Simple Geometric Brownian Motion Based Pricing Model

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Abstract: ***This paper presents some Excel-based simulation exercises that are suitable for use in financial modeling courses. Such exercises are based on a stochastic process of stock price movements, called geometric Brownian motion. Guidance is provided in assigning appropriate values of the drift parameter in the stochastic process for such exercises. Some further simulation***

Keywords: Geometric Brownian Motion, Option Price, Daily Return

1. Introduction

Collection of the option price S (y), $0 \leq y < \infty$ is said to follow the Brownian motion the drift parameter μ and variance σ^2 if for all non-negative values y and t, the random variable:

$$S(t + y) - S(y)$$

are independent for all grades until the time y, and also a variable normal random with mean μt and variance $\sigma^2 t$. Brownian motion model's weakness to model the movement of stock prices is theoretically be worth negative, and the price difference within a certain time period has a similar normal distribution for any initial value. Therefore, the model of the stock pricing movement more precisely follows Geometric Brownian Motion models that can be written,

$$dSt = St(\mu dt + \_dWt) \qquad (1)$$

with S random variables which states stock price, μ and σ^2 constant, t is time. Equation (1) is known as Geometric Brownian motion with μ is parameter drift and σ^2 is the volatility parameter. Changes in stock prices are known as a return.

Brewer, et al (2012) explained that,

$$E(S(t)) = S(0)exp(\mu t)$$

$$Var(S(t)) = S(0)^2[exp(2\mu t)][exp(\sigma^2 t) - 1]$$
$$= [E(S(t))]^2[exp(\sigma^2 t) - 1]$$

2. Daily Stock and Option Price Movements

Although geometric Brownian motion is a stochastic process in continuous time, its implementation in simulation exercises requires that it be approximated in a discrete time setting. We assume for now that a day as a proportion of a year is short enough for such an approximation to work well. The issue as to whether there is any need for using a shorter time interval and, if so, how the Excel-based simulation exercises as described in the next section can be revised accordingly.

To simulate the time paths of daily stock and option prices, from the day of an option investment to the expiry date of an option, we need an explicit expression of the stock price on each day in terms of the stock price a day earlier. Such an expression is a recursive version. Specifically, if we use t and t + Δt; instead of 0 and T > 0; to indicate two successive points in time, it can be written as

$$S_{t+\Delta t} = S_t \exp\left[\left(\mu - \frac{\sigma^2}{2}\right)\Delta t + \sigma\epsilon\sqrt{\Delta t}\right].$$

Now, let n be the number of days in a year. Here, the number can be based on calendar days or trading days; however, the latter is more common in practice. The time interval Δt between two adjacent days is the proportion 1/n of a year. For notational convenience, let S_t and S_{t+1} be the stock prices on two adjacent days, for t = 0,1,2,… until the expiry date of the option that the stock underlies. Provided that μ and σ are stated in annual terms, we can write equation as

$$S_{t+1} = S_t \exp\left[\left(\mu - \frac{\sigma^2}{2}\right)\frac{1}{n} + \frac{\sigma}{\sqrt{n}}\epsilon\right].$$

For a given initial price S_0 and given constant values of μ and σ, the above equation will allow S1, S2, S3,… to be generated. The idea is to use that equation recursively, starting from day 0; for each day, we generate a new random draw of ϵ from the standardized normal distribution for the equation to simulate the stock price of the next day. These simulated daily stock prices, in turn, will allow the corresponding call and put option prices, C1, C2, C3,… and P1, P2, P3,… to be computed successively until the expiry date of each option.

Given the stochastic nature of price movements as characterized by geometric Brownian motion, each set of simulated time paths of stock and option prices will inevitably differ from any other set as generated repeatedly in simulation runs. From a statistical perspective, we are interested in knowing what simulated prices can be expected and how widely dispersed are such prices. We can compute the expected stock price and the standard deviation of stock prices, respectively, on each day until the expiry of the option that the stock underlies. With t being a day label, we simply substitute T on the right hand side of each of the two equations with t=n; for t = 1,2,… until the expiry date of the option; that is,

$$E(S_t) = S_0 \exp\left(\frac{\mu t}{n}\right)$$

$$\sqrt{Var(S_t)} = E(S_t)\sqrt{\exp\left(\frac{\sigma^2 t}{n}\right) - 1}.$$

Suppose that, for some given values of S_0, μ and σ, we have the results of a set of simulation runs. On each day t; the sample average of the simulated stock prices and the sample standard deviation of such prices can easily be computed with Excel.

3. Simulation

Uncertain Variables
The uncertain variables are in cells D16:D266. This model uses the PsiNormal distribution to compute the uncertainty in each time period (each day).

Statistical Functions
Cells H15:H266 contain the statistical function, PsiMean(). Each function is taking the mean of the predicted daily price (column F) over all 1000 trial values. For example, H16 shows the average of all 1000 trial values for cell F16.

Uncertain Functions
The uncertain functions are in cells F15:F266 and represent the predicted daily price for the stock.

Date	Close Price	Actual Daily Returns	Std. Normal Variables	Predicted Daily Price	Mean Predicted Daily Price
1/2/2009	20.33			20.33	#N/A
1/5/2009	20.52	0.009302393	1.165070689	20.55854739	#N/A
1/6/2009	20.76	0.011628038	-0.100179217	20.5429171	#N/A
1/7/2009	19.51	-0.062100904	0.660380863	20.67550912	#N/A
1/8/2009	20.12	0.030787191	-0.420639279	20.59693875	#N/A
1/9/2009	19.52	-0.030274764	-0.646318191	20.47457323	#N/A
1/12/2009	19.47	-0.002564762	0.373780368	20.55106007	#N/A
1/13/2009	19.82	0.01781671	-1.106483555	20.3392591	#N/A
1/14/2009	19.09	-0.037526891	-1.039568266	20.14255148	#N/A
1/15/2009	19.24	0.007826807	-0.926379861	19.96937345	#N/A
1/16/2009	19.71	0.024134676	0.335090737	20.03664406	#N/A
1/20/2009	18.48	-0.064437055	-0.299917666	19.98344675	#N/A
1/21/2009	19.38	0.04755254	0.165416628	20.01860083	#N/A
1/22/2009	17.11	-0.124578519	0.055221617	20.03289112	#N/A
1/23/2009	17.20	0.005246296	-0.460551333	19.94917823	#N/A
1/26/2009	17.63	0.024692613	-0.447528555	19.86827957	#N/A
1/27/2009	17.66	0.001700199	0.235018109	19.91634888	#N/A
1/28/2009	18.04	0.021289319	1.285419251	20.16298309	#N/A

Num Days	250	
Mean Daily Return	0.00019	0.001194
Daily Return Std. Dev.	0.009486	0.02773
Volatility	0.149987	
Appreciation Rate	0.058748	

4. Conclusion

In spite of the fact that the choice of the value of the drift parameter does depend on the subjective view of the investor involved, the choice should not be entirely arbitrary and unguided. As explained in this paper, neither should it simply be set equal to the risk-free interest rate, which tends to favour the writer when simulating the probability of an option investment. Using the idea that a rational individual never willingly chooses to invest for an expected loss, as well as various other ideas, this paper has provided some guidance in setting appropriate values of the drift parameter for simulation runs.

References

[1] Brewer, Kevin. D., et al., Geometric Brownian Motion, Option Pricing, and Simulation: Some Spreadsheet-Based Exercises in Financial Modeling, Spreadsheets in Education (eJSiE), Volume 5 — Issue 3, 2012.
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