

# Theoretical Overview of Basic Image Edge Detectors

Johnbosco I.E. Anosike<sup>1</sup>, Mariam Evarist<sup>2</sup>

<sup>1</sup>Department of Signal and Information Processing, School of Electronics Engineering, Tianjin University of Technology and Education, 1310 Dagunan Road, Hexi District, Tianjin, P. R. China

<sup>2</sup>Department of Signal and Information Processing, School of Electronics Engineering, Tianjin University of Technology and Education, 1310 Dagunan Road, Hexi District, Tianjin, P. R. China

**Abstract:** Through the application of edge detection procedures to an image, the amount of data to be processed can be considerably reduced by filtering out and discarding information that are of less significance, and retaining the essential structural properties of the image. There are several techniques for detecting and processing image edge. The choice of any method relies on need of the end application. In some cases more than one technique is required to meet a targeted image processing output. The purpose of this paper is to comparatively introduce the basic edge detection methodologies by making analysis of their respective concepts and observing their tradeoffs in some conditions so that decision making criteria are established for guidance in application developments.

**Keywords:** image operators, image gradient, Laplacian operators, noise, hysteresis

## 1. Introduction

Edges play quite an important role in many applications of image processing, in particular for machine vision systems that analyze scenes of man-made objects under controlled illumination conditions [1]. Edges constitute significant local changes in an image and are important features for analyzing images. For example, the boundary of an object usually produces step edges because the image intensity of the object is different from the image intensity of the background [2]. This intensity change are noticeable in the color variation characteristics of the pixels that make up the image. When presented in 2D, an edge detector identifies the edges by the pixels with a high gradient. A fast rate of change of intensity at some direction is given by the angle of the gradient vector which is usually observed at the edge pixels (See fig. 1 below. The circle in the image indicates the location of the pixel).

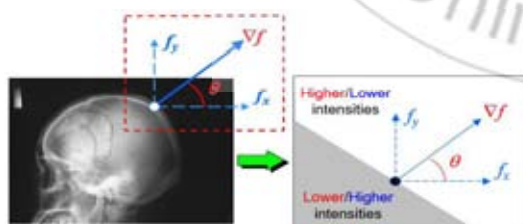


Figure 1: Image edge gradient and pixel.

### 1.1 The concept of Edge Detection

Edge detection is an elementary procedure used in image processing, basically for feature detection and extraction because it aims at detecting significant local changes in an image [2]. The purpose of edge detection is to significantly reduce the amount of data in an image while preserving the structural properties for further image processing. It is difficult to detect edges in a noisy image and this is because both real edge and noise can mix up their contents in high frequency resulting in unclear and distorted result. Therefore

edge detection is tasked with localizing these variations and to identify the physical phenomena which produce them. Hence, it is expected to be efficient and dependable because the validity and possibility of the completion of subsequent image processing stages rely on it.

Overtime, different edge detection algorithms methodologies or techniques, which are used interchangeably in this paper) where developed to facilitate identifying the edge of images for further processing. Their usage and performance however depends on the target and the quality of the image being processed. Evaluation of some images have shown that under noisy conditions Canny, Laplacian of Gaussian, Robert, Prewitt and Sobel methodologies exhibit better performance in this respective order [3].

- 1.2. Edge detection algorithms can be divided into four major stages [4]
  - 1.2.1. Smoothing: subdue all possible noise, while preserving the real edges.
  - 1.2.2. Enhancement or sharpening: improving the image edge quality through filtering.
  - 1.2.3. Detection: determine and select pixel for keeping or discarding as real or noise respectively.
  - 1.2.4. Localization or estimation: Edge thinning and linking are employed on the pixels in this stage to determine the edges.

In the course of this paper, equations may be repeated for emphasis. The rest of the paper covers the elaboration of the defined branches of the edge detection category tree illustrated in fig 2 below, followed by a **Conclusion** which is supported by an **annex**.

## 2. Edge Detecting Methodologies

These are in two categories with subdivision as illustrated in fig. 2 below

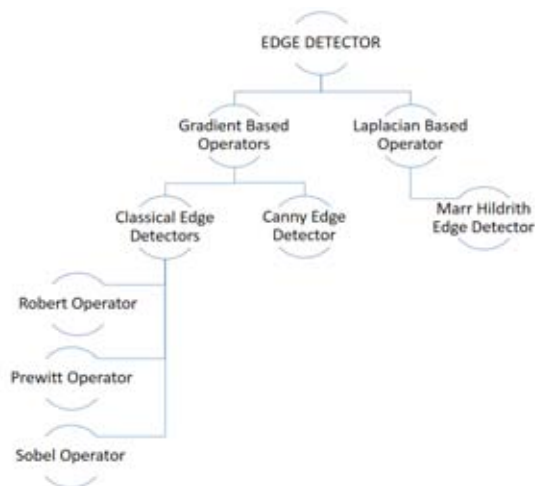


Figure 2: Edge detector category tree

## 2.1 Gradient based operators [7], [8]:

This method usually looks in the first derivative for the maximum and minimum in an image [5]. For ideal continuous image gradient estimate "Roberts, Sobel and Prewitt" operators can be used.

If edge point of a 2D image is given by  $f(x, y)$ , the gradient estimate operators smoothens it in one direction and differentiates it in the other. The maximum change of the contrast in a 2D picture function  $f(x, y)$  occurs along the direction of the gradient of the function (see fig. 1, above).

The image gradient is defined by the formula:

$$G = \nabla f(x, y) = x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} \quad (1)$$

The gradient magnitude (edge strength) is computed as  $|G| = (M_x^2 + M_y^2)^{1/2}$

$$|G| = \sqrt{M_x^2 + M_y^2} \quad (2)$$

Where;  $M_x \Rightarrow \frac{\partial f}{\partial x}$  Magnitude in x direction, and

$M_y \Rightarrow \frac{\partial f}{\partial y}$  Magnitude in y direction.

The Gradient direction (perpendicular to the edge direct orientation) is calculated as:

$$\theta = \arctan\left(\frac{M_y}{M_x}\right) \quad (3)$$

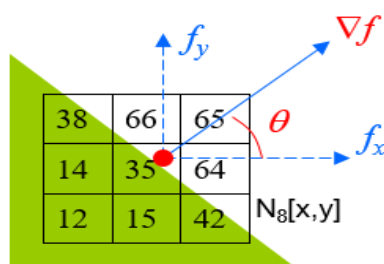


Figure 3: A practical sample image edge pixel.

### 2.1.1 Classical operators

The three operators under classical detectors Robert, Prewitt and Sobel. These classical operators have common characteristics in operation, one of which is the use of Mask

for estimating edge. They are easy to handle but differs respectively in values required for their executions. The one thing that disadvantages them is their nature of being highly sensitive to noise and hence may not produce sharp edges.

Main steps in edge detection using classical operators [4].

- Smooth the input image: This involves image reading and use of filter for convolution.
- Convolve the output image in x-axis with the choice operator's gradient mask.
- Convolve the output image in y-axis with the choice operator's gradient mask.
- Set a threshold (T).
- Compute the magnitude of the gradient.

$$\nabla f = \sqrt{M_x^2 + M_y^2} \approx |M_x| + |M_y| \quad (4)$$

- If magnitude ( $\nabla f$ ) > Threshold (T), then it is a possible edge point.

- Obtain the direction

$$\theta = \arctan\left(\frac{M_y}{M_x}\right) \quad (5)$$

### A. Robert Operator [5], [9], [10]

The Robert operator highlights by its performance a quick to compute areas of high frequency corresponding to 2D edge spatial gradient measurement. The input as well as the output of the operator is commonly a grayscale image. It computes using differentiation. First the summation between the squares of the difference between two adjacent pixels in diagonal position, and then, the image approximate gradient is calculated. Using the default kernels (Mask) of the operator, the input image is convolved and gradient magnitude and directions are computed.

$$M_x = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \quad M_y = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Figure 4: Masks used by Robert Operator

Practically, Robert masks (Fig 4) respond maximally to edges running at 45° to the grid of the pixel according to design. Each two perpendicular orientation is for one kernel; the masks  $M_x$  and  $M_y$  (Fig 4) are respectively overlaid on the image (Fig. 3) neighborhood  $N_8[x, y]$  so that each intensity  $N_{xy}$  can be multiplied by weight  $M_{xy}$ . The products obtained are summed apart and then finally, the Gradient magnitude ( $\nabla f$ ) is computed.

$$\frac{\partial f}{\partial x} \approx (M_x \circ N_8[x, y]) = f(x, y) - f(x+1, y+1) \quad (6)$$

$$\frac{\partial f}{\partial y} \approx (M_y \circ N_8[x, y]) = f(x+1, y) - f(x, y+1) \quad (7)$$

Combining  $M_x$  and  $M_y$ , the absolute magnitude of the gradient at each point and the orientation of that gradient can be obtained as

$$|G| = \sqrt{M_x^2 + M_y^2} \quad (8)$$

With an approximate magnitude as

$$|G| = |M_x| + |M_y| \quad (9)$$

This is much faster when approximating to the magnitude. The angle of orientation of the edge relative to the pixel grid orientation is defined by:

$$\theta = \arctan\left(\frac{M_y}{M_x}\right) - \frac{3\pi}{4} \quad (10)$$

Robert operator is renowned for being simple, very sensitive to noise and has small kernel. It is however not well compatible with today's technology.

### B. Prewitt Operator

Prewitt operator detects vertical and horizontal image edges [10]. It uses the following kernels:

$$M_x = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \quad M_y = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$$

**Figure 5:** Masks used by Prewitt Operator

Mathematically, it convolves its kernels with the original image to calculate approximations of the derivatives for horizontal and vertical changes. If we let **A** be a source image, the computation is given as Fig 6.

$$M_x \cdot A \quad M_y \cdot A$$

**Figure 6:** Prewitt Mask with convolution element

Relatively speaking, the approximation of the generated gradient can be crude, more in cases where there is high frequency variations in the image [12]. Prewitt is derived by assuming that white noise is additive and image surfaces are linear [11]. Compared to the next operator to be discussed Prewitt edge operator performs better. Because those pixels at the center of the masks are not given greater importance [7].

### C. Sobel Operator

The Sobel operator is one of the most commonly used edge detectors [2]. It is employed to determine the approximate edge strength at each point in an input grayscale image. The result shows how "abruptly" or "smoothly" at the point of analysis the image varies. The Sobel operator consists of a pair of 3x3 convolution kernels shown in fig. 7, one estimating the gradient in the x-direction (columns) and the other estimating the gradient in the y-direction (rows). Very similar to the Roberts Cross operator [12], One kernel is simply the other rotated by 90°.

$$M_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \quad M_y = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

**Figure 7:** Masks used by Sobel Operator

A 3x3 neighborhood matrix is used for the gradient calculations as a way to avoid having the gradient calculated about an interpolated point between pixels. This operator emphasizes on pixels that are closer to the center of the mask. The magnitude or edge strength of the gradient is approximated using the formula:

$$|G| = |M_x| + |M_y| \quad (11)$$

In practice, calculation of the likelihood of an edge is easier to interpret than that of the direction.

Sobel operator's computation ability is slower when compared to Robert operator; it has a large kernel and is less sensitive to noise.

### D. Canny Edge detector

This is one of the standard gradient based edge detectors, very popular (widely used in computer vision) and effective operator (Gaussian smoothing + Sobel). It detects edges by first separating noise from the image without disturbing the features of the edges in the image afterwards [5]. The Canny operator's first approach is smoothening the intensity of an image, then producing an extended contour segments by following high gradient magnitudes from one neighborhood to another [13]. Basing his analysis on "step-edges" corrupted by "additive Gaussian noise [4]", Canny has shown that the first derivative of the Gaussian closely approximates the operator that optimizes the product of signal-to-noise ratio and localization.

The basic objectives of Canny's approach is to obtain an algorithm that ensures optimality by following standards [9], [8]:

- 1) Low error rate in detection: This implies maximizing the signal-to-noise ratio. In other words the probability of detecting all and real edge points should be maximized while reducing significantly the possibility of detection non-edge points.
- 2) Localization of Edge points: The edges located must be as close as possible to the real edges. That is, the distances between a point marked as an edge by the detector and the center of the true edge should be minimum.
- 3) Edge point response: The detector should return only one point for each true edge point. That is, the number of local maxima around the true edge should be minimum. This means that the detector should not identify multiple edge pixels where only a single edge point exists. (A school of thought may argue the implicit presence of this in the first criterion).

Cannys mathematical formulation of these criteria is optimal for a class of edges known as step edges [5].

Based on these standard criteria, the following are steps in Canny's Algorithm [2] [4] [12]:

- i. Smooth the image to remove noise with a Gaussian filter:

Compute  $f_x$  and  $f_y$

$$f_x = \frac{\partial}{\partial x}(f * G) = \frac{\partial}{\partial x}G = f * G_x \quad (13)$$

$$f_y = \frac{\partial}{\partial y}(f * G) = \frac{\partial}{\partial y}G = f * G_y \quad (14)$$

$G(x, y)$  is the Gaussian function

$G_x(x, y)$  is the derivate of  $G(x, y)$  with respect to  $x$ :

$$G_x(x, y) = \frac{-x}{\sigma^2}(x, y) \quad (15) \quad G_y(x, y) \text{ is the derivate of } G(x, y)$$

with respect to  $y$ :

$$G_y(x, y) = \frac{-y}{\sigma^2}(x, y) \quad (16)$$

ii. Compute the gradient magnitude and orientation using finite-difference approximations for the partial derivatives:

$$\text{mag}(i, j) = \sqrt{f_x^2 + f_y^2} \quad (17)$$

The Sobel operator can be employed in this stage to perform a 2-D spatial gradient measurement on the image, in which case the edge strength of the gradient is then approximated using the formula:

$$|G| = |G_x| + |G_y| \quad (18)$$

where  $G_x$  and  $G_y$  are the gradients in the x and y directions respectively. Where the gradient of image has a big magnitude, the edge is marked. Gradient magnitudes often indicate the edges quite clearly. However, the edges are typically broad and thus do not indicate exactly where the edges are. To make it possible to determine this, the direction of the edges should be ascertained

$$\theta = \arctan\left(\frac{G_y}{G_x}\right) \quad (19)$$

iii. Apply non-maxima suppression to the gradient magnitude. Only local maxima should be marked as edge.

iv. Use the hysteresis threshold algorithm to detect and link edges. This will give a thin line in the output image. It makes use of both a high threshold ( $t_{\text{high}}$ ) and a low threshold ( $t_{\text{low}}$ ) to avoid the problem of streaking when a single threshold is used.

Conditions to detect pixel as edge given a pixel M (i, j) with G (gradient magnitude):

- If  $G < t_{\text{low}}$  then discard the edge.
- If  $G > t_{\text{high}}$  keep the edge
- Keep the edge if  $t_{\text{low}} < G < t_{\text{high}}$  and a gradient magnitude greater than  $t_{\text{high}}$  is recorded for any of its 3x3 region.
- If high gradient magnitudes is not recorded for any of the pixel (x, y)'s neighbors but at least any is found between  $t_{\text{low}}$  and  $t_{\text{high}}$  then the 5x5 region is checked for any of these pixels that have a magnitude greater than  $t_{\text{high}}$ . If found, keep the edge.
- Discard the edge if otherwise.

## 2.2 Laplacian based operator: Laplacian of Gaussian (LoG) or Marr Hildrith operator [3] [8] [14] [15]:

The Laplacian looks in the second derivative (the Laplacian  $\nabla^2$ ) for what is called zero crossing to find the edges in an image file [6]. It marks the image pixel as an edge where the second derivative is zero. For a 2D image  $f(x, y)$  the operator is defined by

$$\nabla^2 f = x \frac{\partial^2 f}{\partial x^2} + y \frac{\partial^2 f}{\partial y^2} \quad (20)$$

Before Canny proposed his algorithm, The Marr-Hildrith edge detector (LoG) had been a popularly used operator. But the edge point obtained are very sensitive to noise, therefore it combines both Gaussian and Laplacian operator to reduce the noise and detect the sharp edges respectively.

The Gaussian function is defined by the formula:

$$G(x, y) = \frac{1}{\sqrt{2\pi}\sigma^2} \exp\left(-\frac{x^2+y^2}{2\sigma^2}\right) \quad (21)$$

where  $\sigma$  is a standard deviation which determines the degree of smoothing and increment in mask size.

The Laplacian of Gaussian (LoG) operator is computed as:

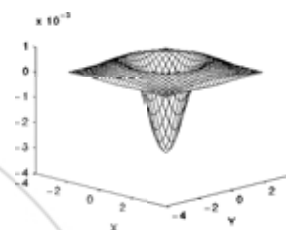
$$\begin{aligned} \text{LoG} &= \frac{\partial^2}{\partial x^2} G(x, y) + \frac{\partial^2}{\partial y^2} G(x, y) \\ &= \frac{x^2 + y^2 - 2\sigma^2}{\sigma^4} \exp\left(-\frac{x^2+y^2}{2\sigma^2}\right) \end{aligned} \quad (22)$$

❖ Fundamental steps of the LoG edge detector are

- Smooth the image by convolving it with Gaussian filter. This reduces the noise so that isolated noise points will be filtered out.
- Examine the second derivative. This is known as the enhancement step.

$$\nabla^2 f = x \frac{\partial^2 f}{\partial x^2} + y \frac{\partial^2 f}{\partial y^2}, \quad (23)$$

This Laplacian is rotation invariant, and owing to its shape, it is referred to as the “Mexican Hat operator”. See the fig. 8 below



**Figure 9:** The two dimension of LoG “Mexican Hat”

- Check for sign changes by looping through each pixel in the smoothed image's Laplacian. The pixel is marked as edge if a sign change is observed and the gradient across is greater than some threshold. As described in canny, the edge gradient change can be run through a hysteresis to give better connected edges [16].

The Marr–Hildreth operator is limited by false edge response, and sometimes severe localization error at curved edges. But in low signal to noise ratio, it is comparatively better than the classical operators. To its credit, it usually does produce a spotty and thick edges [16].

## 3. Conclusion

Since edge detection is a fundamental step in object recognition, this paper has presented in a survey manner the fundamental areas of interest towards understanding the functioning of various edge detection algorithms. Though many edge detectors have been developed, there is still no well-defined metric in selecting the appropriate edge detector for an application [2]. Undoubtedly however, this survey assists in easily deciding what technique best suits a particular application area. Classic operator have shown to be simple in usage but have also demonstrated how outdated it stands in the world of image processing, being in itself very sensitive to noise. In Marr-Hildreth, locality is not so good and the edges are usually not thin. Canny's method is preferred for being less sensitive to noise, adaptive in nature, resolving the problem of streaking, providing good localization and detecting sharper edges as compared to others. It is hence considered the most optimal technique that can be easily adopted to handle image edge detection problems. However, as a matter of emphasis, the detection needs of an application is mostly the deciding factor as to which detector to apply.

As a recommendation, we advise that algorithms be generated with designated software like MATLAB for practical understanding. A quick guide on the majorly observed tradeoffs of the selected detectors are tabulated in the annexed table (Annex 1).

## References

- [1] O. Vincent, O. Folorunso, "A descriptive algorithm for Sobel image edge detection" Proceedings of Informing Science & IT Education Conference (InSITE), 2009
- [2] R. Jain, R. Kasturi, B. Schunck, "Machine vision" Chapter 5. Edge Detection, pp. 140-185, 1995
- [3] E. Argyle. "Techniques for edge detection," Proc. IEEE, vol. 59, pp. 285-286, 1971
- [4] Trucco, Chapt 4 and Jain et al., Chapt 5, "Edge detection", unpublished
- [5] J. Canny, "A Computational Approach to Edge Detection," in IEEE Transactions on pattern Analysis and Machine Intelligence, vol. PAMI-8, no. 6, pp. 679-698, Nov. 1986.
- [6] L. Chen, "Laplacian embedded regression for scalable manifold regularization", Neural Networks and Learning Systems, IEEE Transactions, Volume: 23, pp. 902 – 915, June 2012.
- [7] G. Shrivakshan1, Dr. C. Chandrasekar, "A comparison of various edge detection techniques used in image processing", IJCSI International Journal of Computer Science Issues, Vol. 9, Issue 5, No 1, September 2012
- [8] R. Gonzalez and R. Woods, "Digital image processing", Third Edition pp. 706 – 723, Jan 2002
- [9] A. Jain, "Fundamentals of digital image processing", pp. 347-353, 1988
- [10] R. Maini and Dr. H. Aggarwal, "Study and comparison of various image edge detection techniques" International Journal of Image Processing (IJIP), Volume (3) : Issue (1), 2006
- [11] D. Ziou, S. Tabbone "Edge detection techniques - an overview", June 2000
- [12] N. Senthilkumaran and R. Rajesh, "Edge detection techniques for image segmentation - a survey", Proceedings of the International Conference on Managing Next Generation Software Applications (MNGSA-08), pp.749-760,2008
- [13] L. Shapiro, G. Stockman, "Computer Vision", Prentice Hill, 2001
- [14] Rashmi, M. Kumar, and R. Saxena "Algorithm and technique on various edge detection: a survey" Signal & Image Processing International Journal (SIPIJ) Vol.4, No.3, June 2013
- [15] E. Nadernejad, S. Sharifzadeh and H. Hassanpour "Edge detection techniques: evaluations and comparisons", Applied Mathematical Sciences, Vol. 2, no. 31, 1507 – 1520, 2008
- [16] S. Saini, B. Kasliwal, S. Bhatia, "Comparative study of image edge detection algorithms", unpublished

## Annex 1:

**Table1:** Summary of advantages and disadvantages deduced from the various surveyed detection methodologies:

Operator	Advantages	Disadvantages
Classical (Sobel, Prewitt, Robert)	<ul style="list-style-type: none"> <li>Simple to use</li> <li>Easily detects edges and their orientations</li> </ul>	<ul style="list-style-type: none"> <li>Very sensitivity to noise and so prone increased level of inaccuracy</li> <li>Not much compatible with today's technology</li> <li>Does not provide sharp edge</li> <li>Fixed kernel filter and coefficients size: cannot be easily adapted to any given image</li> <li>Costly because of need of an adaptive edge detection algorithm for a robust solution that is adaptable to the varying image noise.</li> </ul>
Laplacian of Gaussian (LoG) (Marr-Hildreth)	<ul style="list-style-type: none"> <li>Easily finds the right edge locations.</li> <li>Can examine a wider area around pixels</li> <li>Zero-crossings of LoG offer better localization than gradient based when the edges are not very sharp</li> </ul>	<ul style="list-style-type: none"> <li>Malfunctions at the corners, curves and where the gray level intensity function varies.</li> <li>Usage of the Laplacian filter limits finding the orientation of edge</li> <li>LoG operator can be highly sensitive to noise in comparison to canny operator</li> </ul>
Canny	<ul style="list-style-type: none"> <li>Provides edge gradient orientation which results into good localization.</li> <li>Less sensitive to Noise using Gaussian filter which removes noise at a great extent.</li> <li>Adopts hysteresis technique and so removes streaking problem caused by using a single threshold.</li> <li>Easily manipulative in nature to improve result since it depends on adjustable parameters like the standard deviation (<math>\sigma</math>) of Gaussian filter, the threshold <math>t_{low}</math> and <math>t_{high}</math>.</li> </ul>	<ul style="list-style-type: none"> <li>Computationally costly compared to Sobel, Prewitt and Robert's operator</li> <li>Complex Computations,</li> <li>False zero crossing tendencies</li> <li>Time consuming</li> </ul>

## Authors Profile



**Johnbosco I.E Anosike** holds a B.Sc. degree in Informatics Engineering from “Instituto Superior Politecnico, Jose Antonio Echeverria” Havana, Cuba 2011. He is presently a M.Sc. research scholar at School of Electronics Engineering, Tianjin University of Technology and Education (TUTE), China 2017. His current research focus is on Antenna Design and Fabrication at Microwave Technology establishment of TUTE. Other research interests are in machine vision and learning.



**Mariam Evarist** received B.Sc. in Electronics and Communications Engineering from St. Joseph University in Tanzania, 2013. Currently she is pursuing MSc in Signals and Information Processing at Tianjin University of technology and Education, Tianjin, China, with research interest on innovative memristive devices and thin film semiconductor design.

