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Results on Quasi Contraction Random Operators

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Abstract: ***In this paper, we prove the existence of common random fixed point for two continuous random operators under quasi contraction condition in a complete p-normed spaceX(with whose dual separates the point of X). Also, the well-posedness problem of random fixed points is studied. Our results, essentially cover special cases.***

Keywords: p -Normed spaces, Common random fixed point, Random operators, Well-posed problem

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1. Introduction and Preliminaries

Let X be a linear space and $\| \|_p$ be a real valued function on X with $0 < p \le 1$. The ordered pair (X, $\| \|_p$) is called a p-normed space[16] if for all x, y in X and scalars λ :

i. $\|x\|_p \ge 0$ and $\|x\|_p = 0$ iff $x = 0$

ii. $\|\lambda x\|_p = |\lambda|^p \|x\|_p$

iii. $\|x + y\|_p \le \|x\|_p + \|y\|_p$

for more details about p-normed spaces, see [5] or [14] .Throughout this article X will be complete p-normed space whose dual separates the points of it ,$\emptyset \ne A \subseteq X$ be a separable closed , (Ω, Σ)be the measurable space with Σ a sigma algebra of subsets of Ω .

Definition(1.1):[11]
A mapping $F\colon \ \Omega \to X$ is called measurable if, for open subset B of ,$F^{-1}(B) = \{\gamma \in \Omega : F(\gamma) \cap B \ne \emptyset\} \in \Sigma$.

Definition(1.2):[11]
A mapping $h\colon \Omega \times X \to X$is called a random operator if for any $x \in X$, $h(.,x)$is measurable .

Definition (1.3):[19]
A measurable mapping $\lambda\colon \Omega \to A$ is called random fixed point of a random operator $h\colon \Omega \times X \to X$if for every $\gamma \in \Omega$, $\lambda(\gamma) = h(\gamma, \lambda(\gamma))$.

Definition (1.4): [8]
A measurable mapping $\lambda\colon \Omega \to A$ is called common random fixed point of a random operator $h\colon \Omega \times A \to X$ and $G\colon \Omega \times A \to A$ iffor all $\gamma \in \Omega$

$$\lambda(\gamma) = h(\gamma, \lambda(\gamma)) = G(\gamma, \lambda(\gamma)).$$

Definition (1.5):[20]
A random operator $h\colon \Omega \times A \to X$ is called continuous (weakly continuous) if for each $\gamma \in \Omega$, $h(\gamma, .)$is continuous (weakly continuous).

The stochastic generalization of fixed point theory is random fixed point theory. Many researchers are interesting in this subject and it's applications in best approximations, integral equations and differential equations such as [7],[12],[10],[15],[1],[2],[3].

Saluj [18] establish some common random fixed point theorems under contractive type condition in the framework of cone random metric spaces. Rashwan and Albaqeri [17] obtained common random fixed point theorems for six weakly compatible random operators defined on a nonempty closed subset of a separable Hilbert space. In 2013, Arunchaiand Plubtieng [4] proved some random fixed point theorem for the some of weakly-strongly continuous random operators and nonexpansive random operators in Banach spaces. Singh, Rathore, Dubey and Singh [21] obtain a common random fixed point theorems for four continuous random operators in separable Hilbert spaces. Vishwakarme and Chauhan [22] proved common random fixed point theorems for weakly compatible random operators in symmetric spaces. Khanday, Jain and Badshah [13] proved the existence of common random fixed point theorems of two random multivalued generalized contractions by using functional expressions. Chanhan [9] obtained common random fixed point theorems for four continuous random operators satisfying certain contractive conditions in separable Hilbert spaces.

Now, we define a new type of random operators

Definition (1.6):
Let A be a nonempty subset of a p-normed space , let (Ω, Σ) be a measurable space and let $h, G\colon \ \Omega \times A \longrightarrow A$ be tow random operators . The random operator h is called

1. quasi contraction (qc) random operator if

$$\| h(\gamma,x) - h(\gamma,y) \|_p \le k \max\{ \| x - y \|_p, \| x - h(\gamma,x) \|_p, \| y - h(\gamma,y) \|_p, \| x - h(\gamma,y) \|_p, \| y - h(\gamma,x) \|_p\} \qquad (1.1)$$

For all $x, y \in A$, $\gamma \in \Omega$and $0 \le k < 1/2$.

2. G -quasi contraction (G -qc) random operator if

$$\| h(\gamma,x) - G(\gamma,y) \|_p \le k \max\{ \| x - y \|_p, \| x - h(\gamma,x) \|_p, \| y - G(\gamma,y) \|_p, \| x - G(\gamma,y) \|_p, \| y - h(\gamma,x) \|_p\} \qquad (1.2)$$

For all $x, y \in A$, $\gamma \in \Omega$and $0 \le k < 1/2$.

2. Common Random Fixed Point theorem

Theorem (2.1):
Let $\emptyset \neq A \subseteq X$ for fixed $\in \Omega$, h, G satisfy the condition (2.2) . Then $h$and G have a unique common random fixed point .

Proof
Let $\lambda_\circ : \Omega \longrightarrow A$ be arbitrary measurable mapping . We construct a sequence of measurable mappings $< \lambda_n >$ on Ω to A as follows

Let $\lambda_1, \lambda_2 : \Omega \longrightarrow A$ be tow measurable mappings such that $h(\gamma, \lambda_\circ(\gamma)) = \lambda_1(\gamma)andG(\gamma, \lambda_1(\gamma)) = \lambda_2(\gamma)$

By induction , we construct sequence of measurable mappings $\lambda_n : \Omega \longrightarrow A$ such that $h(\gamma, \lambda_{2n-1}(\gamma)) = \lambda_{2n}(\gamma)$ and $G(\gamma, \lambda_{2n}(\gamma)) = \lambda_{2n+1}(\gamma)$ …………(2.1)
From (2.1) and (1.2) , we have

$$\| \lambda_{2n}(\gamma) - \lambda_{2n+1}(\gamma) \|_p = \| h(\gamma, \lambda_{2n-1}(\gamma)) - G(\gamma, \lambda_{2n}(\gamma)) \|_p$$
$$\leq k \max\{ \| \lambda_{2n-1}(\gamma) - \lambda_{2n}(\gamma) \|_p, \| \lambda_{2n-1}(\gamma) - h(\gamma, \lambda_{2n-1}(\gamma)) \|_p, \| \lambda_{2n}(\gamma) - G(\gamma, \lambda_{2n}(\gamma)) \|_p, \| \lambda_{2n-1}(\gamma) - G(\gamma, \lambda_{2n}(\gamma)) \|_p, \| \lambda_{2n}(\gamma) - h(\gamma, \lambda_{2n-1}(\gamma)) \|_p \}$$
$$= k \max\{ \| \lambda_{2n-1}(\gamma) - \lambda_{2n}(\gamma) \|_p, \| \lambda_{2n-1}(\gamma) - \lambda_{2n}(\gamma) \|_p, \| \lambda_{2n}(\gamma) - \lambda_{2n+1}(\gamma) \|_p, \| \lambda_{2n-1}(\gamma) - \lambda_{2n+1}(\gamma) \|_p, \| \lambda_{2n}(\gamma) - \lambda_{2n}(\gamma) \|_p \}$$
$$= k \max\{ \| \lambda_{2n-1}(\gamma) - \lambda_{2n}(\gamma) \|_p, \| \lambda_{2n}(\gamma) - \lambda_{2n+1}(\gamma) \|_p, \| \lambda_{2n-1}(\gamma) - \lambda_{2n+1}(\gamma) \|_p \}$$

Using triangle inequality , we get

$$\| \lambda_{2n}(\gamma) - \lambda_{2n+1}(\gamma) \|_p \leq k \max\{ \| \lambda_{2n-1}(\gamma) - \lambda_{2n}(\gamma) \|_p, \| \lambda_{2n}(\gamma) - \lambda_{2n+1}(\gamma) \|_p, \| \lambda_{2n-1}(\gamma) - \lambda_{2n}(\gamma) \|_p + \| \lambda_{2n}(\gamma) - \lambda_{2n+1}(\gamma) \|_p \}$$
$$= k[\| \lambda_{2n-1}(\gamma) - \lambda_{2n}(\gamma) \|_p + \| \lambda_{2n}(\gamma) - \lambda_{2n+1}(\gamma) \|_p]$$

hence ,$\| \lambda_{2n}(\gamma) - \lambda_{2n+1}(\gamma) \|_p \leq \lambda \| \lambda_{2n-1}(\gamma) - \lambda_{2n}(\gamma) \|_p$
Where $= (k/1-k) < 1$.
By similar way , we have

$$\| \lambda_{2n-1}(\gamma) - \lambda_{2n}(\gamma) \|_p \leq \lambda \| \lambda_{2n-2}(\gamma) - \lambda_{2n-1}(\gamma) \|_p$$

therefore ,

$$\| \lambda_{2n}(\gamma) - \lambda_{2n+1}(\gamma) \|_p \leq \lambda \| \lambda_{2n-1}(\gamma) - \lambda_{2n}(\gamma) \|_p \leq \lambda^2 \| \lambda_{2n-2}(\gamma) - \lambda_{2n-1}(\gamma) \|_p$$

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$$\| \lambda_{2n}(\gamma) - \lambda_{2n+1}(\gamma) \|_p \leq \lambda^{2n} \| \lambda_0(\gamma) - \lambda_1(\gamma) \|_p .$$

To prove $< \lambda_n >$ is Cauchy sequence , for $n, m \in \mathbb{N}$, $n > m$

$$\| \lambda_n(\gamma) - \lambda_m(\gamma) \|_p \leq \| \lambda_n(\gamma) - \lambda_{n-1}(\gamma) \|_p + \| \lambda_{n-1}(\gamma) - \lambda_{n-2}(\gamma) \|_p + \dots + \| \lambda_{m+1}(\gamma) - \lambda_m(\gamma) \|_p$$
$$\leq (\lambda^{n-1} + \lambda^{n-2} + \dots + \lambda^m) \| \lambda_0(\gamma) - \lambda_1(\gamma) \|_p$$
$$\leq (\lambda^m / 1 - \lambda) \| \lambda_0(\gamma) - \lambda_1(\gamma) \|_p$$

Let $\epsilon > 0$ be given , choose a natural number K large enough such that
$\lambda^m \| \lambda_1(\gamma) - \lambda_0(\gamma) \|_p < \epsilon$for every $m \geq K$.
Hence $\| \lambda_n(\gamma) - \lambda_m(\gamma) \|_p < \epsilon$ for every $n > m \geq K$.

So, $\{\lambda_n(\gamma)\}$ is a Cauchy sequence in , and completeness of X implise that there exists $\lambda(\gamma) \in X$ such that $\lambda_n(\gamma) \rightarrow \lambda(\gamma)$ as $n \rightarrow \infty$.

To show that λ is a common random fixed point of $h$and G, coinsider the following by using triangle inequality, (2.1) and (1.2)

$$\|\lambda(\gamma) - h(\gamma, \lambda(\gamma))\|_p \leq \|\lambda(\gamma) - \lambda_{2n+2}(\gamma)\|_p + \|\lambda_{2n+2}(\gamma) - h(\gamma, \lambda(\gamma))\|_p$$
$$= \|\lambda(\gamma) - \lambda_{2n+2}(\gamma)\|_p + \|h(\gamma, \lambda(\gamma)) - G(\gamma, \lambda_{2n+1}(\gamma))\|_p$$
$$\leq \|\lambda(\gamma) - \lambda_{2n+2}(\gamma)\|_p + k \max\{ \|\lambda(\gamma) - \lambda_{2n+1}(\gamma)\|_p, \|\lambda(\gamma) - h(\gamma, \lambda(\gamma))\|_p, \|\lambda_{2n+1}(\gamma) - G(\gamma, \lambda_{2n+1}(\gamma))\|_p, \|\lambda_{2n+1}(\gamma) - h(\gamma, \lambda(\gamma))\|_p, \|\lambda_{2n+1}(\gamma) - h(\gamma, \lambda(\gamma))\|_p \}$$
$$= \|\lambda(\gamma) - \lambda_{2n+2}(\gamma)\|_p + k \max\{ \|\lambda(\gamma) - \lambda_{2n+1}(\gamma)\|_p, \|\lambda(\gamma) - h(\gamma, \lambda(\gamma))\|_p, \|\lambda_{2n+1}(\gamma) - \lambda_{2n+2}(\gamma)\|_p, \|\lambda_{2n+1}(\gamma) - h(\gamma, \lambda(\gamma))\|_p, \|\lambda_{2n+1}(\gamma) - h(\gamma, \lambda(\gamma))\|_p \}$$

taking the limit as $n \rightarrow \infty$ in the above inequality , getting that

$$\|\lambda(\gamma) - h(\gamma, \lambda(\gamma))\|_p \leq k \|\lambda(\gamma) - h(\gamma, \lambda(\gamma))\|_p$$

this implies that $(1-k)\|\lambda(\gamma) - h(\gamma, \lambda(\gamma))\|_p \leq 0$ ……………..(2.2)
since $0 \leq k < 1/2$, (2.2) must be true only $\|\lambda(\gamma) - h(\gamma, \lambda(\gamma))\|_p = 0$, thus $\lambda(\gamma) = h(\gamma, \lambda(\gamma))$ …………..(2.3)
Similarly , we can show that $\lambda(\gamma) = G(\gamma, \lambda(\gamma))$…………………..(2.4).
hence$\lambda : \Omega \longrightarrow A$ is a common random fixed point of h and G .
For uniqueness , let $\alpha(\gamma)$ be another common random fixed point of S and T , that is for all $\gamma \in \Omega$, $\alpha(\gamma) = h(\gamma, \alpha(\gamma)) = G(\gamma, \alpha(\gamma))$.
Then for all $\in \Omega$, we have

$$\|\lambda(\gamma) - \alpha(\gamma)\|_p = \|h(\gamma, \lambda(\gamma)) - G(\gamma, \alpha(\gamma))\|_p$$

From (2.3) ,(2.4) and (1.2) , we have

$$\|\lambda(\gamma) - \alpha(\gamma)\|_p \leq k \max\{\|\lambda(\gamma) - \alpha(\gamma)\|_p, \|\lambda(\gamma) - h(\gamma, \lambda(\gamma))\|_p, \|\alpha(\gamma) - G(\gamma, \alpha(\gamma))\|_p, \|\lambda(\gamma) - G(\gamma, \alpha(\gamma))\|_p\|_p, \|\alpha(\gamma) - h(\gamma, \lambda(\gamma))\|_p\}$$
$$= k \max\{\|\lambda(\gamma) - \alpha(\gamma)\|_p, 0\}$$
$$= k\|\lambda(\gamma) - \alpha(\gamma)\|_p$$
$$< \|\lambda(\gamma) - \alpha(\gamma)\|_p$$

Which is contraction . Hence $\lambda : \Omega \longrightarrow A$ is a unique common random fixed point of $h$and .■

Corollary (2.2):
If A and h as in theorem (2.1)and for each $\in \Omega$, $h(\gamma, .) : A \rightarrow A$is (gqc) :
Then there is a random fixed point of h .

Corollary (2.3):
If ,A , h , G as in theorem (2.1) and for each $\gamma \in \Omega$, $h(\gamma, .), G(\gamma, .) : A \rightarrow A$satisfies one of the following conditions :

1. $\| h(\gamma, x) - G(\gamma, y) \|_p \leq k \max\{ \| x - y \|_p, \| x - h(\gamma, x) \|_p, \| y - G(\gamma, y) \|_p \}$;
2. $\| h(\gamma, x) - G(\gamma, y) \|_p \leq k \max\{ \| x - h(\gamma, x) \|_p, \| y - G(\gamma, y) \|_p \}$.

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3. $\| h(\gamma,x) - G(\gamma,y) \|_p \leq k \max\{ \| x-y \|_p, \| x - h(\gamma,x) \|_p,$
$\| y - G(\gamma,x) \|_p, 1/2[\|x - G(\gamma,y)\|_p + \|y - h(\gamma,x)\|_p \}$
4. $\| h(\gamma,x) - G(\gamma,y) \|_p \leq k \max\{ \| x-y \|_p, 1/2[\|x - h(\gamma,x)\|_p + y - G(\gamma,y) \|_p], 1/2[\|x - G(\gamma,y)\|_p + \|y - h(\gamma,x)\|_p]\}$.
For all $x,y \in X; 0 < k < 1/2$ .Then h and G have a unique common random fixed point .

3. Well-Posed Problem

Definition (3.1):
Let $(X, \| \quad \|_p)$ be a p-normed space and $T: \Omega \times X \longrightarrow X$ a random mapping . the random fixed point problem of T is said to be well- posed if :
i.T has a unique random fixed point $\lambda : \Omega \longrightarrow X$;
ii. for any sequence $\{\lambda_n(\gamma)\}$ of measurable mappings in X such that $\lim_{n\to\infty} \|T(\gamma, \lambda_n(\gamma)) - \lambda_n(\gamma)\|_p = 0$, we have
$\lim_{n\to\infty} \|\lambda_n(\gamma) - \lambda(\gamma)\|_p = 0$.

Definition (3.2):
Let $(X, \| \quad \|_p)$ be a p-normed space and let $\mathcal{T}$ be a set of a random operators in X . The random fixed point of $\mathcal{T}$ is said to be well-posed if :
i.$\mathcal{T}$ has a unique random fixed point $\lambda : \Omega \longrightarrow X$;
ii. for any sequence $\{\lambda_n(\gamma)\}$ of measurable mappings in X such that $\lim_{n\to\infty} \|T(\gamma, \lambda_n(\gamma)) - \lambda_n(\gamma)\|_p = 0$, $\forall\, T \in \mathcal{T}$ we have
$\lim_{n\to\infty} \|\lambda_n(\gamma) - \lambda(\gamma)\|_p = 0$.

Theorem (3.3):
If A, h , G as in theorem (2.1) and for each $\gamma \in \Omega$, $h(\omega,.), G(\omega,.): A \to A$ satisfies (1.2) , then the common random fixed point for the set of random operators $\{h, G\}$ is well-posed .

Proof:
By theorem (2.1) , the random operators h and G have a unique common random fixed point $\lambda: \Omega \longrightarrow A$. Let $\{\lambda_n(\gamma)\}$ be a sequence of measurable mappings in A such that

$$\lim_{n\to\infty} \|h(\gamma, \lambda_n(\gamma)) - \lambda_n(\gamma)\|_p = \lim_{n\to\infty} \|G(\gamma, \lambda_n(\gamma)) - \lambda_n(\gamma)\|_p = 0$$

By the triangle inequality , (1.2) ,(2.3) and (2.4) , we have

$$\begin{aligned} \|\lambda(\gamma) - \lambda_n(\gamma)\|_p &\leq \|h(\gamma, \lambda(\gamma)) - G(\gamma, \lambda_n(\gamma))\|_p \\ &+ \|G(\gamma, \lambda_n(\gamma)) - \lambda_n(\gamma)\|_p \\ &\leq h\, max\{\|\lambda(\gamma) - \lambda_n(\gamma)\|_p, \|\lambda_n(\gamma) \\ &- G(\gamma, \lambda_n(\gamma))\|_p, \|\lambda(\gamma) \\ &- G(\gamma, \lambda_n(\gamma))\|_p, \|\lambda_n(\gamma) - h(\gamma, \lambda(\gamma))\|_p\} \\ &+ \|G(\gamma, \lambda_n(\gamma)) - \lambda_n(\gamma)\|_p \\ \leq h\left[\|\lambda(\gamma) - G(\gamma, \lambda_n(\gamma))\|_p + \|G(\gamma, \lambda_n(\gamma)) - \lambda_n(\gamma)\|_p\right] \\ &+ \|G(\gamma, \lambda_n(\gamma)) - \lambda_n(\gamma)\|_p \\ = h\|\lambda(\gamma) - G(\gamma, \lambda_n(\gamma))\|_p + (1 \\ &+ h)\|G(\gamma, \lambda_n(\gamma)) - \lambda_n(\gamma)\|_p \end{aligned}$$

By the triangle inequality , we get

$$\begin{aligned} \|\lambda(\gamma) - \lambda_n(\gamma)\|_p \leq h\Big[\|\lambda(\gamma) - \lambda_n(\gamma)\|_p \\ + \|\lambda_n(\gamma) - G(\gamma, \lambda_n(\gamma))\|_p\Big] \\ +(1+h)\|G(\gamma, \lambda_n(\gamma)) - \lambda_n(\gamma)\|_p \\ = h\|\lambda(\gamma) - \lambda_n(\gamma)\|_p + (1+2h)\|G(\gamma, \lambda_n(\gamma)) - \lambda_n(\gamma)\|_p \\ (1-h)\|\lambda(\gamma) - \lambda_n(\gamma)\|_p \\ \leq (1+2h)\|G(\gamma, \lambda_n(\gamma)) - \lambda_n(\gamma)\|_p \end{aligned}$$

thus we have , $\lim_{n\to\infty} \|\lambda(\gamma) - \lambda_n(\gamma)\|_p = 0$, it follows that the common random fixed point for the set of random operators $\{h, G\}$ is well-posed .■

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