International Journal of Science and Research (IJSR)
ISSN (Online): 2319-7064
Index Copernicus Value (2015): 78.96 | Impact Factor (2015): 6.391

Volatility of Option Pricing Model with Brown Geometric Motion Method

Mika Debora Br Barus, Hevlie Winda Nazry S, Sarina

Department of Mathematics, University of North Sumatera, Indonesia

Abstract: ***Basically, the option is defined as a contract between two parties (Writer and holder) in which the writer gives the right but not the obligation to holder to buy (call option) or sell (put option) a stock by the agreed price in the future. This will obviously lead loss for the writer. To avoid this, the writer must give the price of the option. Many researchers have discussed the nature of the moment on the model option pricing method Brownian motion. But the method of Brownian motion has weaknesses in modeling the movement of the price of options. Therefore, in the paper will discuss how to determine the nature of the moment on the option pricing model with the using Geometric Brownian Motion.***

Keywords : Geometric Brownian Motion, Option Price, Personality moment

1. Introduction

Collection of the option price S (y), $0 \leq y < \infty$ is said to follow the Brownian motion the drift parameter μ and variance σ^2 if for all non-negative values y and t, the random variable:

$$S(t + y) - S(y)$$

are independent for all grades until the time y, and also a variable normal random with mean μt and variance $\sigma^2 t$. Brownian motion model's weakness to model the movement of stock prices is theoretically be worth negative, and the price difference within a certain time period has a similar normal distribution for any initial value. Therefore, the model of the stock pricing movement more precisely follows Geometric Brownian Motion models that can be written,

$$dSt = St(\mu dt + \_dWt) \qquad (1)$$

with S random variables which states stock price, μ and σ^2 constant, t is time. Equation (1) is known as Geometric Brownian motion with μ is parameter drift and σ^2 is the volatility parameter. Changes in stock prices are known as a return.

2. Volatility of Option Pricing

Suppose given another option pricing model. Suppose given price option at time t is S (t), and h> 0 as in equation (2), then:

$$S(t+h) = S(t)\exp\left[\left(\mu - \frac{1}{2}\sigma^2\right)h + \sigma[z(t+h) - z(t)]\right], h \geq 0$$

So, it can be searched for conditional volatility:

(1) $Var[S(t+h)|S(t)]$:

$$\begin{aligned} Var[S(t+h)|S(t)] &= Var[S(t)exp[(\mu - \frac{1}{2}\sigma^2)h + \sigma[z(t+h) - z(t)]]|S(t)] \\ &= [S(t)]^2 exp[2(\mu - \frac{1}{2}\sigma^2)h] + Var[exp[\sigma[z(t+h) - z(t)]]|S(t)] \\ &= [S(t)]^2 exp[2(\mu - \frac{1}{2}\sigma^2)h] + Var[exp[\sigma z(h)]] \\ &= [S(t)]^2 exp[2(\mu - \frac{1}{2}\sigma^2)h] + e^{h\sigma^2}(e^{h\sigma^2} - 1) \end{aligned}$$

(2) $Var[S(t+dt)|S(t)]$:

$$\begin{aligned} Var[S(t+dt)|S(t)] &= Var[S(t+dt) - S(t)|S(t)] \\ &= Var[dS(t)|S(t)] \\ &= Var[\mu S(t)dt + \sigma S(t)dZ(t)|S(t)] \\ &= Var[\sigma S(t)dZ(t)|S(t)] \\ &= [\sigma S(t)]^2 Var[dZ(t)|S(t)] \\ &= [\sigma S(t)]^2 Var[Z(t+dt) - Z(t)|S(t)] \\ &= [\sigma S(t)]^2 Var[Z(dt)] \\ &= [\sigma S(t)]^2 dt \end{aligned}$$

(3) $Var[\frac{dS(t)}{S(t)}|Z(t)]$:

$$\begin{aligned} Var[\frac{dS(t)}{S(t)}|Z(t)] &= Var[\mu dt + \sigma dZ(t)|Z(t)] \\ &= Var[\sigma dZ(t)|Z(t)] \\ &= \sigma^2 Var[dZ(t)|Z(t)] \\ &= \sigma^2 Var[dZ(t)], (\because kenaikanbebas) \\ &= \sigma^2 dt \end{aligned}$$

$$\begin{aligned} Var[\frac{dS(t)}{S(t)}] &= Var[\mu dt + \sigma dZ(t)] \\ &= Var[\sigma dZ(t)] \\ &= \sigma^2 Var[dZ(t)] \\ &= \sigma^2 dt \end{aligned}$$

$$\therefore Var[\frac{dS(t)}{S(t)}|Z(t)] = Var[\frac{dS(t)}{S(t)}] = \sigma^2 dt$$

Generally, we can know the volatility of any conditional events and then we can get the decision about option pricing by Brownian Motion Method.

References

[1] Brewer, Kevin. D., et al., Geometric Brownian Motion, Option Pricing, and Simulation: Some Spreadsheet-Based Exercises in Financial Modeling, Spreadsheets in Education (eJSiE), Volume 5 — Issue 3, 2012.
[2] Hull, John C. Options,Futures, And Other Derivatives, 7th Edition. New Jersey: Pearson Education, 2009.