum_{i=0}^{k} X_i$$

$G_k(.)$:The Distribution function of sum of k independent and non-identically distributed exponential random variables.

$g_k(.)$: The probability density function of S_k

$$G_k(t) = \sum_{i=1}^{k} c_i(1- e^{-\alpha_i t}),\ g_k(t) = \sum_{i=1}^{k} c_i \alpha_i e^{-\alpha_i t}$$

$$g_k^*(s) = \sum_{i=1}^{k} c_i \frac{\alpha_i}{\alpha_i+s},\ \text{Where } c_i= \prod_{\substack{j=1\\ j\neq i}}^{k} \frac{\alpha_j}{\alpha_j-\alpha_i}, i=1,2,3\ldots k$$

U_i : The inter-decision times are independent and non-identically distributed exponential random variables between (i-1) and i^{th} decisions with parameters β_i (β_i>0)

$F_i(.)$: The Distribution function of U_i

$f_i(.)$: The probability density function of U_i with mean $\frac{1}{\beta_i}$ (β_i >0)

R_k : The waiting time upto k decisions

$F_k(.)$: The Distribution function of R_k

$f_k(.)$: The probability density function of R_k

$$F_k(t) = \sum_{i=1}^{k} b_i(1- e^{-\beta_i t}),\ f_k(t) = \sum_{i=i}^{k} b_i\beta_i e^{-\beta_i t},$$

$$f_k^*(s) = \sum_{i=1}^{k} b_i \frac{\beta_i}{\beta_i+s}\ \text{Where } b_i= \prod_{\substack{j=1\\ j\neq i}}^{k} \frac{\beta_j}{\beta_j-\beta_i}, i=1,2,3\ldots k$$

Y_1,Y_2: The continuous random variables denoting the optional thresholds levels for the grade 1 and grade 2 follows exponential distribution with parameters λ_1 & λ_2 respectively.

Z_1,Z_2 : The continuous random variables denoting the mandatory thresholds levels for the grade 1 and grade 2 follows exponential distribution with parameters μ_1 & μ_2 respectively.

It is assumed that $Y_1<Z_1$ and $Y_2<Z_2$.

$H_1(.)$:Distribution function of Y_1

$H_2(.)$:Distribution function of Y_2

$H_3(.)$:Distribution function of Z_1

$H_4(.)$:Distribution function of Z_2

W: The continuous random variable denoting the time to recruitment in the organization.

P: The probability that the organization is not going for recruitment whenever the total loss of manpower crosses the optional threshold Y

$V_k(t)$: The probability that exactly k decisions are taken in [0,t)

L(.) : Distribution function of W

l(.) : The probability density function of W

$l^*(.)$: The laplace transform of l(.)

E(W) :The expected time to recruitment

4. Main Results

The distribution H_1 has the SCBZ property from Walker(1968),H_1 (y) is a distribution ofY_1 have SCBZ property with parameter(θ_1,θ_2,λ_1).

$$H_1(y) = \begin{cases} 1-e^{-\theta_1 y}, y<\tau_0 \\ 1-e^{-\theta_1\tau_0}e^{-\theta_2(y-\tau_0)}, \tau_0<y \end{cases} \quad (1)$$

For some fixed $\tau = \tau_0$

Assume that the truncation level τ_0 itself a randam variable such that τ follows exponential distribution with parameter λ_1 for grade 1 and λ_2 for grade 2 respectively.

By the law of total probability and using (1)

$H_1(y) = P(Y_1 \le y) = \int_0^\infty P(Y_1 \le y/\tau{=}x) f_\tau(x)dx$ Where $f_\tau(x) = \lambda_1 e^{-\lambda_1 x}$

$$= 1 - e^{-(\theta_1+\lambda_1)y}\left[\frac{\theta_1-\theta_2}{\theta_1-\theta_2+\lambda_1}\right] - \frac{\lambda_1}{\theta_1-\theta_2+\lambda_1} e^{-\theta_2 y}$$

$$H_1(y) = 1- p_1 e^{-(\theta_1+\lambda_1)y} - q_1 e^{-\theta_2 y} \quad (2)$$

Where $p_1 = \frac{\theta_1-\theta_2}{\theta_1-\theta_2+\lambda_1}$ $q_1 = \frac{\lambda_1}{\theta_1-\theta_2+\lambda_1} = 1-p_1, p_1+q_1 = 1$

Similarly $H_2(y)$ is a distribution function of Y_2 have SCBZ property with parameter $(\theta_3, \theta_4, \lambda_2)$.

$$H_2(y) = 1- p_2 e^{-(\theta_3+\lambda_2)y} - q_2 e^{-\theta_4 y} \quad (3)$$

Where $p_2 = \frac{\theta_3-\theta_4}{\theta_3-\theta_4+\lambda_2}$ $q_2 = \frac{\lambda_2}{\theta_3-\theta_4+\lambda_2} = 1-p_2, p_2+q_2 = 1$

$H_3(z)$ is a distribution function of Z_1 have SCBZ property with parameter $(\theta_5, \theta_6, \mu_1)$

$$H_3(z) = 1 - p_3 e^{-(\theta_5+\mu_1)z} - q_3 e^{-\theta_6 z} \quad (4)$$

Where $p_3 = \frac{\theta_5-\theta_6}{\theta_5-\theta_6+\mu_1}$, $q_3 = \frac{\mu_1}{\theta_5-\theta_6+\mu_1} = 1 - p_3$, $p_3 + q_3 = 1$ $H_4(z)$ is a distribution function of Z_2 have SCBZ property with parameter $(\theta_7, \theta_8, \mu_2)$.

$$H_4(z) = 1 - p_4 e^{-(\theta_7+\mu_2)z} - q_4 e^{-\theta_8 z} \quad (5)$$

Where $p_4 = \frac{\theta_7-\theta_8}{\theta_7-\theta_8+\mu_2}$ $q_4 = \frac{\mu_2}{\theta_7-\theta_8+\mu_2} = 1 - p_4$, $p_4 + q_4 = 1$

The survival function of W is given by

$P(W>t) = \sum_{k=o}^{\infty}$ [Probability that exactly k-decisions are taken in [0,t),k=0,1,2…][Probability that the total number of exits in these k-decisions does not cross the optional level Y or the total number of exits in these k-decisions crosses the optional level Y but lies below the mandatory level Z and the organization is not making recruitment]

$$P(W > t) = \sum_{k=0}^{\infty} V_k(t) P(S_k < Y) + \sum_{k=0}^{\infty} V_k(t) P(S_k \geq Y) P(S_k < Z) p \quad (6)$$

Case(i):

For maximum case, consider $P(S_k < Y)$, conditioning upon S_k and using the law of total probability,

We get

$$P(S_k<Y) = \int_0^\infty P(S_k < Y / S_k = x) g_k(x)dx$$
$$= \int_0^\infty (1 - H(x))\, g_k(x)dx$$
$$= \int_0^\infty [1 - H_1(x)\, H_2(x)\,] g_k(x)dx$$
$$= p_1 \int_0^\infty e^{-(\theta_1+\lambda_1)x} g_k(x)dx + p_2 \int_0^\infty e^{-(\theta_3+\lambda_2)x} g_k(x)dx$$
$$+ q_1 \int_0^\infty e^{-\theta_2 x} g_k(x)dx + q_2 \int_0^\infty e^{-\theta_4 x} g_k(x)dx$$
$$- p_1 p_2 \int_0^\infty e^{-(\theta_1+\theta_3+\lambda_1+\lambda_2)x} g_k(x)dx$$
$$- p_1 q_2 \int_0^\infty e^{-(\theta_1+\theta_4+\lambda_1)x} g_k(x)dx(x)dx$$
$$- q_1 p_2 \int_0^\infty e^{-(\theta_2+\theta_3+\lambda_2)x} g_k$$
$$- q_1 q_2 \int_0^\infty e^{-(\theta_2+\theta_4)x} g_k(x)dx$$

$$P(S_k<Y) = p_1 g_k^*(\theta_1 + \lambda_1) + p_2 g_k^*(\theta_3 + \lambda_2) + q_1 g_k^*(\theta_2) + q_2 g_k^*(\theta_4) - p_1 p_2 g_k^*(\theta_1 + \theta_2 + \lambda_1 + \lambda_2) - p_1 q_2 g_k^*(\theta_1 + \theta_4 + \lambda_1) - q_1 p_2 g_k^*(\theta_2 + \theta_3 + \lambda_2) - q_1 q_2 g_k^*(\theta_2 + \theta_4) \quad (7)$$

$$P(S_k < Y) = p_1 N_1 + p_2 N_2 + q_1 N_3 + q_2 N_4 - p_1 p_2 N_5 - p_1 q_2 N_6 - q_1 p_2 N_7 - q_1 q_2 N_8 \quad (8)$$

Where $N_1 = g_k^*(\theta_1 + \lambda_1)$, $N_2 = g_k^*(\theta_3 + \lambda_2)$, $N_3 = g_k^*(\theta_2)$, $N_4 = g_k^*(\theta_4)$, $N_5 = g_k^*(\theta_1 + \theta_3 + \lambda_1 + \lambda_2)$, $N_6 = g_k^*(\theta_1 + \theta_4 + \lambda_1)$, $N_7 = g_k^*(\theta_2 + \theta_3 + \lambda_2)$, $N_8 = g_k^*(\theta_2 + \theta_4)$

Similarly

$$P(S_k<Z) = p_3 \int_0^\infty e^{-(\theta_5+\mu_1)x} g_k(x)dx$$
$$+ p_4 \int_0^\infty e^{-(\theta_7+\mu_2)x} g_k(x)dx$$
$$+ q_3 \int_0^\infty e^{-\theta_6 x} g_k(x)dx$$
$$+ q_4 \int_0^\infty e^{-\theta_8 x} g_k(x)dx$$
$$- p_3 p_4 \int_0^\infty e^{-(\theta_5+\theta_7+\mu_1+\mu_2)x} g_k(x)dx$$
$$- p_3 q_4 \int_0^\infty e^{-(\theta_5+\theta_8+\mu_1)x} g_k(x)dx$$
$$- q_3 p_4 \int_0^\infty e^{-(\theta_6+\theta_7+\mu_2)x} g_k(x)dx$$
$$- q_3 q_4 \int_0^\infty e^{-(\theta_6+\theta_8)x} g_k(x)dx$$

$$P(S_k < Z) = p_3 g_k^*(\theta_5 + \mu_1) + p_4 g_k^*(\theta_7 + \mu_2) + q_3 g_k^*(\theta_6) + q_4 g_k^*(\theta_8) - p_3 p_4 g_k^*(\theta_5 + \theta_7 + \mu_1 + \mu_2) - p_3 q_4 g_k^*(\theta_5 + \theta_8 + \mu_1) - q_3 p_4 g_k^*(\theta_6 + \theta_7 + \mu_2) - q_3 q_4 g_k^*(\theta_6 + \theta_8) \quad (9)$$

$$P(S_k < Z) = p_3 N_9 + p_4 N_{10} + q_3 N_{11} + q_4 N_{12} - p_3 p_4 N_{13} - p_3 q_4 N_{14} - q_3 p_4 N_{15} - q_3 q_4 N_{16} \quad (10)$$

Where $N_9 = g_k^*(\theta_5 + \mu_1)$, $N_{10} = g_k^*(\theta_7 + \mu_2)$, $N_{11} = g_k^*(\theta_6)$, $N_{12} = g_k^*(\theta_8)$, $N_{13} = g_k^*(\theta_5 + \theta_7 + \mu_1 + \mu_2)$, $N_{14} = g_k^*(\theta_5 + \theta_8 + \mu_1)$, $N_{15} = g_k^*(\theta_6 + \theta_7 + \mu_2)$, $N_{16} = g_k^*(\theta_6 + \theta_8)$

Substituting (8) & (10) in (7) ,we get

$$P(W>t) = \sum_{k=0}^{\infty} V_k(t)\{(p_1N_1+p_2N_2+q_1N_3+q_2N_4- p_1p_2N_5 - p_1q_2N_6-q_1p_2N_7-q_1q_2N_8)+p(1-(p_1N_1+p_2N_2 +q_1N_3+q_2N_4- p_1p_2N_5-p_1q_2N_6-q_1p_2N_7-q_1q_2N_8) (p_3N_9+p_4N_{10}+q_3N_{11}+q_4N_{12}- p_3p_4N_{13} \backslash -p_3q_4N_{14} - q_3p_4N_{15}-q_3q_4N_{16})$$

$$P(W>t) = \sum_{k=0}^{\infty} V_k(t)\{(p_1N_1+p_2N_2+q_1N_3+q_2N_4- p_1p_2N_5 -p_1q_2N_6-q_1p_2N_7-q_1q_2N_8)\ (1-(p_3N_9+p_4N_{10} +q_3N_{11}+q_4N_{12}- p_3p_4N_{13}-p_3q_4N_{14}-q_3p_4N_{15}-q_3q_4N_{16}\ )) +p(p_3N_9+p_4N_{10}+q_3N_{11}+q_3N_{12}- p_3p_4N_{13} -p_3q_4N_{14}-q_3p_4N_{15}-q_3q_4N_{16})\}$$

$$P(W>t) = \sum_{k=0}^{\infty} V_k(t)\{B_k(1-pC_k)+pC_k\}$$

Where $B_k = p_1N_1+p_2N_2+q_1N_3+q_2N_4- p_1p_2N_5 -p_1q_2N_6-q_1p_2N_7-q_1q_2N_8$

$C_k = p_3N_9+p_4N_{10}+q_3N_{11}+q_4N_{12}- p_3p_4N_{13} -p_3q_4N_{14}-q_3p_4N_{15}-q_3q_4N_{16}$

$P(W>t) = \sum_{k=0}^{\infty} V_k(t)A_k$, Where $A_k = B_k(1-pC_k)+pC_k$,

From renewal theory (Medhi),

$V_k(t) = F_k(t) - F_{k+1}(t)$ with $F_0(t) = 1$

$$P(W>t) = \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)]A_k \quad (11)$$

$$P(W>t) = \sum_{k=0}^{\infty} F_k(t)A_k - \sum_{k=0}^{\infty} F_{k+1}(t)A_k$$

Since $L(t) = 1- P(W>t)$ and $l(t) = \frac{d}{dt}L(t)$,we get

$$L(t) = 1 - \sum_{k=0}^{\infty} F_k(t)A_k + \sum_{k=0}^{\infty} F_{k+1}(t)A_k$$

$$l(t) = -\sum_{k=0}^{\infty} f_k(t)A_k + \sum_{k=0}^{\infty} f_{k+1}(t)A_k \quad (12)$$

Taking Laplace Transform on both sides , we get

$$l^*(s) = \sum_{k=0}^{\infty} f_{k+1}^*(s)\, A_k - \sum_{k=0}^{\infty} f_k^*(s)\, A_k \quad (13)$$

Where $f_k^*(s) = \sum_{i=1}^{k} \frac{b_i\beta_i}{s+\beta_i}$

The mean time to recruitment, is known that

$$E(W) = -\frac{d}{ds}(l^*(s))_{s=0},\ r = 1,2,3\ldots \quad (14)$$

Now

$$-\frac{d}{ds} f_k^*(s)|_{s=0} = E[U_1+U_2+U_3+.......+U_k] = \sum_{i=1}^{k} \frac{1}{\beta_i} \quad (15)$$

From the equations (13),(14) & (15) ,we get

$$E(W) = \sum_{k=0}^{\infty} \sum_{i=1}^{k+1} \frac{1}{\beta_i} A_k - \sum_{k=0}^{\infty} \sum_{i=1}^{k} \frac{1}{\beta_i} A_k$$

$$E(W) = \sum_{k=0}^{\infty} \frac{1}{\beta_{k+1}} A_k \quad (16)$$

Equation (16) gives the mean time to recruitment for maximum case.

Where $A_k=[p_1N_1+p_2N_2+q_1N_3+q_2N_4- p_1p_2N_5-p_1q_2N_6 -q_1p_2N_7- q_1q_2N_8][1-p(p_3N_9+p_4N_{10}+q_3N_{11}+q_4N_{12} - p_3p_4N_{13}- p_3q_4N_{14}-q_3p_4N_{15}-q_3q_4N_{16})] +p[p_3N_9+p_4N_{10}+q_3N_{11}+q_4N_{12}- p_3p_4N_{13} -p_3q_4N_{14}-q_3p_4N_{15}-q_3q_4N_{16})]$

Case (ii):

For minimum case, consider $P(S_k < Y)$, conditioning upon S_k and using the law of total probability,

We get

$$P(S_k<Y) = \int_0^\infty P(S_k < Y/S_k = x) g_k(x)dx$$

$$= \int_0^\infty \left(1 - H(x)\right) g_k(x)dx$$

$$= \int_0^\infty (1 - H_1(x))\,(1 - H_2(x)) g_k(x)dx$$

$$= \int_0^\infty p_1p_2 e^{-(\theta_1+\theta_3+\lambda_1+\lambda_2)x}\, g_k(x)dx$$

$$+\int_0^\infty p_1q_2 e^{-(\theta_1+\theta_4+\lambda_1)x}\, g_k(x)dx$$

$$+\int_0^\infty q_1p_2 e^{-(\theta_2+\theta_3+\lambda_2)x}\, g_k(x)dx$$

$$+\int_0^\infty q_1q_2 e q_1 q_2^{-(\theta_2+\theta_4)x}\, g_k(x)dx$$

$P(S_k < Y) = p_1p_2 g_k^*(\theta_1 + \theta_3+\lambda_1+\lambda_2$

$$+ p_1q_2 g_k^*(\theta_1 + \theta_4 + \lambda_1)$$

$$+ q_1p_2 g_k^*(\theta_2 + \theta_3 + \lambda_2)$$

$$+ q_1q_2 g_k^*(\theta_2 + \theta_4) \quad (17)$$

$$P(S_k < Y) = p_1p_2N_5 + p_1q_2N_6 + q_1p_2N_7 + q_1q_2N_8 \quad (18)$$

Where $N_5 = g_k^*(\theta_1 + \theta_2 + \lambda_1 + \lambda_2)$,

$N_6 = g_k^*(\theta_1 + \theta_4 + \lambda_1)$

$N_7 = g_k^*(\theta_2 + \theta_3 + \lambda_2)$,

$N_8 = g_k^*(\theta_2 + \theta_4)$

Similarly

$$P(S_k<Z) = p_3p_4 \int_0^\infty e^{-(\theta_5+\theta_7+\mu_1+\mu_2)x}\, g_k(x)dx$$

$$+p_3q_4 \int_0^\infty e^{-(\theta_5+\theta_8+\mu_1)x}\, g_k(x)dx$$

$$+q_3p_4 \int_0^\infty e^{-(\theta_6+\theta_7+\mu_2)x}\, g_k(x)dx$$

$$+q_3q_4 \int_0^\infty e^{-(\theta_6+\theta_8)x}\, g_k(x)dx \quad (19)$$

$$P(S_k < Z) = p_3p_4 g_k^*(\theta_5 + \theta_7 + \mu_1 + \mu_2)$$

$$+p_3q_4 g_k^*(\theta_5 + \theta_8 + \mu_1)$$

$$+q_3p_4 g_k^*(\theta_6 + \theta_7 + \mu_2)$$

$$+q_3q_4 g_k^*(\theta_6 + \theta_8) \quad (20)$$

$$P(S_k < Z) = p_3p_4N_{13} + p_3q_4N_{14} + q_3p_4N_{15} + q_3q_4N_{16} \quad (21)$$

Where $N_{13} = g_k^*(\theta_5 + \theta_7 + \mu_1 + \mu_2)$,,

$N_{14} = g_k^*(\theta_5 + \theta_8 + \mu_1)$

$N_{15} = g_k^*(\theta_6 + \theta_7 + \mu_2)$

$N_{16} = g_k^*(\theta_6 + \theta_8)$

Substituting (18) & (21) in (6) ,we get

$$P(W > t) = \sum_{k=0}^{\infty} V_k(t)\{(p_1p_2N_5+p_1q_2N_6+q_1p_2N_7+q_1q_2N_8)$$

$$(1\text{-}p(p_3p_4N_{13}+p_3q_4N_{14}+q_3p_4N_{15}+q_3q_4N_{16}))$$

$$+p(p_3p_4N_{13}+p_3q_4N_{14}+q_3p_4N_{15}+q_3q_4N_{16})\}$$

$$P(W > t) = \sum_{k=0}^{\infty} V_k(t)\{B_k(1\text{-}pC_k)+pC_k\}$$

Where $B_k = p_1p_2N_5+p_1q_2N_6+q_1p_2N_7+q_1q_2N_8$

$C_k = p_3p_4N_{13}+p_3q_4N_{14}+q_3p_4N_{15}+q_3q_4N_{16}$

$$P(W > t) = \sum_{k=0}^{\infty} V_k(t)A_k, \text{ Where } A_k = B_k(1\text{-}pC_k)+pC_k,$$

As we found in case (i),we get

$$E(W) = \sum_{k=0}^{\infty} \frac{1}{\beta_{k+1}} A_k \quad (22)$$

Equation (22) gives the mean time to recruitment for minimum case.

Where $A_k= [p_1p_2N_5+p_1q_2N_6+q_1p_2N_7+q_1q_2N_8] [1-p(p_3p_4N_{13}+p_3q_4N_{14}+q_3p_4N_{15}+q_3q_4N_{16})] +p[p_3p_4N_{13}+p_3q_4N_{14}+q_3p_4N_{15}+q_3q_4N_{16}]$

5. Special Case

Description of this case is similar to that of previous case, except the condition on inter-decision times. The analytical results for the survival function of time to recruitment when the loss of manpower is independent and non-identically distributed exponential random variables with parameters α_i and inter-decision times are independent and identically distributed exponential random variables with parameter β and optional, mandatory thresholds having SCBZ property.

(i) For maximum case, mean time to recruitment is

$$E(W) = \frac{1}{\beta} \sum_{k=0}^{\infty} A_k \quad (23)$$

Where A_k as in maximum case.

(ii) For minimum case, mean time to recruitment is

$$E(W) = \frac{1}{\beta} \sum_{k=0}^{\infty} A_k \quad (24)$$

Where A_k as in minimum case.

6. Conclusion

The manpower planning model developed in this paper more general compare to earlier work in this direction and it can be used to plan for the adequate provision of manpower for the organization at graduate, professional and management level in the context of attrition. There is a scope for studying the applicability of the designed model using simulation. Further, by collecting relevant data, one can test the goodness of fit for the distributions assumed in this paper. The findings given in this paper enable one to estimate manpower gap in future, thereby facilitating the assessment of manpower profile in predicting future manpower development not only on industry but also in a wider domain. The present work can be studied for a two grade manpower system. This work also can be extended in two sources of depletion for two graded manpower system.

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