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A Mathematical Model of a Criminal-Prone Society

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Abstract: ***It is well known that mathematical models are useful for controlling many physical, biological and engineering systems. In this paper, it is shown that mathematical models are also very useful to control crime in a society. Models may be useful to predic the evolution of the crime in a society depending upon different social factors and it may be useful to develop policies for controlling crime. Recently, Nuno et al. [1] proposed and analysed a mathematical model of a criminal-prone self-protected society. We have modified the interaction function between criminals and guards in Nuno et al. [1] model and investigated the impacts of this modification. We have modified the model of Nuno et al. [1] modifying the number of arrests per unit time. The steady state solutions of the proposed model are determined and their stability nature are discussed. Numerical simulation reuslts are presented to verify the analytically obtained stability conditions.***

Keywords: Mathematical Model, Social crime, Stability Analysis

1. Introduction

Dynamical system and chaos theories are useful to model allmost all natural phenomena eg biological systems [2],[3],[4], synchronization behaviour [5],[6],[7], astrophysical system [8],economics [9], social sciences [10]. Criminal behaviours and violence may be modelled using the concepts of population dynamics. Population dynamics can be modelled and analysed with the help of nonlinear dynamics [11],[12],[13],[14]. Predator prey models are used to model dynamics of criminals. In such a model criminals prey upon the public. Compartmental models have been used to study the influence of sociological and economical factors on the evolution of criminality. Such studies will be useful to determine possible strategies for reducing and controlling crime. Criminal behaviour and violence can also be modelled by using infectious disease model [15].Agent based model may also be suitable to investigate spread of criminal behaviour [16].

Recently Nuno et al. [2] has proposed a mathematical model of a criminal prone society. In this paper, we have modified the interaction function between criminals and guards to formulate a new model. The effects of modification on the dynamics of the model are investigated. Results are compaired with the existing results. In Section 2, the model is formulated. In Section 3 the steady states of the model are determined and their stability nature are discussed. In section 4 impacts of social measures and police actions are reported with the help of numerical simulation results. In Section 5 conclusions are drawn.

2. Mathematical Model

We consider the following structured subpopulation in the society :

1) At time t, the number of criminals represent $Y(t)$.
2) $G(t)$ represent the number of guards are hired at time t.
3) The number of individuals that are in prison represented by $J(t)$ at time t.
4) X_1 and X_2 are the non-criminal population which is divided into two subclass from criminal.The wealth of each class contributes in a different way i.e, $W(t)$, at time t,

$$W(t)=W_1(t)+W_2(t). \tag{1}$$

Now we consider a closed society without criminals nor guards. The flow of social classes is:

$$\dot{X}_i(t)=\alpha_{i-1}X_{i-1}(t)-(\alpha_i+\beta_i)X_i(t)+\beta_i+1X_i+1(t)$$
(2)

Where α_i,β_i are the rate of social promotion and relegation for i-th class.In particular,we assume that α_i and β_i are non-negative increasing functions and non-negative decreasing function with respect to W(t). Under assumptions (1) and (2),the equilibrium distribution of individuals among the different classes corresponds to the solutions of:

$$-\alpha_1X_1+\beta_2X_2=0,$$
$$\alpha_{i-1}X_{i-1}-(\alpha_i+\beta_i)X_i+\beta_i+1X_i+1=0,$$
$$for\, n=2,....,n-1$$
$$\alpha_{n-1}X_{n-1}-\beta_nX_n=0$$

Next we examine by argument and considered the result of the personal appearance of criminals and guards.The system is closed because there is neither input nor output flux and criminals come from any social classes. We assume the interaction between criminals and security guards of the following type :

5) $K(t)$,crime rate regarded as to be proportional to the number of encounter between criminals Y and targets, X_i. i.e,

$$K(t)=\sum\theta_iX_i(t)Y(t) \tag{3}$$

The term θ_i is decreasing function of X_i, e.g.

$$\theta_i(t) = \frac{a_i}{b_i + X_i(t)} \quad (4)$$

When the number of targets i.e, X_i is large the number of crimes of i-th population is proportional the number of criminals.

6) The recruitment of criminals from the population classes X_i which to the purpose only lower term of i. Particular, we collect the class X_1 that are social familiar risk of becoming criminals. Under examination X_1 is the only class relation with Y and the prisoners J. $R(t)$,the criminal recruitment term is assumed to be given by:

$$R(t) = k_1 X_1(t) Y(t). \quad (5)$$

Here, we make the simplifying of equation (5) is given by :

$$k_1 = kW(t) \quad (6)$$

The inner meaning of the equation (6) is the total number of wealth of the society is equivalent to the wealth of the targets.So obviously,the recruitment of criminals depends social classes,social promotion and age.So, when the guards are not present then the equation for $Y(t)$ is:

$$\dot{Y}(t) = kW(t)X_1(t)Y(t) - \rho Y(t) - \sigma Y^2(t)$$

Negative terms unite with decay and effect of an intraspecific competition.If there are no criminals that mean initially $Y(0)$ = 0, the above system is free of them, i.e. $Y(t)$ = 0 for all $t >$ 0.In the next Section we will see that this situation is unstable.

7) Per unit time $A(t)$,the number of arrests (i.e. removing criminals) is proportional to the number of criminals and guards.so ,

$$A(t) = \frac{mY(t)^\alpha G(t)}{M + Y(t)^\alpha} \quad (7)$$

8) Dynamics of the population in jail is governed by the following equation:

$$\dot{J}(t) = A(t) - \tau J(t) \quad (8)$$

9) The following factors to be governed by the evolution of the number of guards;

(i)Mathematically modelled of physiological term of retirement can be written as:

$$p(t) = -qG(t) \quad (9)$$

(ii) The rate of hiring new guards make for a non criminal population is proportional to the number of crimes $K(t)$:

$$H(t) = h\frac{a_1}{b_1 + X_1(t)} X_1(t)Y(t) + h\frac{a_2}{b_2 + X_2(t)} X_2(t)Y(t) \quad (10)$$

(iii) We assume the number of casualties suffered by guards in the following form :

$$L(t) = l\frac{Y(t)^\alpha G(t)}{M + Y(t)^\alpha} \quad (11)$$

Where L is neutrilization of guards by corruption. Note that Nuno et al. [1] assumed the form of $L(t)$ as $L(t) = l\frac{Y(t)G(t)}{M + Y(t)}$.

10)Now lastly we reached the model of the wealth W of the system of crimes and the cost of maintaining the security service.We assume some condition on it ;

(i) The sum of the number of crimes committed per unit time and to the total wealth of the system is proportional to the negative effect of the crimes on W .

$$C(t) = -\lambda W(t)K(t) = -\lambda W(t)Y(t)\left(\frac{a_1}{b_1 + X_1(t)} X_1(t) + \frac{a_2}{b_2 + X_2(t)} X_2(t)\right)$$

(ii) The guards are maintained given by the cost:

$$\gamma(t) = -g(W)G(t) \quad (12)$$

Here g is the unit cost that the population is capable to bear depends on the total wealth W .

Putting these value in section (2) we are the following system of ODEs:

$$\begin{aligned}
\dot{Y} &= kWX_1Y - \rho Y - \sigma Y^2 - m\frac{Y^\alpha G}{M + Y^\alpha} \\
\dot{G} &= -qG + hY\left(\frac{a_1}{b_1 + X_1(t)} X_1 + \frac{a_2}{b_2 + X_2(t)} X_2\right) - l\frac{Y^\alpha G}{M + Y^\alpha} \\
\dot{J} &= m\frac{Y^\alpha G}{M + Y^\alpha} - \tau J \\
\dot{X}_1 &= -\alpha_1 X_1 + \beta_2 X_2 + \tau J - kWX_1Y + \rho Y + \sigma Y^2 \\
\dot{X}_2 &= \alpha_1 X_1 - \beta_2 X_2
\end{aligned} \quad (13)$$

The evolution of the total wealth:

$$\dot{W} = (C_1X_1 + C_2X_2 - W) - \lambda YW\left(\frac{a_1}{b_1 + X_1(t)} X_1 + \frac{a_2}{b_2 + X_2(t)} X_2\right) - g(W)G$$

where the total wealth is equal to $c_i X_i$ and c_i are nonnegative constants for all i = 1,2.

$$\overline{W} = C_1X_1 + C_2X_2 \quad (14)$$

3. Analysis of the Model

Once a model for a criminal-prone self-protected society has been derived, a natural question that arises is that of comparing on it the two main strategies that are commonly considered to control crime, namely:

(i) improving the strength of police to catch criminals and remove them from the society and
(ii) hindering the recruitment of new criminals from the society, mainly from the lower classes at risk.

The first approach requires either an increase of the police size or a larger effectiveness or both. The second one can be achieved either by social promotion of the class at risk (i.e. increasing α_1 and decreasing β_2) or by reducing the appeal of criminal behavior (i.e. by decreasing the value of k, say by improving education and social consciousness, etc). To ascertain which strategy is better in a given socio-economical context is an open problem of great interest for policy makers.

To shed some light on the related effect of social promotion and police repression, we next discuss a particular situation in which only two classes exist and the police size is kept constant, G. Also, jailed population is not taken into account. Essentially, we assume that the time scale we consider is an intermediate one between the average time spend in jail by prisoners and the period over which the policy of guards recruitment changes. Instead of equations (13) we now have:

$$\dot{X}_1 = -\alpha_1 X_1 + \beta_2 X_2 + \tau J - kWX_1Y + \rho Y + \sigma Y^2 + m\frac{Y^\alpha G}{M+Y^\alpha}$$
$$\dot{X}_2 = \alpha_1 X_1 - \beta_2 X_2$$
$$\dot{Y} = kWX_1Y - \rho Y - \sigma Y^2 - m\frac{Y^\alpha G}{M+Y^\alpha} \qquad (15)$$

In the equation of $\dot{X}_1$ in (15) the extra term added with respect to the equation in (13) come from the criminal population.We introduced to close dynamic system, so that $X_1 + X_2 + Y = N$. Also, it is assumed α_1 and β_2 are independent. Now the equation of the total wealth:

$$\frac{dW}{dt} = c_1X_1 + c_2X_2 - W - \lambda YW\left(\frac{a_1}{b_1+X_1}X_1 + \frac{a_2}{b_2+X_2}X_2\right) - gWG$$

where $g(W) = gW, g > 0$. We choose parameters k and G measure the appearance of criminals and the size of the police forces. So our 3D model is given by ,

$$\dot{W} = c_1X_1 + c_2X_2 - W - \lambda YW\left(\frac{a_1}{b_1+X_1}X_1 + \frac{a_2}{b_2+X_2}X_2\right) - gWG$$
$$\dot{Y} = kWX_1Y - \rho Y - \sigma Y^2 - m\frac{Y^\alpha G}{M+Y^\alpha}$$

$$\dot{X}_1 = -\alpha_1 X_1 + \beta_2 X_2 + \tau J - kWX_1Y + \rho Y + \sigma Y^2 + m\frac{Y^\alpha G}{M+Y^\alpha}$$

4. Impacts of social measures and police actions

Case 1 : $\alpha = 1$
In this case ,the model is same as Nuno et al. [1].

$$\dot{W} = c_1X_1 + c_2X_2 - W - \lambda YW\left(\frac{a_1}{b_1+X_1}X_1 + \frac{a_2}{b_2+X_2}X_2\right) - gWG$$
$$\dot{Y} = kWX_1Y - \rho Y - \sigma Y^2 - m\frac{YG}{M+Y} \qquad (17)$$

$$\dot{X}_1 = -\alpha_1 X_1 + \beta_2 X_2 + \tau J - kWX_1Y + \rho Y + \sigma Y^2 + m\frac{YG}{M+Y}$$

To discuss this let us see a particular example we set $N = 1$ and $\lambda = 1$ and we assume values for the remaining parameters:

$$M = 1; \rho = 0.01; g = 0.1; a_1 = 1; b_1 = 1; a_2 = 1; b_2 = 1; m = 0.1$$
$$c_1 = 0.1; c_2 = 1; \alpha_1 = 0.1; \beta_2 = 0.01; \sigma = 0 \qquad (18)$$

Our observation is that,the system (17) has at least the free-of-criminals stationary state:

$$Y = 0; X_1 \approx 0.091; \qquad (19)$$

i.e, wealth depends on the value of G :

$$W \approx \frac{9.181}{10+G} \qquad (20)$$

The free-of-criminals state analytically by linearization.The jacobian matrix calculate is given by:

$$J = \begin{pmatrix} -1-0.1G & -1-\frac{5.1375}{10+G} & -0.9 \\ 0 & \frac{0.835471}{10+G}k - 0.01 - 0.1G & 0 \\ 0 & -\frac{0.835471}{10+G}k + 0.1G & -0.11 \end{pmatrix}$$

whose eigenvalues are:

$$\lambda_1 = -0.11; \lambda_2 = -1 - 0.1G \qquad (22)$$

And,

$$\lambda_3 = -\frac{-0.835471k + 0.1 + 0.01G + G + 0.1G^2}{10 + G} \qquad (23)$$

λ_3 to be negative either k must be small or G must be large.If we choose that the size of security forces is fixed i.e, $G = G_0$ then the criminal population k must be smaller than

$$k_c = 0.120 + 0.012G_0 + 0.120G_0^2 \qquad (24)$$

Now We try to concentrate changing α_1 on the evolution of the criminal population and compare with change of police size and keep constant value of k = 0.5. Remaining parameter are shown in (18) and a free-of-criminals state exist:

$$Y = 0; x_1 = \frac{1}{100\alpha_1 + 1}$$

$$W = \frac{1 + 1000\alpha_1}{1000\alpha_1 + 100\alpha_1 G + 10 + G}$$

Stability of this criminal free state depends on the sign of the eigenvalue which is shown in figure1.

$$\lambda = -\frac{-0.4 - 480\alpha_1 + 10^3\alpha_1^2 + G(10100\alpha_1^2 + 202\alpha_1 + 1.01) + G^2(10^3\alpha_1^2 + 20\alpha_1 + 0.1)}{10^5\alpha_1^2 + 2*10^3\alpha_1 + 10 + G(10^4\alpha_1^2 + 2*10^2\alpha_1 + 1)} \qquad (26)$$

The eigenvalue is positive if $A_1 < \alpha_1 < A_2$ for given G and therefore criminality is stop either social promotion is too low or too high. These values of α depends range $G \in [0, 0.2]$. For the value G = 0.1:

$$A_1 \approx 0.00; A_2 \approx 0.22 \qquad (27)$$

our criminal-free society is unstable for $\alpha_1 \in (A_1, A_2)$.

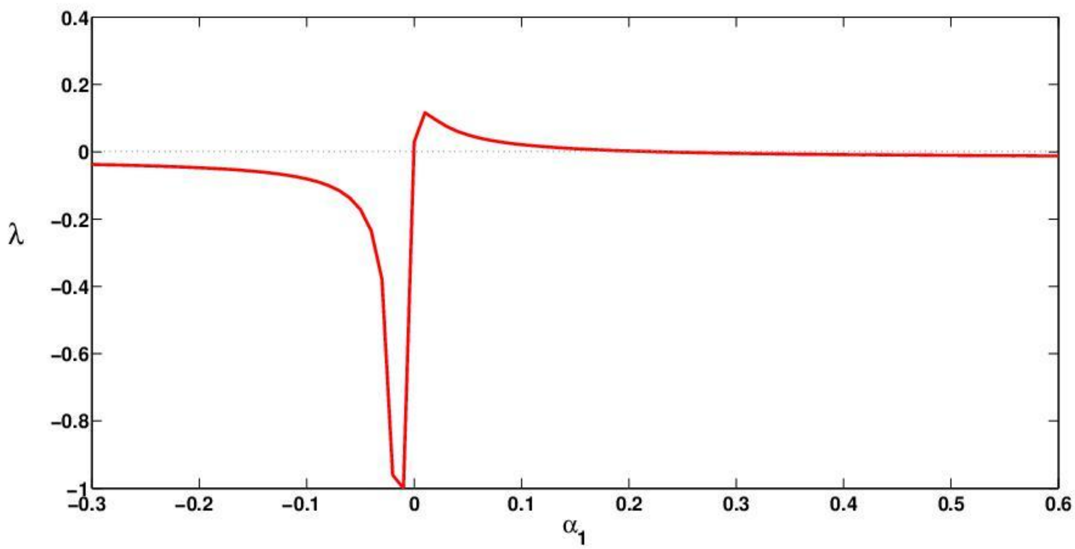

Figure 1: Eigenvalue as a function of α_1 when α = 1 of the system (26)

Now, we try to concentrate changing β_2 on the evolution of the criminal population and compare with change of police size and keep constant the value of k, k = 0.5. Remaining parameters are shown in (18) and free-of-criminals state exist:

$$Y = 0; x_1 = \frac{10\beta_2}{10\beta_2 + 1}; W = \frac{10 + 10\beta_2}{100\beta_2 + 10\beta_2 G + 10 + G} \qquad (28)$$

Stability of this criminal free state depends on the sign of the eigenvalue which is shown in figure2 :

$$\lambda = -\frac{0.1 - 48\beta_2 - 40\beta_2^2 + G(101\beta_2^2 + 20.2\beta_2 + 1.01) + G^2(10\beta_2^2 + 2\beta_2 + 0.1)}{10^3\beta_2^2 + 200\beta_2 + 10 + G(100\beta_2^2 + 20\beta_2 + 1)} \qquad (29)$$

This eigenvalue is positive if $\beta_2 < A_1; \beta_2 > A_2$ for given G and therefore criminality is stop either social promotion is too low or too high.The values of α depend range $G \in [0, 0.2]$. For the value $G = 0.1$:

$$A_1 \approx -1.5467; A_2 \approx 0.0044 \quad (30)$$

our criminal-free society is unstable for $\beta_2 < A_1; \beta_2 > A_2$

Also, here for $\alpha = 1$ time evolution of Y and X_1 of the system (17) are plotted in figure 3.

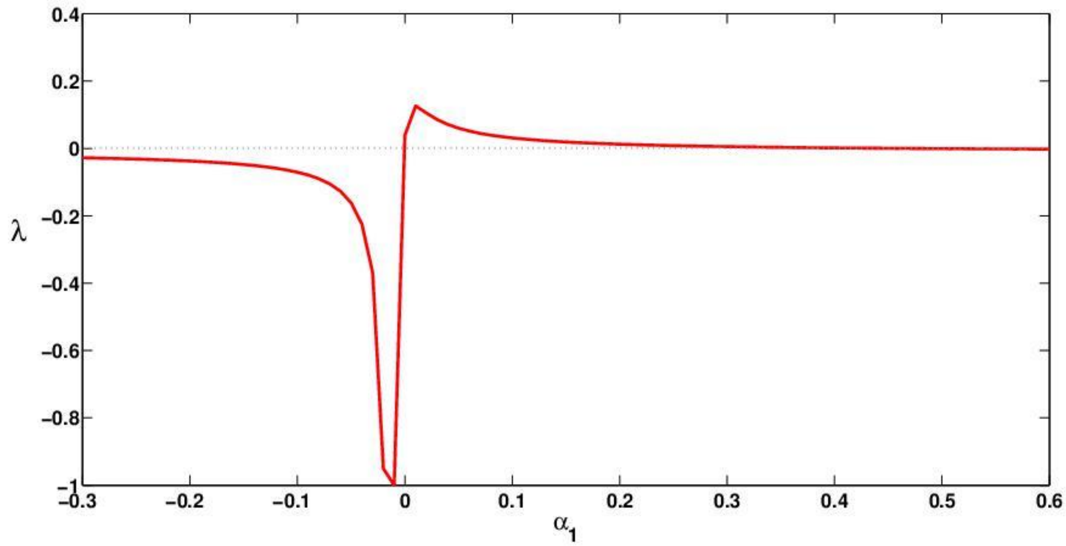

Figure 2: Eigenvalue as a function of β_2 when $\alpha = 1$ of the system (29)

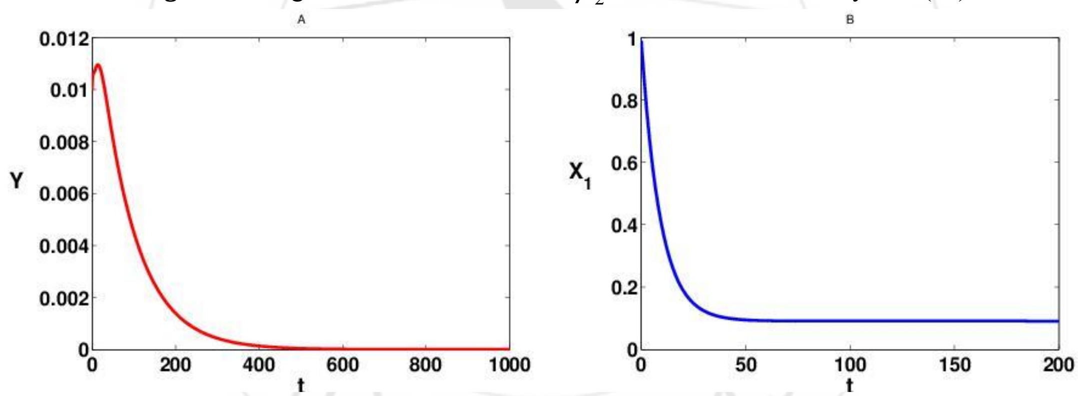

Figure 3: Time evolution of the system (17) for $\alpha = 1.0$, (A) time evolution of Y ,(B) the time evolution of X_1 are plotted.

Case 2 : $\alpha \neq 1$

In this case, the model takes the following form:

$$\dot{W} = c_1 X_1 + c_2 X_2 - W - \lambda Y W \left(\frac{a_1}{b_1 + X_1} X_1 + \frac{a_2}{b_2 + X_2} X_2 \right) - gWG$$

$$\dot{Y} = kWX_1 Y - \rho Y - \sigma Y^2 - m \frac{Y^\alpha G}{M + Y^\alpha} \quad (31)$$

$$\dot{X}_1 = -\alpha_1 X_1 + \beta_2 X_2 + \tau J - kWX_1 Y + \rho Y + \sigma Y^2 + m \frac{Y^\alpha G}{M + Y^\alpha}$$

To analyse the model numerically, we set $N = 1$ and $\lambda = 1$ and we assume values for the remaining parameters as :

$$M = 1; \rho = 0.01; g = 0.1; a_1 = 1; b_1 = 1; a_2 = 1; b_2 = 1; m = 0.1$$

$$c_1 = 0.1; c_2 = 1; \alpha_1 = 0.1; \beta_2 = 0.01; \sigma = 0 \quad (32)$$

Our observation by the above condition , the system (30) has at least the free-of-criminals stationary state:

$$Y = 0; X_1 \approx 0.091; \quad (33)$$

with a total wealth that depends on the value of G :

$$W \approx \frac{9.181}{10+G} \tag{34}$$

The free-of-criminals state analytically by linearization.The Jacobian matrix calculate is given by:

$$J = \begin{pmatrix} -1-0.1G & -1-\frac{5.1375}{10+G} & -0.9 \\ 0 & \frac{0.835471}{10+G}k-0.01 & 0 \\ 0 & -\frac{0.835471}{10+G}k & -0.11 \end{pmatrix} \tag{35}$$

whose eigenvalues are:

$$\lambda_1 = -0.11; \lambda_2 = -1-0.1G$$
$$\text{and} \lambda_3 = -\frac{-0.835471k+0.1+0.01G}{10+G}.$$

Now, λ_3 become negative when either k is small or G is large. If we choose that the size of security forces is fixed i.e, $G = G_0$ then the criminal population k must be smaller than

$$k_c = 0.120 + 0.0120G_0 \tag{36}$$

Similarly,We try to concentrate changing α_1 on the evolution of the criminal population and compare with change of the police size and keep constant the value k = 0.5.Remaining parameters are shown in (31) and a free-of-criminals state exist:

$$Y = 0; x_1 = \frac{1}{100\alpha_1+1}; W = \frac{1+1000\alpha_1}{1000\alpha_1+100\alpha_1 G+10+G} \tag{37}$$

Stability of this criminal free state depends on the sign eigenvalue which is shown in figure3.

$$\lambda = -\frac{-0.4-480\alpha_1+10^3\alpha_1^2+G(100\alpha_1^2+2\alpha_1+1.01)}{10^5\alpha_1^2+2*10^3\alpha_1+10+G(10^4\alpha_1^2+2*10^2\alpha_1+1)} \tag{38}$$

This eigenvalue is positive if $A_1 < \alpha_1 < A_2$ and therefore criminality is stop either social promotion is too low or too high .The value of α depends range $G \in [0, 0.2]$. For the value $G = 0.1$:

$$A_1 \approx 0.00; A_2 \approx 0.47 \tag{39}$$

our criminal-free society is unstable for $\alpha_1 \in (A_1, A_2)$.

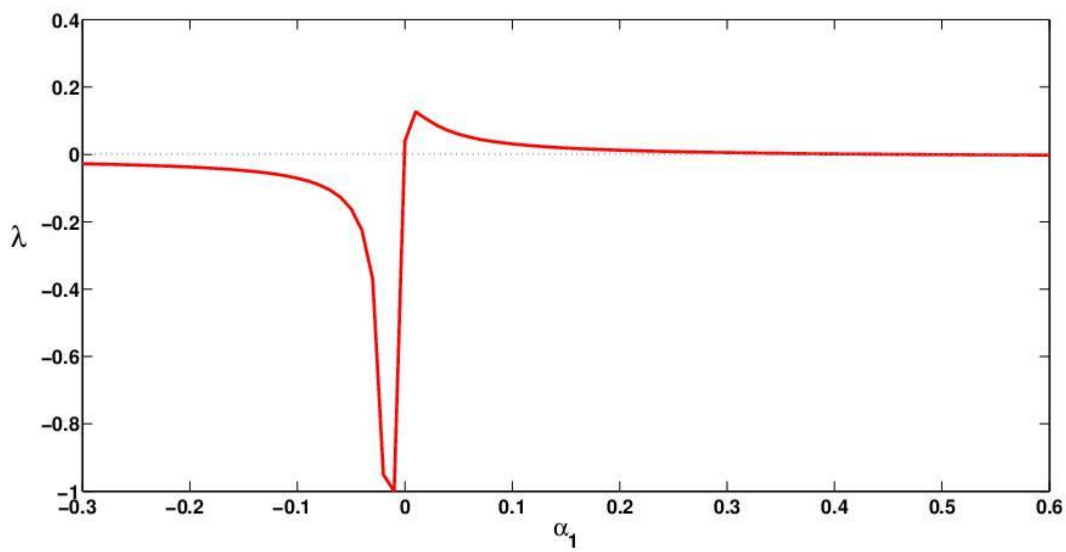

Figure 4: Eigenvalue as a function of α_1 when $\alpha \neq 1$ of the system (38)

We try to concentrate changing β_2 on the evolution of the criminal population and compare with change of police size and keep constant $k = 0.5$ .Remaining parameters are shown in (31) and a free-of-criminals state exist:

$$Y = 0, \quad x_1 = \frac{10\beta_2}{10\beta_2+1}, \quad W = \frac{10+10\beta_2}{100\beta_2+10\beta_2 G+10+G} \tag{40}$$

Stability of this criminal free state depends on the sign of eigenvalue which is shown in figure4 .

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$$\lambda = -\frac{0.1 - 48\beta_2 - 40\beta_2^2 + G(1\beta_2^2 + 0.2\beta_2 + 0.01)}{10^3\beta_2^2 + 200\beta_2 + 10 + G(100\beta_2^2 + 20\beta_2 + 1)} \quad (41)$$

This eigenvalue is positive if $\beta_2 < A_1; \beta_2 > A_2$ for given G and therefore criminality is stop either social relegation is too low or too high.The values of β_2 depends range of $G \in [0,0.2]$. For the value $G = 0.1$:

$$A_1 \approx -1.20; A_2 \approx 0.00 \quad (42)$$

our criminal-free society is unstable for $\beta_2 < A_1; \beta_2 > A_2$

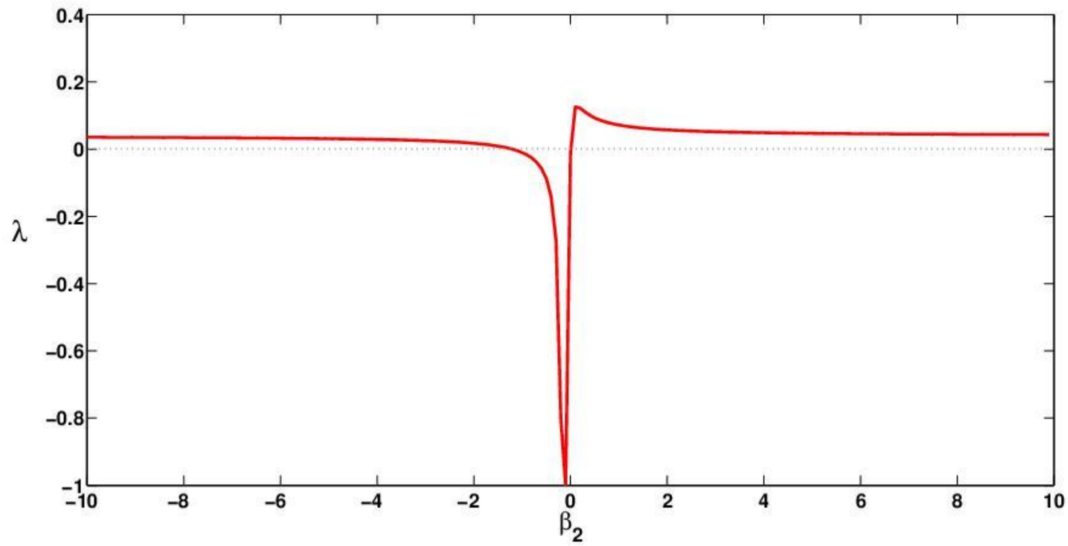

Figure 5: Eigenvalue as a function of β_2 when $\alpha \neq 1$ of the system (41)

Also,here for $\alpha = 0.6, 1.6$ time evolution of Y and X_1 of the system (30) are plotted in figure 6 and 7 respectively.

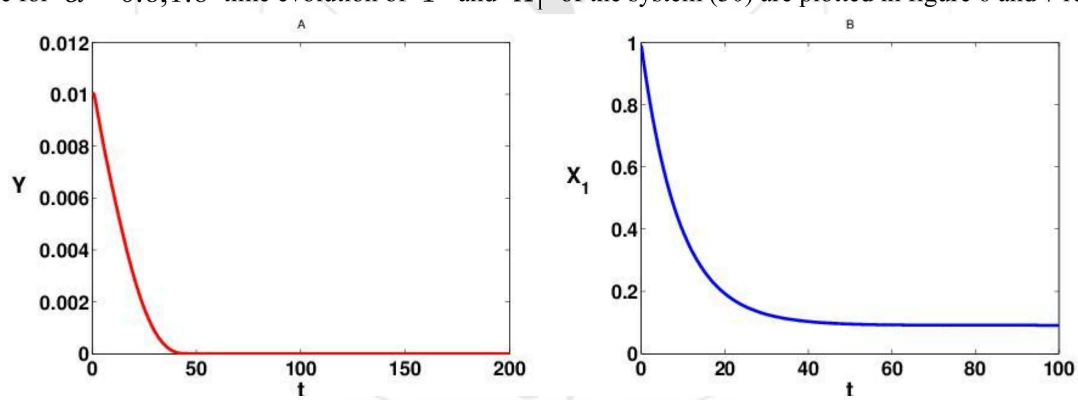

Figure 6: Time evolution of the system (31) for $\alpha = 0.6$, (A) time evolution of Y ,(B) the time evolution of X_1 are plotted

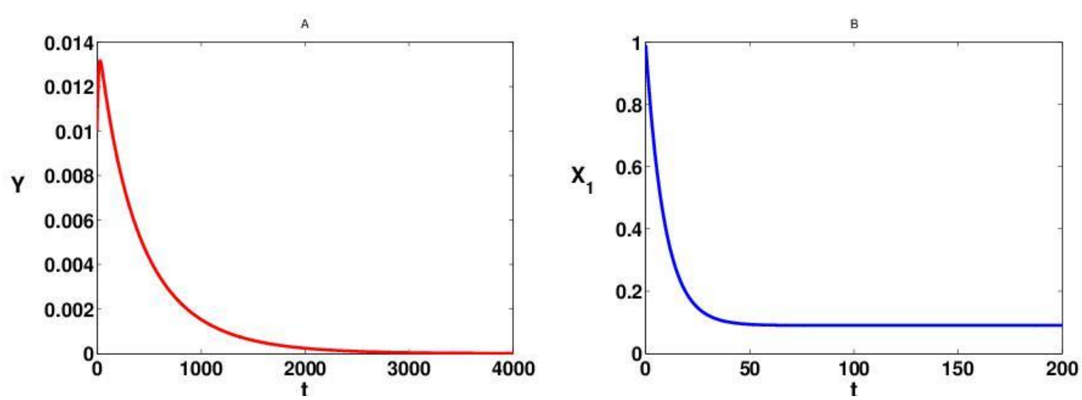

Figure 7: Time evolution of the system (31) for $\alpha = 1.6$, (A) time evolution of Y ,(B) the time evolution of X_1 are plotted.

5. Conclusions

A mathematical model of criminal prone society has been formulated modifing the number of arrest function of the model of Nuno et al. [1]. It is assumed in our model that the people are divided into different classes according to their contribution to the total wealth of the system. The equilibrium points of the model are determined and their stability nature are analysed. We have shown that the free-of-criminals steady state becomes asymptotically stable when criminal is decreased (i.e, to decrease the k-value) or α_1 i.e, social promotion is increased or when the size of the police forces is also increased.In other word criminality may decreased even when $\alpha_1 < A_1$ (i.e, low social promotion exist).We have also seen that approach increscent police forces could be less effective that increscent social polices due to quadratic dependence on G of the determinant eigenvalues(23) and (37). To get free of criminal society if α is increased then it take more time. For $\alpha < 1$ the number of criminal decreased and for $\alpha \geq 1$ the number of criminal first increased then decreased. Population of the lowest class decreases if there is social promotion. This study will be useful to control crime in the society.

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