International Journal of Science and Research (IJSR)
ISSN (Online): 2319-7064
Index Copernicus Value (2015): 78.96 | Impact Factor (2015): 6.391

Supra-Approximation Spaces Using Mixed Degree Systems in Graph Theory

Dr. Y. Y. Yousif[1], S. S. Obaid[2]

Department of Mathematics, Faculty of Education for Pure Science (Ibn Al-Haitham), Baghdad University, Baghdad-Iraq,

Abstract: ***This paper is concerned with introducing and studying the o-space by using out degree system (resp. i-space by using in degree system) which are the core concept in this paper. In addition, the m-lower approximations, the m-upper approximations and o-space and i-space. Furthermore, we introduce near supraopen (near supraclosed) d. g.'s. Finally, the supra-lower approximation, supra-upper approximation, supra-accuracy are defined and some of its properties are investigated.***

Keywords: *o*-space, *i*-space, supra-approximation space, near supra-approximation space, *m*-lower approximation and *m*-upper approximation

2000 Mathematics Subject Classification: 04A05, 54A05, 05C20

1. Introduction and Preliminaries

We study the approximation spaces and use some of near open d. g.'s to introduce new definitions and levels of the upper and lower approximations and this leading to many new results. Also, in light of these results we reach to different levels of the boundary regions. In addition we reach to a several levels of accuracy for the d. g. 's. We study some of the results in [2], [13], [17] and [18]. Furthermore, we gave new generalization of some definitions in d. g. 's to the some known definitions in topology which are near open d. g. 's. We built on some of the results in [10], [11], [12] and [13]. Many works have appeared recently, for example in structural analysis [19].

A direct graph or d. g. [16] is pair $D = (V(D), E(D))$ where $V(D)$ is a non-empty set (called vertex set) and $E(D)$ of ordered pairs of elements of $V(D)$ (called edge set). An edge of the from (ϖ, ϖ) is called a loop. If $\varpi \in V(D)$, the out-degree of ϖ is $|\{u \in V(D) : (\varpi, u) \in E(D)\}|$ and in-degree of ϖ is $|\{u \in V(D) : (u, \varpi) \in E(D)\}|$. The out-degree set of ϖ is denoted by OD and defined by: $OD = \{u \in V(D) : (\varpi, u) \in E(D)\}$ and the in-degree set of ϖ is denoted by ID and defined by: $ID = \{u \in V(D) : (u, \varpi) \in E(D)\}$. The out-degree system (resp. in-degree system) of a vertex $\varpi \in V(D)$ is denoted by $ODS(\varpi)$ (resp. $IDS(\varpi)$) and defined by $ODS(\varpi) = \{OD\}$ (resp. $IDS(\varpi) = \{ID\}$). A d. g. is symmetric if $(\varpi, u) \in E(D)$ implies $(u, \varpi) \in E(D)$. A subd. g. of a d. g. D is a d. g. each of whose vertices belong to $V(D)$ and each of whose edges belong to $E(D)$. A subfamily μ of X is said to supratopology [8] on X if (i) $X, \phi \in \mu$ (ii)if $A_i \in \forall i \in j$ then $\cup A_i \in \mu$. (X, μ) is called supratopology space.

A subset A of a topological space (X, τ) is called
(a) Regular open [15] (briefly R-open) if $A = Int(Cl(A))$,
(b) Semi-open [6] (briefly S-open) if $A \subseteq Cl(Int(A))$,
(c) Pre-open [7] (briefly p-open) if $A \subseteq Int(Cl((A))$,
(d) γ-open [5] (briefly b-open [3]) if $A \subseteq Cl(Int(A)) \cup Int(Cl(A))$,
(e) α-open [9] if $A \subseteq Int(Cl(Int(A))$ and
(f) β-open [1] (= semi-pre-open [4]) if $A \subseteq Cl(Int(Cl(A))$.

2. *m*-Lower and *m*-Upper Approximations

In this section, we introduce two topological spaces, namely *o*-space and *i*-space. The open d. g. in the first one is called *o*-open and in the other one is called *i*-open. We defined *o*-interior, *i*-interior, *o*-closure and *i*-closure. Hence we define the lower approximations in generalized rough set theory by using *m*-interior (resp. *o*-interior and *i*-interior). Also, the upper approximations are defined by using *m*-closure (resp. *o*-closure and *i*-closure). Furthermore, a theorem is introduced which proves that the approximations based on (*m*-interior and *m*-closure) are accurate more than the approximations based on either (*o*-interior and *o*-closure) or (*i*-interior and *i*-closure) or both of them. By using both of them, we are defining the upper approximation of$H$ by intersection of *o*-closure of H and *i*-closure of H and, on the other hand, the lower approximation of H by union of *o*-interior of H and *i*-interior of H is defined.

Definition 2.1 Let $D = (V(D), E(D))$ be a d. g. and suppose that $\xi_o : V(D) \longrightarrow P(P(V(D)))$ (resp. $\xi_i : V(D) \longrightarrow P(P(V(D))))$ is a mapping which assigns for each ϖ in $V(D)$ it's out (resp. in) degree system in $P(P(V(D)))$. The pair (D, ξ_o) (resp. (D, ξ_i)) is called an *o*-space (resp. *i*-space).

Remark 2.2 If D is a symmetric d. g., then $D = D^{-1}$ and so $\xi_o = \xi_i = \xi_m$. Hence the *o*-space, *i*-space and *m*-space are identical.

Definition 2.3 Let (D, ξ_o) be an *o*-space and (D, ξ_i) be an *i*-space and let $H \subseteq D$. Then

(a) The *o*-derived and *i*-derived of a d. g.H are defined respectively by:

$$[V(H)]_o = \{\varpi \in V(D) \,;\, OD(\varpi), OD(\varpi) \cap (V(H) - \{\varpi\}) \neq \phi\},$$
$$[V(H)]_i = \{\varpi \in V(D) \,;\, ID(\varpi), ID(\varpi) \cap (V(H) - \{\varpi\}) \neq \phi\},$$

(b) The classes of *o*-closed and *i*-closed respectively by:

$$F_{\xi_o} = \{H \subseteq D; [V(H)]_o \subseteq V(H)\},$$
$$F_{\xi_i} = \{H \subseteq D; [V(H)]_i \subseteq V(H)\},$$

(c) The classes of *o*-open and *i*-open respectively by:

$$\Omega_{\xi_o} = \{O \subseteq D; V(O) = V(D) - V(H) \text{ such that } V(H) \in F_{\xi_o}\},$$

$\Omega_{\xi_i} = \{O\subseteq D;\ V(O) = V(D) - V(H)$ such that $V(H)\in F_{\xi i}\}$,

(d) The o-interior and i-interior of a d. g.H are defined respectively by:

$$Int_o(V(H)) = \cup\ \{V(O)\in\Omega_{\xi_o}\ ;\ V(O)\subseteq V(H)\},$$
$$Int_i(V(H)) = \cup\ \{V(O)\in\Omega_{\xi_i}\ ;\ V(O)\subseteq V(H)\},$$

(e) The o-closure and i-closure of a d. g.H are defined respectively by:

$$Cl_o(V(H)) = \cap\ \{V(K)\in F_{\xi o};\ V(H)\subseteq V(K)\},$$
$$Cl_i(V(H)) = \cap\ \{V(K)\in F_{\xi i};\ V(H)\subseteq V(K)\},$$

(f) The o-boundary and i-boundary of a d. g.H are defined respectively by:

$$[V(H)]_o^b = Cl_o(V(H)) - Int_o(V(H)),$$
$$[V(H)]_i^b = Cl_i(V(H)) - Int_i(V(H)).$$

Theorem 2.4 If $(D,\ \xi_o)$ (resp. $(D,\ \zeta_i)$) is an o-space (resp. i-space) and $H\subseteq D$, then H is an o-open (resp. i-open) if and only if it contains the out degree (resp. in degree) of each of its vertices.

Proof. The proof is similar to the proof of Theorem (4.7) in [19].

Definition 2.5 Let $D= (V(D),\ E(D))$ be an approximation space and $H\subseteq D$. Then H is called out composed (resp. in composed) if H contains the out degree (resp. in degree) of each of its vertices i.e. for each $\varpi\in V(H)$, $OD(\varpi)\subseteq V(H)$ (resp. for each $\varpi\in V(H)$, $ID(\varpi)\subseteq V(H)$).

Definition 2.6 Let $D= (V(D),\ E(D))$ be an approximation space, then the class of all out composed (resp. in composed) d. g. 's are denoted by T_o (resp. T_i) and defined by:

$T_o = \{H\subseteq D$; for each $\varpi\in V(H),\ OD(\varpi)\subseteq V(H)\}$
$T_i = \{H\subseteq D$; for each $\varpi\in V(H),\ ID(\varpi)\subseteq V(H)\}$.

Proposition 2.7 Let $D= (V(D),\ E(D))$ be an approximation space, then T_o (resp. T_i) forms a topology on D.

Theorem 2.8 Let $D = (V(D),\ E(D))$ be an approximation space, then T_o is the complement topology of T_i and vice versa.

Proposition 2.9 Let $D = (V(D),\ E(D))$ be an approximation space, then $\Omega_{\xi_o} = T_o$ and $\Omega_{\xi_i} = T_i$.

Proof. The proof is immediately follows from Theorem (2.4), Definition (2.5) and Definition(2.6).

Remark 2.10 An immediate consequence of Proposition (2.9), Theorem (2.8) and Proposition (2.7) we have Ω_{ξ_o} and Ω_{ξ_i} form topologies on D. Furthermore, Ω_{ξ_o} is the complement topology on Ω_{ξ_i} and vice versa.

Definition 2.11 Let $D = (V(D),\ E(D))$ be an approximation space and Ω_{ξ_o}, Ω_{ξ_i} and Ω_{ξ_m} be the supratopologies induced by $D$and let $H\subseteq D$. Then

(a) The o-lower (resp.o-upper) approximations of H are defined respectively by:
$L_o(V(H)) = Int_o(V(H)),U_o(V(H)) = Cl_o(V(H))$,

(b) The i-lower (resp.i-upper) approximations of H are defined respectively by:
$L_i(V(H)) = Int_i(V(H)),U_i(V(H)) = Cl_i(V(H))$,

(c) The m-lower (resp.m-upper) approximations of H are defined respectively by:
$L_m(V(H)) = Int_m(V(H)),U_m(V(H)) = Cl_m(V(H))$.

Definition 2.12 Let $D = (V(D),\ E(D))$ be an approximation space, Ω_{ξ_o}, Ω_{ξ_i} and Ω_{ξ_m} be the supratopologies induced by D and let $H\subseteq D$. Then

(a) The o-boundary (resp.o-positive and o-negative) regions of H are defined respectively by:

$$Bd_o(V(H)) = U_o(V(H)) - L_o(V(H)),POS_o(V(H)) = L_o(V(H)),$$
$$NEG_o(V(H)) = V(D) - U_o(V(H)),$$

(b) The i-boundary (resp.i-positive and i-negative) regions of H are defined respectively by:

$$Bd_i(V(H)) = U_i(V(H)) - L_i(V(H)),POS_i(V(H)) = L_i(V(H)),NEG_i(V(H)) = V(D) - U_i(V(H)).$$

(c) The m-boundary (resp. m-positive and m-negative) regions of H are defined respectively by:

$$Bd_m(V(H)) = U_m(V(H)) - L_m(V(H)),POS_m(V(H)) = L_m(V(H)),$$
$$NEG_m(V(H)) = V(D) - U_m(V(H)).$$

Definition 2.13 Let $D= (V(D),\ E(D))$ be an approximation space. The accuracy of the approximation of $H\subseteq D$ using (ξ_o, ξ_i and ξ_m) are defined respectively by:

$$\eta_o(V(H)) = 1 - \frac{|Bd_o(V(H))|}{|V(D)|},\eta_i(V(H)) = 1 - \frac{|Bd_i(V(H))|}{|V(D)|},\eta_m(V(H)) = 1 - \frac{|Bd_m(V(H))|}{|V(D)|}.$$

It is obvious that $0 \leq \eta_o(V(H))\leq 1$, $0 \leq \eta_i(V(H))\leq 1$ and $0 \leq \eta_m(V(H))\leq 1$. Moreover, if $\eta_o(V(H)) = 1$ or $\eta_i(V(H)) = 1$ or $\eta_m(V(H)) = 1$ then H is called H-definable (H-exact) d. g. Otherwise, it is called H-rough.

Example 2.14 Let $D = (V(D), E(D))$: $V(D) = \{\varpi_1,\ \varpi_2,\ \varpi_3,\ \varpi_4,\ \varpi_5\}$, $E(D) = \{(\varpi_1,\ \varpi_2),\ (\varpi_1,\ \varpi_4),\ (\varpi_2,\varpi_2),\ (\varpi_2,\ \varpi_3),\ (\varpi_2,\ \varpi_4),\ (\varpi_4,\ \varpi_5),\ (\varpi_4,\ \varpi_3),\ (\varpi_5,\ \varpi_2),\ (\varpi_5,\ \varpi_5)\}$.

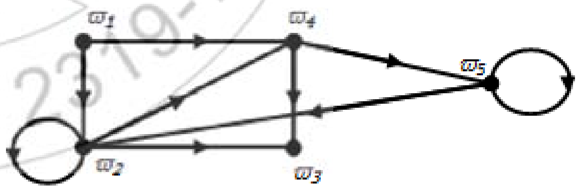


Figure 2.1: d. g. D given in Example 2.14.

$\xi_m(\varpi_1) = \{\{\varpi_2,\ \varpi_4\},\ \phi\}$, $\xi_m(\varpi_2) = \{\{\varpi_2,\ \varpi_3,\ \varpi_4\},\ \{\varpi_1,\ \varpi_2,\ \varpi_5\}\}$, $\xi_m(\varpi_3) = \{\phi,\ \{\varpi_2,\ \varpi_4\}\}$, $\xi_m(\varpi_4) = \{\{\varpi_3,\ \varpi_5\},\ \{\varpi_1,\ \varpi_2\}\}$, $\xi_m(\varpi_5) = \{\{\varpi_2,\ \varpi_5\},\ \{\varpi_4,\ \varpi_5\}\}$.

$\Omega_{\xi o} = \{V(D),\ \phi,\ \{\varpi_3\},\ \{\varpi_2,\varpi_3,\ \varpi_4,\ \varpi_5\}\}$,$\Omega_{\xi i} = \{V(D),\ \phi,\ \{\varpi_1\},\ \{\varpi_1,\varpi_2,\ \varpi_4,\ \varpi_5\}\}$ and $\Omega_{\xi m} = \{V(D),\ \phi,\ \{\varpi_1\},\ \{\varpi_3\},\ \{\varpi_1,\varpi_3\},\ \{\varpi_1,\varpi_2,\ \varpi_5\},\ \{\varpi_3,\ \varpi_4,\ \varpi_5\},\ \{\varpi_1,\varpi_2,\ \varpi_3,\ \varpi_4\},\ \{\varpi_1,\varpi_2,\ \varpi_3,\ \varpi_5\},\ \{\varpi_1,\varpi_2,\ \varpi_4,\ \varpi_5\},\ \{\varpi_1,\varpi_3,\ \varpi_4,\ \varpi_5\},\ \{\varpi_2,\varpi_3,\ \varpi_4,\ \varpi_5\}\}$

We can get the following four tables

Table 2.1: $L_o(V(H))$, $L_i(V(H))$ and $L_m(V(H))$ for all $H\subseteq D$.

$V(H)$	$L_o(V(H))$	$L_i(V(H))$	$L_m(V(H))$
$\{\varpi_1\}$	ϕ	$\{\varpi_1\}$	$\{\varpi_1\}$
$\{\varpi_2\}$	ϕ	ϕ	ϕ

$\{\varpi_3\}$	$\{\varpi_3\}$	ϕ	$\{\varpi_3\}$
$\{\varpi_4\}$	ϕ	ϕ	ϕ
$\{\varpi_5\}$	ϕ	ϕ	ϕ
$\{\varpi_1, \varpi_2\}$	ϕ	$\{\varpi_1\}$	$\{\varpi_1\}$
$\{\varpi_1, \varpi_3\}$	$\{\varpi_3\}$	$\{\varpi_1\}$	$\{\varpi_1, \varpi_3\}$
$\{\varpi_1, \varpi_4\}$	ϕ	$\{\varpi_1\}$	$\{\varpi_1\}$
$\{\varpi_1, \varpi_5\}$	ϕ	$\{\varpi_1\}$	$\{\varpi_1\}$
$\{\varpi_2, \varpi_3\}$	$\{\varpi_3\}$	ϕ	$\{\varpi_3\}$
$\{\varpi_2, \varpi_4\}$	ϕ	ϕ	ϕ
$\{\varpi_2, \varpi_5\}$	ϕ	ϕ	ϕ
$\{\varpi_3, \varpi_4\}$	$\{\varpi_3\}$	ϕ	$\{\varpi_3\}$
$\{\varpi_3, \varpi_5\}$	$\{\varpi_3\}$	ϕ	$\{\varpi_3\}$
$\{\varpi_4, \varpi_5\}$	ϕ	ϕ	ϕ
$\{\varpi_1, \varpi_2, \varpi_3\}$	$\{\varpi_3\}$	$\{\varpi_1\}$	$\{\varpi_1, \varpi_3\}$
$\{\varpi_1, \varpi_2, \varpi_4\}$	ϕ	$\{\varpi_1\}$	$\{\varpi_1\}$
$\{\varpi_1, \varpi_2, \varpi_5\}$	ϕ	$\{\varpi_1\}$	$\{\varpi_1, \varpi_2, \varpi_5\}$
$\{\varpi_1, \varpi_3, \varpi_4\}$	$\{\varpi_3\}$	$\{\varpi_1\}$	$\{\varpi_1, \varpi_3\}$
$\{\varpi_1, \varpi_3, \varpi_5\}$	$\{\varpi_3\}$	$\{\varpi_1\}$	$\{\varpi_1, \varpi_3\}$
$\{\varpi_1, \varpi_4, \varpi_5\}$	ϕ	$\{\varpi_1\}$	$\{\varpi_1\}$
$\{\varpi_2, \varpi_3, \varpi_4\}$	$\{\varpi_3\}$	ϕ	$\{\varpi_3\}$
$\{\varpi_2, \varpi_3, \varpi_5\}$	$\{\varpi_3\}$	ϕ	$\{\varpi_3\}$
$\{\varpi_2, \varpi_4, \varpi_5\}$	ϕ	ϕ	ϕ
$\{\varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_3\}$	ϕ	$\{\varpi_3, \varpi_4, \varpi_5\}$
$\{\varpi_1, \varpi_2, \varpi_3, \varpi_4\}$	$\{\varpi_3\}$	$\{\varpi_1\}$	$\{\varpi_1, \varpi_2, \varpi_3, \varpi_4\}$
$\{\varpi_1, \varpi_2, \varpi_3, \varpi_5\}$	$\{\varpi_3\}$	$\{\varpi_1\}$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$
$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	ϕ	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$
$\{\varpi_1, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_3\}$	$\{\varpi_1\}$	$\{\varpi_1, \varpi_3, \varpi_4, \varpi_5\}$
$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_1, \varpi_3, \varpi_4, \varpi_5\}$	ϕ	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$
$V(D)$	$V(D)$	$V(D)$	$V(D)$
ϕ	ϕ	ϕ	ϕ

Table 2.2: $U_o(V(H))$, $U_i(V(H))$ and $U_m(V(H))$ for all $H \subseteq D$.

$V(H)$	$U_o(V(H))$	$U_i(V(H))$	$U_m(V(H))$
$\{\varpi_1\}$	$\{\varpi_1\}$	$V(D)$	$\{\varpi_1\}$
$\{\varpi_2\}$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_2\}$
$\{\varpi_3\}$	$V(D)$	$\{\varpi_3\}$	$\{\varpi_3\}$
$\{\varpi_4\}$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_4\}$
$\{\varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_5\}$
$\{\varpi_1, \varpi_2\}$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	$V(D)$	$\{\varpi_1, \varpi_2\}$
$\{\varpi_1, \varpi_3\}$	$V(D)$	$V(D)$	$V(D)$
$\{\varpi_1, \varpi_4\}$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	$V(D)$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$
$\{\varpi_1, \varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	$V(D)$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$
$\{\varpi_2, \varpi_3\}$	$V(D)$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$
$\{\varpi_2, \varpi_4\}$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_4, \varpi_5\}$
$\{\varpi_2, \varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_4, \varpi_5\}$
$\{\varpi_3, \varpi_4\}$	$V(D)$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_3, \varpi_4\}$
$\{\varpi_3, \varpi_5\}$	$V(D)$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$
$\{\varpi_4, \varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_4, \varpi_5\}$

$\{\varpi_1, \varpi_2, \varpi_3\}$	$V(D)$	$V(D)$	$V(D)$
$\{\varpi_1, \varpi_2, \varpi_4\}$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	$V(D)$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$
$\{\varpi_1, \varpi_2, \varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	$V(D)$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$
$\{\varpi_1, \varpi_3, \varpi_4\}$	$V(D)$	$V(D)$	$V(D)$
$\{\varpi_1, \varpi_3, \varpi_5\}$	$V(D)$	$V(D)$	$V(D)$
$\{\varpi_1, \varpi_4, \varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	$V(D)$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$
$\{\varpi_2, \varpi_3, \varpi_4\}$	$V(D)$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$
$\{\varpi_2, \varpi_3, \varpi_5\}$	$V(D)$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$
$\{\varpi_2, \varpi_4, \varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_4, \varpi_5\}$
$\{\varpi_3, \varpi_4, \varpi_5\}$	$V(D)$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$
$\{\varpi_1, \varpi_2, \varpi_3, \varpi_4\}$	$V(D)$	$V(D)$	$V(D)$
$\{\varpi_1, \varpi_2, \varpi_3, \varpi_5\}$	$V(D)$	$V(D)$	$V(D)$
$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	$V(D)$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$
$\{\varpi_1, \varpi_3, \varpi_4, \varpi_5\}$	$V(D)$	$V(D)$	$V(D)$
$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$	$V(D)$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$
$V(D)$	$V(D)$	$V(D)$	$V(D)$
ϕ	ϕ	ϕ	ϕ

Table 2.3: $B_o(V(H))$, $B_i(V(H))$ and $B_m(V(H))$ for all $H \subseteq D$.

$V(H)$	$B_o(V(H))$	$B_i(V(H))$	$B_m(V(H))$
$\{\varpi_1\}$	$\{\varpi_1\}$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$	ϕ
$\{\varpi_2\}$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_2\}$
$\{\varpi_3\}$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	$\{\varpi_3\}$	ϕ
$\{\varpi_4\}$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_4\}$
$\{\varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_5\}$
$\{\varpi_1, \varpi_2\}$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_2\}$
$\{\varpi_1, \varpi_3\}$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_4, \varpi_5\}$
$\{\varpi_1, \varpi_4\}$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_4, \varpi_5\}$
$\{\varpi_1, \varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_4, \varpi_5\}$
$\{\varpi_2, \varpi_3\}$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_4, \varpi_5\}$
$\{\varpi_2, \varpi_4\}$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_4, \varpi_5\}$
$\{\varpi_2, \varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_4, \varpi_5\}$
$\{\varpi_3, \varpi_4\}$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_4\}$
$\{\varpi_3, \varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_4, \varpi_5\}$
$\{\varpi_4, \varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_4, \varpi_5\}$
$\{\varpi_1, \varpi_2, \varpi_3\}$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_4, \varpi_5\}$
$\{\varpi_1, \varpi_2, \varpi_4\}$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_4, \varpi_5\}$
$\{\varpi_1, \varpi_2, \varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_4\}$
$\{\varpi_1, \varpi_3, \varpi_4\}$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_4, \varpi_5\}$
$\{\varpi_1, \varpi_3, \varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_4, \varpi_5\}$
$\{\varpi_1, \varpi_4, \varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_4, \varpi_5\}$
$\{\varpi_2, \varpi_3, \varpi_4\}$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_4, \varpi_5\}$
$\{\varpi_2, \varpi_3, \varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_4, \varpi_5\}$
$\{\varpi_2, \varpi_4, \varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_4, \varpi_5\}$
$\{\varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_2\}$
$\{\varpi_1, \varpi_2, \varpi_3, \varpi_4\}$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_5\}$
$\{\varpi_1, \varpi_2, \varpi_3, \varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_4\}$
$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	$\{\varpi_3\}$	ϕ
$\{\varpi_1, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_2\}$
$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_1\}$	$V(D)$	ϕ
$V(D)$	ϕ	ϕ	ϕ
ϕ	ϕ	ϕ	ϕ

Paper ID: ART2017834 DOI: 10.21275/ART2017834

Table 2.4: $\eta_o(V(H))$, $\eta_i(V(H))$ and $\eta_m(V(H))$ for all $H \subseteq D$.

$V(H)$	$\eta_o(V(H))$	$\eta_i(V(H))$	$\eta_m(V(H))$
$\{\varpi_1\}$	4/5	1/5	1
$\{\varpi_2\}$	1/5	1/5	4/5
$\{\varpi_3\}$	1/5	4/5	1
$\{\varpi_4\}$	1/5	1/5	4/5
$\{\varpi_5\}$	1/5	1/5	4/5
$\{\varpi_1, \varpi_2\}$	1/5	1/5	4/5
$\{\varpi_1, \varpi_3\}$	1/5	1/5	2/5
$\{\varpi_1, \varpi_4\}$	1/5	1/5	2/5
$\{\varpi_1, \varpi_5\}$	1/5	1/5	2/5
$\{\varpi_2, \varpi_3\}$	1/5	1/5	2/5
$\{\varpi_2, \varpi_4\}$	1/5	1/5	2/5
$\{\varpi_2, \varpi_5\}$	1/5	1/5	2/5
$\{\varpi_3, \varpi_4\}$	1/5	1/5	4/5
$\{\varpi_3, \varpi_5\}$	1/5	1/5	2/5
$\{\varpi_4, \varpi_5\}$	1/5	1/5	2/5
$\{\varpi_1, \varpi_2, \varpi_3\}$	1/5	1/5	2/5
$\{\varpi_1, \varpi_2, \varpi_4\}$	1/5	1/5	2/5
$\{\varpi_1, \varpi_2, \varpi_5\}$	1/5	1/5	4/5
$\{\varpi_1, \varpi_3, \varpi_4\}$	1/5	1/5	2/5
$\{\varpi_1, \varpi_3, \varpi_5\}$	1/5	1/5	2/5
$\{\varpi_1, \varpi_4, \varpi_5\}$	1/5	1/5	2/5
$\{\varpi_2, \varpi_3, \varpi_4\}$	1/5	1/5	2/5
$\{\varpi_2, \varpi_3, \varpi_5\}$	1/5	1/5	2/5
$\{\varpi_2, \varpi_4, \varpi_5\}$	1/5	1/5	2/5
$\{\varpi_3, \varpi_4, \varpi_5\}$	1/5	1/5	4/5
$\{\varpi_1, \varpi_2, \varpi_3, \varpi_4\}$	1/5	1/5	4/5
$\{\varpi_1, \varpi_2, \varpi_3, \varpi_5\}$	1/5	1/5	4/5
$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	1/5	4/5	1
$\{\varpi_1, \varpi_3, \varpi_4, \varpi_5\}$	1/5	1/5	4/5
$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$	4/5	0	1
$V(D)$	1	1	1
ϕ	1	1	1

Proposition 2.15 Let $D = (V(D), E(D))$ be an approximation space and $F_{\xi o}$, $F_{\xi i}$ and $F_{\xi m}$ be the classes of o-closed, i-closed and m-closed graphs induced by D. Then any o-closed (or i-closed) d. g. is m-closed.

Proof. Let $H \subseteq D$ be an o-closed d. g., then $[V(H)]'_o \subseteq V(H)$. Since $[V(H)]'_o = \{\varpi \in V(D) : OD(\varpi) \cap (V(H) - \{\varpi\}) \neq \phi\}$ and $[V(H)]'_m = \{\varpi \in V(D)$: for all $MD(\varpi)$, $MD(\varpi) \cap (V(H) - \{\varpi\}) \neq \phi\} = \{\varpi \in V(D) : OD(\varpi) \cap (V(H) - \{\varpi\}) \neq \phi$ and $ID(\varpi) \cap (V(H) - \{\varpi\}) \neq \phi\}$.

Consequently, we have $[V(H)]'_m \subseteq [V(H)]'_o$ and so $[V(H)]'_m \subseteq V(H)$ which implies H is m-closed. Therefore any o-closed d. g. is m-closed. Similarly, we can prove that any i-closed is m-closed.

Proposition 2.16 Let $D = (V(D), E(D))$ be an approximation space and Ω_{ξ_o}, Ω_{ξ_i} and Ω_{ξ_m} be the supratopologies induced by D. Then any o-open (or i-open) d. g. is m-open.

Proof. Let $O \subseteq D$ be o-open d. g. and $F = D - O$. So F is o-closed d. g. and by using Proposition (2.15), F is m-closed. Hence $O = D - F$ is m-open. Accordingly, any o-open d. g. is m-open. By the same manner we can prove that any i-open d. g. is m-open.

Proposition 2.17 Let $D = (V(D), E(D))$ be an approximation space and $H \subseteq D$. Then

(a) $L_m(V(H)) \supseteq L_o(V(H)) \cup L_i(V(H))$,
(b) $U_m(V(H)) \subseteq U_o(V(H)) \cap U_i(V(H))$,
(c) $B_m(V(H)) \subseteq B_o(V(H)) \cap B_i(V(H))$,
(d) $\eta_m(V(H)) \geq \max\{\eta_o(V(H)), \eta_i(V(H))\}$.

Proof

(a) Since $L_o(V(H)) = \cup \{V(O) \in \Omega_{\xi_o} ; V(O) \subseteq V(H)\}$. Hence $L_o(V(H)) \subseteq V(H)$ and $L_o(V(H))$ is o-open since the union of any family of o-open d. g. 's is o-open. Because $L_i(V(H)) = \cup \{V(N) \in \Omega_{\xi_i} ; V(N) \subseteq V(H)\}$. So $L_i(V(H)) \subseteq V(H)$ and $L_i(V(H))$ is i-open since the union of any family of i-open d. g. 's is i-open. Because $L_o(V(H))$ is o-open, then by Proposition (2.16), it is m-open and since $L_i(V(H))$ is i-open then, by Proposition (2.16), it is also m-open. Hence $L_o(V(H)) \cup L_i(V(H))$ is m-open and $L_o(V(H)) \cup L_i(V(H)) \subseteq V(H)$. But $L_m(V(H)) = \cup \{V(M) \in \Omega_{\xi_m} ; V(M) \subseteq V(H)\}$. Consequently, $L_o(V(H)) \cup L_i(V(H)) \subseteq L_m(V(H))$.

(b) Since $U_o(V(H)) = \cap \{V(F) \in F_{\xi o}; V(H) \subseteq V(F)\}$. Hence $V(H) \subseteq U_o(V(H))$ and $U_o(V(H))$ is o-closed since the intersection of any family of o-closed d. g. 's is o-closed. Because $U_i(V(H)) = \cap \{V(Z) \in F_{\xi i}; V(H) \subseteq V(Z)\}$. Thus $V(H) \subseteq U_i(V(H))$ and $U_i(V(H))$ is i-closed since the intersection of any family of i-closed d. g. 's is i-closed. Because $U_o(V(H))$ is o-closed then, by Proposition (2.15), it is m-closed and since $U_i(V(H))$ is i-closed then, by Proposition(2.15), it is also m-closed. Hence $U_o(V(H)) \cap U_i(V(H))$ is m-closed and $V(H) \subseteq U_o(V(H)) \cap U_i(V(H))$. But $U_m(V(H)) = \cap \{V(K) \in F_{\xi m}; V(H) \subseteq V(K)\}$. According $U_m(V(H)) \subseteq U_o(V(H)) \cap U_i(V(H))$.

(c) Let $\varpi \in B_m(V(H))$, then $\varpi \in (U_m(V(H)) - L_m(V(H)))$ and so $\varpi \in U_m(V(H)) \wedge \varpi \notin L_m(V(H))$. Since $U_m(V(H)) \subseteq U_o(V(H)) \cap U_i(V(H))$ and $L_m(V(H)) \supseteq L_o(V(H)) \cup L_i(V(H))$. Then $\varpi \in (U_o(V(H)) \cap U_i(V(H))) \wedge \varpi \notin (L_o(V(H)) \cup L_i(V(H)))$, this imply $(\varpi \in U_o(V(H)) \wedge \varpi \in U_i(V(H))) \wedge (\varpi \notin L_o(V(H)) \wedge \varpi \notin L_i(V(H)))$, this imply $(\varpi \in U_o(V(H)) \wedge \varpi \notin L_o(V(H))) \wedge (\varpi \in U_i(V(H)) \wedge \varpi \notin L_i(V(H)))$, this imply $(\varpi \in U_o(V(H)) - L_o(V(H))) \wedge (\varpi \in U_i(V(H)) - L_i(V(H)))$, this imply $\varpi \in B_o(V(H)) \wedge \varpi \in B_i(V(H)))$, this imply $\varpi \in (B_o(V(H)) \cap B_i(V(H)))$. Therefore $B_m(V(H)) \subseteq B_o(V(H)) \cap B_i(V(H))$.

(d) By using (c) the proof is obvious.

Remark 2.18 Let $D = (V(D), E(D))$ be an approximation space and $H \subseteq D$. Then the following statements are not necessarily true.

(a) $L_m(V(H)) = L_o(V(H)) \cup L_i(V(H))$,
(b) $U_m(V(H)) = U_o(V(H)) \cap U_i(V(H))$,
(c) $B_m(V(H)) = B_o(V(H)) \cap B_i(V(H))$ and
(d) $\eta_m(V(H)) = \max\{\eta_o(V(H)), \eta_i(V(H))\}$.

The next example shows pervious remark.

Example 2.19 According to Example (2.14),

(a) Let $V(H) = (V(H), E(H))$: $V(H) = \{\varpi_1, \varpi_2, \varpi_5\}$, $E(H) = \{(\varpi_1, \varpi_2), (\varpi_2, \varpi_1), (\varpi_2, \varpi_5), (\varpi_5, \varpi_1)\}$. Then $L_m(V(H))$

$= \{\varpi_1, \varpi_2, \varpi_5\}$, $L_o(V(H)) = \phi$ and $L_i(V(H)) = \{\varpi_1\}$, such that $L_o(V(H)) \cup L_i(V(H)) = \{\varpi_1\}$ and so $L_m(V(H)) \neq L_o(V(H)) \cup L_i(V(H))$.

(b) Let $V(H) = (V(H), E(H))$: $V(H) = \{\varpi_2\}$, $E(H) = \phi$. Then $U_m(V(H)) = \{\varpi_2\}$, $U_o(V(H)) = \{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$ and $U_i(V(H)) = \{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$, such that $U_o(V(H)) \cap U_i(V(H)) = \{\varpi_2, \varpi_4, \varpi_5\}$ and so $U_m(V(H)) \neq U_o(V(H)) \cap U_i(V(H))$.

(c) Let $V(H) = (V(H), E(H))$: $V(H) = \{\varpi_2, \varpi_4, \varpi_5\}$, $E(H) = \{(\varpi_2, \varpi_5), (\varpi_4, \varpi_5)\}$. Then $B_m(V(H)) = \{\varpi_4\}$, $B_o(V(H)) = \{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$ and $B_i(V(H)) = \{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$, such that $B_o(V(H)) \cap B_i(V(H)) = \{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$ and so $B_m(V(H)) \neq B_o(V(H)) \cap B_i(V(H))$.

(d) Let $V(H) = (V(H), E(H))$: $V(H) = \{\varpi_3\}$, $E(H) = \phi$. Then $\eta_m(V(H)) = 1$, $\eta_o(V(H)) = 1/5$ and $\eta_i(V(H)) = 4/5$, such that $\max\{\eta_o(V(H)), \eta_i(V(H))\} = 4/5$ and so $\eta_m(V(H)) \neq \max\{\eta_o(V(H)), \eta_i(V(H))\}$.

3. Near Supraopend. g. 's

In this section, we introduce some new notions, definitions and propositions about near supraopend. g. 's, near supraclosedd. g. 's, near supraboundaryd. g. 's, near supraclosure operators and near suprainterior operators. These concepts are very essential to develop the theoretical foundations of rough set theory. By similar way of definitions of regular open set [15], semi-open set [6], pre-open set [7], γ-open set [5] (=b-open set [3]), α-open set [9] and β-open set [1](=semi-pre-open set [4]), we shall introduce the following definitions.

Definition 3.1 Let (D, Ω) be a supratopology space.$H \subseteq D$ is called

(a) Regular supraopen (briefly r-supraopen) if $V(H) = Int_u(Cl_u(V(H)))$,

(b) α-supraopen if $V(H) \subseteq Int_u(Cl_u(Int_u(V(H))))$,

(c) Semi-supraopen (briefly s-supraopen) if $V(H) \subseteq Cl_u(Int_u(V(H)))$,

(d) Pre-supraopen (briefly p-supraopen) if $V(H) \subseteq Int_u(Cl_u((V(H)))$,

(e) γ-supraopen (briefly b-supraopen) if $V(H) \subseteq Cl_u(Int_u(V(H))) \cup Int_u(Cl_u(V(H)))$ and

(f) β-supraopen (briefly semi-pre-supraopen) if $V(H) \subseteq Cl_u(Int_u(Cl_u(V(H))))$.

Definition 3.2 Let (D, Ω) be a supratopological space. The complement of an r-supraopen (resp. α-supraopen, s-supraopen, p-supraopen, γ-supraopen and β-supraopen) is said to be r-supraclosed (resp. α-supraclosed, s-supraclosed, p-supraclosed, γ-supraclosed and β-supraclosed).

Remark 3.3 Let (D, Ω) be a supratopological space, then

(a) The family of all supracloesdd. g. 's is denoted by F.

(b) The family of all r-supraopen (resp. α-supraopen, s-supraopen,p-supraopen, γ-supraopen and β-supraopen) d. g. 's are denoted by $R\Omega$ (resp. $\alpha\Omega$, $S\Omega$, $P\Omega$, $\gamma\Omega$ and $\beta\Omega$).

(c) The family of all r-supraclosed (resp. α-supraclosed, s-supraclosed,p-supraclosed, γ-supraclosed and β-supraclosed) d. g. 's are denoted by RF (resp. αF, SF, PF, γF and βF).

Example 3.4 Let $D = (V(D), E(D))$: $V(D) = \{\varpi_1, \varpi_2, \varpi_3, \varpi_4, \varpi_5\}$, $E(D) = \{(\varpi_1, \varpi_2), (\varpi_1, \varpi_3), (\varpi_2, \varpi_1), (\varpi_3, \varpi_4), (\varpi_4, \varpi_1), (\varpi_4, \varpi_3)\}$

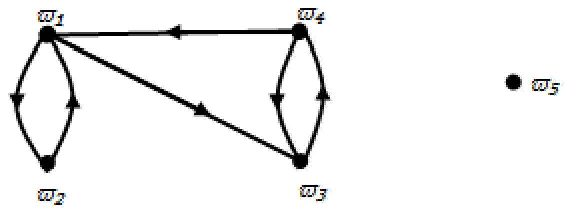


Figure 3.1: d. g. D given in Example 3.4

$\xi_m(\varpi_1) = \{\{\varpi_2, \varpi_3\}, \{\varpi_2, \varpi_4\}\}$, $\xi_m(\varpi_2) = \{\{\varpi_1\}\}$, $\xi_m(\varpi_3) = \{\{\varpi_4\}, \{\varpi_1, \varpi_4\}\}$, $\xi_m(\varpi_4) = \{\{\varpi_1, \varpi_3\}, \{\varpi_3\}\}$ and $\xi_m(\varpi_5) = \{\phi\}$.

$\Omega_{\xi m} = \{V(D), \phi, \{\varpi_5\}, \{\varpi_3, \varpi_4\}, \{\varpi_3, \varpi_4, \varpi_5\}, \{\varpi_1, \varpi_2, \varpi_3, \varpi_4\}\}$,

$R\Omega_{\xi m} = \{V(D), \phi, \{\varpi_5\}, \{\varpi_1, \varpi_2, \varpi_3, \varpi_4\}\}$,

$\alpha\Omega_{\xi m} = \{V(D), \phi, \{\varpi_5\}, \{\varpi_3, \varpi_4\}, \{\varpi_1, \varpi_3, \varpi_4\}, \{\varpi_2, \varpi_3, \varpi_4\}, \{\varpi_3, \varpi_4, \varpi_5\}, \{\varpi_1, \varpi_2, \varpi_3, \varpi_4\}, \{\varpi_1, \varpi_3, \varpi_4, \varpi_5\}, \{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}\}$and

$P\Omega_{\xi m} = \{V(D), \phi, \{\varpi_3\}, \{\varpi_4\}, \{\varpi_5\}, \{\varpi_1, \varpi_3\}, \{\varpi_1, \varpi_4\}, \{\varpi_2, \varpi_3\}, \{\varpi_2, \varpi_4\}, \{\varpi_3, \varpi_4\}, \{\varpi_3, \varpi_5\}, \{\varpi_4, \varpi_5\}, \{\varpi_1, \varpi_2, \varpi_3\}, \{\varpi_1, \varpi_2, \varpi_4\}, \{\varpi_1, \varpi_3, \varpi_4\}, \{\varpi_1, \varpi_3, \varpi_5\}, \{\varpi_1, \varpi_4, \varpi_5\}, \{\varpi_2, \varpi_3, \varpi_4\}, \{\varpi_2, \varpi_3, \varpi_5\}, \{\varpi_2, \varpi_4, \varpi_5\}, \{\varpi_3, \varpi_4, \varpi_5\}, \{\varpi_1, \varpi_2, \varpi_3, \varpi_4\}, \{\varpi_1, \varpi_2, \varpi_3, \varpi_5\}, \{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}, \{\varpi_1, \varpi_3, \varpi_4, \varpi_5\}, \{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}\}$.

Also, the classes of near supraclosedd. g. 's are given as follows:

$F_{\xi m} = \{V(D), \phi, \{\varpi_5\}, \{\varpi_1, \varpi_2\}, \{\varpi_1, \varpi_2, \varpi_5\}, \{\varpi_1, \varpi_2, \varpi_3, \varpi_4\}\}$,

$RF_{\xi m} = \{V(D), \phi, \{\varpi_5\}, \{\varpi_1, \varpi_3, \varpi_4, \varpi_5\}\}$,

$\alpha F_{\xi m} = \{V(D), \phi, \{\varpi_1\}, \{\varpi_2\}, \{\varpi_5\}, \{\varpi_1, \varpi_2\}, \{\varpi_1, \varpi_5\}, \{\varpi_2, \varpi_5\}, \{\varpi_1, \varpi_2, \varpi_5\}, \{\varpi_1, \varpi_2, \varpi_3, \varpi_4\}\}$ and

$PF_{\xi m} = \{V(D), \phi, \{\varpi_1\}, \{\varpi_2\}, \{\varpi_3\}, \{\varpi_4\}, \{\varpi_5\}, \{\varpi_1, \varpi_2\}, \{\varpi_1, \varpi_3\}, \{\varpi_1, \varpi_4\}, \{\varpi_1, \varpi_5\}, \{\varpi_2, \varpi_3\}, \{\varpi_2, \varpi_4\}, \{\varpi_2, \varpi_5\}, \{\varpi_3, \varpi_5\}, \{\varpi_4, \varpi_5\}, \{\varpi_1, \varpi_2, \varpi_3\}, \{\varpi_1, \varpi_2, \varpi_4\}, \{\varpi_1, \varpi_2, \varpi_5\}, \{\varpi_1, \varpi_3, \varpi_5\}, \{\varpi_1, \varpi_4, \varpi_5\}, \{\varpi_2, \varpi_3, \varpi_5\}, \{\varpi_2, \varpi_4, \varpi_5\}, \{\varpi_1, \varpi_2, \varpi_3, \varpi_4\}, \{\varpi_1, \varpi_2, \varpi_3, \varpi_5\}, \{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}\}$.

Example 3.5 Let $D = (V(D), E(D))$: $V(D) = \{\varpi_1, \varpi_2, \varpi_3, \varpi_4, \varpi_5\}$, $E(D) = \{(\varpi_1, \varpi_2), (\varpi_2, \varpi_1), (\varpi_2, \varpi_5), (\varpi_3, \varpi_4), (\varpi_4, \varpi_3), (\varpi_4, \varpi_5), (\varpi_5, \varpi_1), (\varpi_5, \varpi_3)\}$

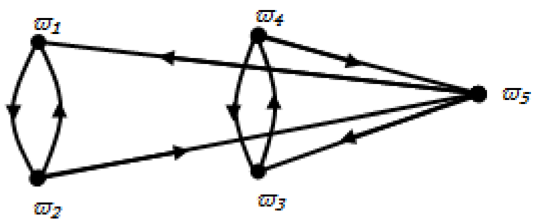


Figure 3.2: d. g. D given in Example 3.5.

$\xi_m(\varpi_1) = \{\{\varpi_2\}, \{\varpi_2, \varpi_5\}\}$, $\xi_m(\varpi_2) = \{\{\varpi_1, \varpi_5\}, \{\varpi_1\}\}$, $\xi_m(\varpi_3) = \{\{\varpi_4\}, \{\varpi_4, \varpi_5\}\}$, $\xi_m(\varpi_4) = \{\{\varpi_3, \varpi_5\}, \{\varpi_3\}\}$ and $\xi_m(\varpi_5) = \{\{\varpi_3, \varpi_1\}, \{\varpi_4, \varpi_2\}\}$.

$\Omega_{\xi m} = \{V(D), \phi, \{\varpi_1, \varpi_2\}, \{\varpi_3, \varpi_4\}, \{\varpi_1, \varpi_2, \varpi_3, \varpi_4\}\}$,

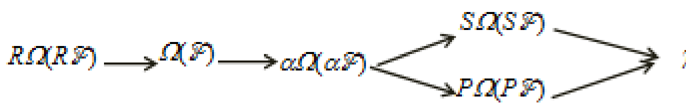
$R\Omega_{\xi m} = \{V(D), \phi, \{\varpi_1, \varpi_2\}, \{\varpi_3, \varpi_4\}\}$,

Paper ID: ART2017834 DOI: 10.21275/ART2017834

$\alpha\Omega_{\xi m} = \{V(D), \phi, \{\varpi_1, \varpi_2\}, \{\varpi_3, \varpi_4\}, \{\varpi_1, \varpi_2, \varpi_3, \varpi_4\}\}$,
$S\Omega_{\xi m} = \{V(D), \phi, \{\varpi_1, \varpi_2\}, \{\varpi_3, \varpi_4\}, \{\varpi_1, \varpi_2, \varpi_5\}, \{\varpi_3, \varpi_4, \varpi_5\}, \{\varpi_1, \varpi_2, \varpi_3, \varpi_4\}\}$,
$p\Omega_{\xi m} = \{V(D), \phi, \{\varpi_1\}, \{\varpi_2\}, \{\varpi_3\}, \{\varpi_4\}, \{\varpi_1, \varpi_2\}, \{\varpi_1, \varpi_3\}, \{\varpi_1, \varpi_4\}, \{\varpi_2, \varpi_3\}, \{\varpi_2, \varpi_4\}, \{\varpi_3, \varpi_4\}, \{\varpi_1, \varpi_2, \varpi_3\}, \{\varpi_1, \varpi_2, \varpi_4\}, \{\varpi_1, \varpi_3, \varpi_4\}, \{\varpi_1, \varpi_3, \varpi_5\}, \{\varpi_1, \varpi_4, \varpi_5\}, \{\varpi_2, \varpi_3, \varpi_4\}, \{\varpi_2, \varpi_3, \varpi_5\}, \{\varpi_2, \varpi_4, \varpi_5\}, \{\varpi_1, \varpi_2, \varpi_3, \varpi_4\}, \{\varpi_1, \varpi_2, \varpi_3, \varpi_5\}, \{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}, \{\varpi_1, \varpi_3, \varpi_4, \varpi_5\}, \{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}\}$,
$\gamma\Omega_{\xi m} = \{V(D), \phi, \{\varpi_1\}, \{\varpi_2\}, \{\varpi_3\}, \{\varpi_4\}, \{\varpi_1, \varpi_2\}, \{\varpi_1, \varpi_3\}, \{\varpi_1, \varpi_4\}, \{\varpi_2, \varpi_3\}, \{\varpi_2, \varpi_4\}, \{\varpi_3, \varpi_4\}, \{\varpi_1, \varpi_2, \varpi_3\}, \{\varpi_1, \varpi_2, \varpi_4\}, \{\varpi_1, \varpi_2, \varpi_5\}, \{\varpi_1, \varpi_3, \varpi_4\}, \{\varpi_1, \varpi_3, \varpi_5\}, \{\varpi_1, \varpi_4, \varpi_5\}, \{\varpi_2, \varpi_3, \varpi_4\}, \{\varpi_2, \varpi_3, \varpi_5\}, \{\varpi_2, \varpi_4, \varpi_5\}, \{\varpi_3, \varpi_4, \varpi_5\}, \{\varpi_1, \varpi_2, \varpi_3, \varpi_4\}, \{\varpi_1, \varpi_2, \varpi_3, \varpi_5\}, \{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}, \{\varpi_1, \varpi_3, \varpi_4, \varpi_5\}, \{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}\}$ and
$\beta\Omega_{\xi m} = \{V(D), \phi, \{\varpi_1\}, \{\varpi_2\}, \{\varpi_3\}, \{\varpi_4\}, \{\varpi_1, \varpi_2\}, \{\varpi_1, \varpi_3\}, \{\varpi_1, \varpi_4\}, \{\varpi_1, \varpi_5\}, \{\varpi_2, \varpi_3\}, \{\varpi_2, \varpi_4\}, \{\varpi_2, \varpi_5\}, \{\varpi_3, \varpi_4\}, \{\varpi_3, \varpi_5\}, \{\varpi_4, \varpi_5\}, \{\varpi_1, \varpi_2, \varpi_3\}, \{\varpi_1, \varpi_2, \varpi_4\}, \{\varpi_1, \varpi_2, \varpi_5\}, \{\varpi_1, \varpi_3, \varpi_4\}, \{\varpi_1, \varpi_3, \varpi_5\}, \{\varpi_1, \varpi_4, \varpi_5\}, \{\varpi_2, \varpi_3, \varpi_4\}, \{\varpi_2, \varpi_3, \varpi_5\}, \{\varpi_2, \varpi_4, \varpi_5\}, \{\varpi_3, \varpi_4, \varpi_5\}, \{\varpi_1, \varpi_2, \varpi_3, \varpi_4\}, \{\varpi_1, \varpi_2, \varpi_3, \varpi_5\}, \{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}, \{\varpi_1, \varpi_3, \varpi_4, \varpi_5\}, \{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}\}$.

Also, the classes of near supraclosedd. g. 's are given as follows:
$F_{\xi m} = \{V(D), \phi, \{\varpi_1, \varpi_2, \varpi_5\}, \{\varpi_3, \varpi_4, \varpi_5\}\}$,
$RF_{\xi m} = \{V(D), \phi, \{\varpi_1, \varpi_2, \varpi_5\}, \{\varpi_3, \varpi_4, \varpi_5\}\}$,
$\alpha F_{\xi m} = \{V(D), \phi, \{\varpi_5\}, \{\varpi_1, \varpi_2, \varpi_5\}, \{\varpi_3, \varpi_4, \varpi_5\}\}$,
$SF_{\xi m} = \{V(D), \phi, \{\varpi_5\}, \{\varpi_1, \varpi_2\}, \{\varpi_3, \varpi_4\}, \{\varpi_1, \varpi_2, \varpi_5\}, \{\varpi_3, \varpi_4, \varpi_5\}\}$,
$PF_{\xi m} = \{V(D), \phi, \{\varpi_1\}, \{\varpi_2\}, \{\varpi_3\}, \{\varpi_4\}, \{\varpi_5\}, \{\varpi_1, \varpi_3\}, \{\varpi_1, \varpi_4\}, \{\varpi_1, \varpi_5\}, \{\varpi_2, \varpi_3\}, \{\varpi_2, \varpi_4\}, \{\varpi_2, \varpi_5\}, \{\varpi_3, \varpi_5\}, \{\varpi_4, \varpi_5\}, \{\varpi_1, \varpi_2, \varpi_5\}, \{\varpi_1, \varpi_3, \varpi_5\}, \{\varpi_1, \varpi_4, \varpi_5\}, \{\varpi_2, \varpi_3, \varpi_5\}, \{\varpi_2, \varpi_4, \varpi_5\}, \{\varpi_3, \varpi_4, \varpi_5\}, \{\varpi_1, \varpi_2, \varpi_3, \varpi_5\}, \{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}, \{\varpi_1, \varpi_3, \varpi_4, \varpi_5\}, \{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}\}$,
$\gamma F_{\xi m} = \{V(D), \phi, \{\varpi_1\}, \{\varpi_2\}, \{\varpi_3\}, \{\varpi_4\}, \{\varpi_5\}, \{\varpi_1, \varpi_2\}, \{\varpi_1, \varpi_3\}, \{\varpi_1, \varpi_4\}, \{\varpi_1, \varpi_5\}, \{\varpi_2, \varpi_3\}, \{\varpi_2, \varpi_4\}, \{\varpi_2, \varpi_5\}, \{\varpi_3, \varpi_4\}, \{\varpi_3, \varpi_5\}, \{\varpi_4, \varpi_5\}, \{\varpi_1, \varpi_2, \varpi_5\}, \{\varpi_1, \varpi_3, \varpi_5\}, \{\varpi_1, \varpi_4, \varpi_5\}, \{\varpi_2, \varpi_3, \varpi_5\}, \{\varpi_2, \varpi_4, \varpi_5\}, \{\varpi_3, \varpi_4, \varpi_5\}, \{\varpi_1, \varpi_2, \varpi_3, \varpi_5\}, \{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}, \{\varpi_1, \varpi_3, \varpi_4, \varpi_5\}, \{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}\}$ and
$\beta F_{\xi m} = \{V(D), \phi, \{\varpi_1\}, \{\varpi_2\}, \{\varpi_3\}, \{\varpi_4\}, \{\varpi_5\}, \{\varpi_1, \varpi_2\}, \{\varpi_1, \varpi_3\}, \{\varpi_1, \varpi_4\}, \{\varpi_1, \varpi_5\}, \{\varpi_2, \varpi_3\}, \{\varpi_2, \varpi_4\}, \{\varpi_2, \varpi_5\}, \{\varpi_3, \varpi_4\}, \{\varpi_3, \varpi_5\}, \{\varpi_4, \varpi_5\}, \{\varpi_1, \varpi_2, \varpi_3\}, \{\varpi_1, \varpi_2, \varpi_4\}, \{\varpi_1, \varpi_2, \varpi_5\}, \{\varpi_1, \varpi_3, \varpi_4\}, \{\varpi_1, \varpi_3, \varpi_5\}, \{\varpi_1, \varpi_4, \varpi_5\}, \{\varpi_2, \varpi_3, \varpi_4\}, \{\varpi_2, \varpi_3, \varpi_5\}, \{\varpi_2, \varpi_4, \varpi_5\}, \{\varpi_3, \varpi_4, \varpi_5\}, \{\varpi_1, \varpi_2, \varpi_3, \varpi_5\}, \{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}, \{\varpi_1, \varpi_3, \varpi_4, \varpi_5\}, \{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}\}$.

Proposition 3.6 Let (D,Ω) be a supratopological space, then the relation between Ω(resp. F) and the families of near supraopen (resp. near supraclosed) d. g. 's are given as follows:



Proof (a)

(1) By definition of r-supraopren, we have $R\Omega \subseteq \Omega$.

(2) Let H be open subd. g., implies $V(H) = Int_u(V(H))$, but $Int_u(V(H)) \subseteq Cl_u(Int_u(V(H)))$, so $V(H) \subseteq Cl_u(Int_u(V(H)))$ and $Int_u(V(H)) \subseteq Int_u(Cl_u(Int_u(V(H))))$. Therefore, $V(H) \subseteq Int_u(Cl_u(Int_u(V(H))))$ and H is α-supraopen.

(3) Let H be α-supraopen, then $V(H) \subseteq Int_u(Cl_u(Int_u(V(H))))$ and $V(H) \subseteq Cl_u(Int_u(V(H)))$. Therefore, H is s-supraopen.

(4) Clear.

(5) Let H be γ-supraopen, then $V(H) \subseteq Cl_u(Int_u(V(H))) \cup Int_u(Cl_u(V(H)))$. In general $V(H) \subseteq Cl_u(V(H))$, then $Cl_u(Int_u(V(H))) \subseteq Cl_u(Int_u(Cl_u(V(H))))$ and $Int_u(Cl_u(V(H))) \subseteq Cl_u(Int_u(Cl_u(V(H))))$, implies $Cl_u(Int_u(V(H))) \cup Int_u(Cl_u(V(H))) \subseteq Cl_u(Int_u(Cl_u(V(H)))) \cup Cl_u(Int_u(Cl_u(V(H)))) = Cl_u(Int_u(Cl_u(V(H))))$. Hence, $V(H) \subseteq Cl_u(Int_u(Cl_u(V(H))))$ which means H is β-supraopen.

(b)

(1) Let H be α-supraopen, then $V(H) \subseteq Int_u(Cl_u(Int_u(V(H))))$, since $Int_u(V(H)) \subseteq V(H)$, this implies $Int_u(Cl_u(Int_u(V(H)))) \subseteq Int_u(Cl_u(V(H)))$. Hence, $V(H) \subseteq Int_u(Cl_u(V(H)))$. Therefore, H is p-supraopen.

(2) Clear.

Remark 3.7 Considering Example (3.5), the two classes $S\Omega$ (resp. SF) and $P\Omega$ (resp. PF) are not comparable because there is no one of them is contained in the other.

Definition 3.8 Let (D, Ω) be a supratopological space and $H \subseteq D$, then the near suprainterior of H is denoted by $Int_{ju}(V(H))$ and defined by:

$Int_{ju}(V(H)) = \cup \{V(O) \subseteq V(H);\ O$ is j-supraopen$\}$, where $j = r, \alpha, s, p, \gamma$ and β.

Definition 3.9 Let (D, Ω) be a supratopological space and $H \subseteq D$, then the near supraclosure of H is denoted by $Cl_{ju}(V(H))$ and defined by:

$Cl_{ju}(V(H)) = \cap \{V(H) \subseteq V(F);\ F$ is j-supraclosed$\}$, where $j = r, \alpha, s, p, \gamma$ and β.

Definition 3.10 Let (D, Ω) be a supratopological space and $H \subseteq D$, then the near supraboundary of H is denoted by $Bd_{ju}(V(H))$ and defined by:

$Bd_{ju}(V(H)) = Cl_{ju}(V(H)) - Int_{ju}(V(H))$, where $j = r, \alpha, s, p, \gamma$ and β.

Remark 3.11 Since every topological space is a supratopological space, then the concepts in Definition (3.2) are special cases of those in Definition (3.1).

Example 3.12 According to Example (3.4). Table (3.1) (resp. Table (3.2) and Table (3.3)) shows $Int_{ju}(V(H))$ (resp. $Cl_{ju}(V(H))$ and $Bd_{ju}(V(H))$) for all $H \subseteq D$ and for all $j = r, \alpha$ and p.

Table 3.1: According to Example (3.4),$Int_m(V(H))$ and $Int_{jm}(V(H))$, where $j \in \{r, \alpha\}$, for all $H \subseteq D$

$V(H)$	$Int_{rm}(V(H))$	$Int_m(V(H))$	$Int_{\alpha m}(V(H))$
$\{\varpi_1\}$	ϕ	ϕ	ϕ
$\{\varpi_2\}$	ϕ	ϕ	ϕ
$\{\varpi_3\}$	ϕ	ϕ	ϕ

www.ijsr.net

Paper ID: ART2017834 DOI: 10.21275/ART2017834

International Journal of Science and Research (IJSR)
ISSN (Online): 2319-7064
Index Copernicus Value (2015): 78.96 | Impact Factor (2015): 6.391

$\{\varpi_4\}$	ϕ	ϕ	ϕ
$\{\varpi_5\}$	$\{\varpi_5\}$	$\{\varpi_5\}$	$\{\varpi_5\}$
$\{\varpi_1,\varpi_2\}$	ϕ	ϕ	ϕ
$\{\varpi_1,\varpi_3\}$	ϕ	ϕ	ϕ
$\{\varpi_1,\varpi_4\}$	ϕ	ϕ	ϕ
$\{\varpi_1,\varpi_5\}$	$\{\varpi_5\}$	$\{\varpi_5\}$	$\{\varpi_5\}$
$\{\varpi_2,\varpi_3\}$	ϕ	ϕ	ϕ
$\{\varpi_2,\varpi_4\}$	ϕ	ϕ	ϕ
$\{\varpi_2,\varpi_5\}$	$\{\varpi_5\}$	$\{\varpi_5\}$	$\{\varpi_5\}$
$\{\varpi_3,\varpi_4\}$	ϕ	$\{\varpi_3,\varpi_4\}$	$\{\varpi_3,\varpi_4\}$
$\{\varpi_3,\varpi_5\}$	$\{\varpi_5\}$	$\{\varpi_5\}$	$\{\varpi_5\}$
$\{\varpi_4,\varpi_5\}$	$\{\varpi_5\}$	$\{\varpi_5\}$	$\{\varpi_5\}$
$\{\varpi_1,\varpi_2,\varpi_3\}$	ϕ	ϕ	ϕ
$\{\varpi_1,\varpi_2,\varpi_4\}$	ϕ	ϕ	ϕ
$\{\varpi_1,\varpi_2,\varpi_5\}$	$\{\varpi_5\}$	$\{\varpi_5\}$	$\{\varpi_5\}$
$\{\varpi_1,\varpi_3,\varpi_4\}$	ϕ	$\{\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_3,\varpi_4\}$
$\{\varpi_1,\varpi_3,\varpi_5\}$	$\{\varpi_5\}$	$\{\varpi_5\}$	$\{\varpi_5\}$
$\{\varpi_1,\varpi_4,\varpi_5\}$	$\{\varpi_5\}$	$\{\varpi_5\}$	$\{\varpi_5\}$
$\{\varpi_2,\varpi_3,\varpi_4\}$	ϕ	$\{\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$
$\{\varpi_2,\varpi_3,\varpi_5\}$	$\{\varpi_5\}$	$\{\varpi_5\}$	$\{\varpi_5\}$
$\{\varpi_2,\varpi_4,\varpi_5\}$	$\{\varpi_5\}$	$\{\varpi_5\}$	$\{\varpi_5\}$
$\{\varpi_3,\varpi_4,\varpi_5\}$	$\{\varpi_5\}$	$\{\varpi_3,\varpi_4,\varpi_5\}$	$\{\varpi_3,\varpi_4,\varpi_5\}$
$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$
$\{\varpi_1,\varpi_2,\varpi_3,\varpi_5\}$	$\{\varpi_5\}$	$\{\varpi_5\}$	$\{\varpi_5\}$
$\{\varpi_1,\varpi_2,\varpi_4,\varpi_5\}$	$\{\varpi_5\}$	$\{\varpi_5\}$	$\{\varpi_5\}$
$\{\varpi_1,\varpi_3,\varpi_4,\varpi_5\}$	$\{\varpi_5\}$	$\{\varpi_3,\varpi_4,\varpi_5\}$	$\{\varpi_1,\varpi_3,\varpi_4,\varpi_5\}$
$\{\varpi_2,\varpi_3,\varpi_4,\varpi_5\}$	$\{\varpi_5\}$	$\{\varpi_3,\varpi_4,\varpi_5\}$	$\{\varpi_2,\varpi_3,\varpi_4,\varpi_5\}$
$V(D)$	$V(D)$	$V(D)$	$V(D)$
ϕ	ϕ	ϕ	ϕ

Table 3.2: According to Example (3.4), $Cl_m(V(H))$ and $Cl_{jm}(V(H))$, where $j \in \{r, \alpha\}$, for all $H \subseteq D$.

$V(H)$	$Cl_{rm}(V(H))$	$Cl_m(V(H))$	$Cl_{\alpha m}(V(H))$
$\{\varpi_1\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2\}$	$\{\varpi_1\}$
$\{\varpi_2\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2\}$	$\{\varpi_2\}$
$\{\varpi_3\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$
$\{\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$
$\{\varpi_5\}$	$\{\varpi_5\}$	$\{\varpi_5\}$	$\{\varpi_5\}$
$\{\varpi_1,\varpi_2\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2\}$	$\{\varpi_1,\varpi_2\}$
$\{\varpi_1,\varpi_3\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$
$\{\varpi_1,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$
$\{\varpi_1,\varpi_5\}$	$V(D)$	$\{\varpi_1,\varpi_2,\varpi_5\}$	$\{\varpi_1,\varpi_5\}$
$\{\varpi_2,\varpi_3\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$
$\{\varpi_2,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$
$\{\varpi_2,\varpi_5\}$	$V(D)$	$\{\varpi_1,\varpi_2,\varpi_5\}$	$\{\varpi_2,\varpi_5\}$
$\{\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$
$\{\varpi_3,\varpi_5\}$	$V(D)$	$V(D)$	$V(D)$
$\{\varpi_4,\varpi_5\}$	$V(D)$	$V(D)$	$V(D)$
$\{\varpi_1,\varpi_2,\varpi_3\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$
$\{\varpi_1,\varpi_2,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$
$\{\varpi_1,\varpi_2,\varpi_5\}$	$V(D)$	$\{\varpi_1,\varpi_2,\varpi_5\}$	$\{\varpi_1,\varpi_2,\varpi_5\}$
$\{\varpi_1,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$
$\{\varpi_1,\varpi_3,\varpi_5\}$	$V(D)$	$V(D)$	$V(D)$
$\{\varpi_1,\varpi_4,\varpi_5\}$	$V(D)$	$V(D)$	$V(D)$
$\{\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$
$\{\varpi_2,\varpi_3,\varpi_5\}$	$V(D)$	$V(D)$	$V(D)$
$\{\varpi_2,\varpi_4,\varpi_5\}$	$V(D)$	$V(D)$	$V(D)$
$\{\varpi_3,\varpi_4,\varpi_5\}$	$V(D)$	$V(D)$	$V(D)$
$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$
$\{\varpi_1,\varpi_2,\varpi_3,\varpi_5\}$	$V(D)$	$V(D)$	$V(D)$
$\{\varpi_1,\varpi_2,\varpi_4,\varpi_5\}$	$V(D)$	$V(D)$	$V(D)$
$\{\varpi_1,\varpi_3,\varpi_4,\varpi_5\}$	$V(D)$	$V(D)$	$V(D)$
$\{\varpi_2,\varpi_3,\varpi_4,\varpi_5\}$	$V(D)$	$V(D)$	$V(D)$
$V(D)$	$V(D)$	$V(D)$	$V(D)$
ϕ	ϕ	ϕ	ϕ

Table 3.3: According to Example (3.4), $Bd_m(V(H))$ and $Bd_{jm}(V(H))$, where $j \in \{r, \alpha\}$, for all $H \subseteq D$.

$V(H)$	$Bd_{rm}(V(H))$	$Bd_m(V(H))$	$Bd_{\alpha m}(V(H))$
$\{\varpi_1\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2\}$	$\{\varpi_1\}$
$\{\varpi_2\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2\}$	$\{\varpi_2\}$
$\{\varpi_3\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$
$\{\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$
$\{\varpi_5\}$	ϕ	ϕ	ϕ
$\{\varpi_1,\varpi_3\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2\}$	$\{\varpi_1,\varpi_2\}$
$\{\varpi_1,\varpi_3\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$
$\{\varpi_1,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$
$\{\varpi_1,\varpi_5\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2\}$	$\{\varpi_1\}$
$\{\varpi_2,\varpi_3\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$
$\{\varpi_2,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$
$\{\varpi_2,\varpi_5\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2\}$	$\{\varpi_2\}$
$\{\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2\}$	$\{\varpi_1,\varpi_2\}$
$\{\varpi_3,\varpi_5\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$
$\{\varpi_4,\varpi_5\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$
$\{\varpi_1,\varpi_2,\varpi_3\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$
$\{\varpi_1,\varpi_2,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$
$\{\varpi_1,\varpi_2,\varpi_5\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2\}$	$\{\varpi_1,\varpi_2\}$
$\{\varpi_1,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2\}$	$\{\varpi_2\}$
$\{\varpi_1,\varpi_3,\varpi_5\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$
$\{\varpi_1,\varpi_4,\varpi_5\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$
$\{\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2\}$	ϕ
$\{\varpi_2,\varpi_3,\varpi_5\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$
$\{\varpi_2,\varpi_4,\varpi_5\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$
$\{\varpi_3,\varpi_4,\varpi_5\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2\}$	$\{\varpi_1,\varpi_2\}$
$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	ϕ	ϕ	ϕ
$\{\varpi_1,\varpi_2,\varpi_3,\varpi_5\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$
$\{\varpi_1,\varpi_2,\varpi_4,\varpi_5\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$
$\{\varpi_1,\varpi_3,\varpi_4,\varpi_5\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2\}$	$\{\varpi_2\}$
$\{\varpi_2,\varpi_3,\varpi_4,\varpi_5\}$	$\{\varpi_1,\varpi_2,\varpi_3,\varpi_4\}$	$\{\varpi_1,\varpi_2\}$	$\{\varpi_1\}$
$V(D)$	ϕ	ϕ	ϕ
ϕ	ϕ	ϕ	ϕ

Example 3.13 According to Example (3.5). Table (3.4) (resp. Table (3.5) and Table (3.6)) shows $Int_{ju}(V(H))$ (resp. $Cl_{ju}(V(H))$ and $Bd_{ju}(V(H))$) for all $H \subseteq D$ and for all $j = r, \alpha, s, p, \gamma$ and β.

Table 3.4: According to Example (3.5), $Int_m(V(H))$ and $Int_{jm}(V(H))$, where $j \in \{s, p, \gamma, \beta\}$, for all $H \subseteq D$.

$V(H)$	$Int_m(V(H))$	$Int_{sm}(V(H))$	$Int_{pm}(V(H))$	$Int_{\gamma m}(V(H))$	$Int_{\beta m}(V(H))$
$\{\varpi_1\}$	ϕ	ϕ	$\{\varpi_1\}$	$\{\varpi_1\}$	$\{\varpi_1\}$

$\{\varpi_2\}$	ϕ	ϕ	$\{\varpi_2\}$	$\{\varpi_2\}$	$\{\varpi_2\}$
$\{\varpi_3\}$	ϕ	ϕ	$\{\varpi_3\}$	$\{\varpi_3\}$	$\{\varpi_3\}$
$\{\varpi_4\}$	ϕ	ϕ	$\{\varpi_4\}$	$\{\varpi_4\}$	$\{\varpi_4\}$
$\{\varpi_5\}$	ϕ	ϕ	ϕ	ϕ	ϕ
$\{\varpi_1, \varpi_2\}$	$\{\varpi_1, \varpi_2\}$	$\{\varpi_1, \varpi_2\}$	$\{\varpi_1, \varpi_2\}$	$\{\varpi_1, \varpi_2\}$	$\{\varpi_1, \varpi_2\}$
$\{\varpi_1, \varpi_3\}$	ϕ	ϕ	$\{\varpi_1, \varpi_3\}$	$\{\varpi_1, \varpi_3\}$	$\{\varpi_1, \varpi_3\}$
$\{\varpi_1, \varpi_4\}$	ϕ	ϕ	$\{\varpi_1, \varpi_4\}$	$\{\varpi_1, \varpi_4\}$	$\{\varpi_1, \varpi_4\}$
$\{\varpi_1, \varpi_5\}$	ϕ	ϕ	$\{\varpi_1, \varpi_5\}$	$\{\varpi_1\}$	$\{\varpi_1, \varpi_5\}$
$\{\varpi_2, \varpi_3\}$	ϕ	ϕ	$\{\varpi_2, \varpi_3\}$	$\{\varpi_2, \varpi_3\}$	$\{\varpi_2, \varpi_3\}$
$\{\varpi_2, \varpi_4\}$	ϕ	ϕ	$\{\varpi_2, \varpi_4\}$	$\{\varpi_2, \varpi_4\}$	$\{\varpi_2, \varpi_4\}$
$\{\varpi_2, \varpi_5\}$	ϕ	ϕ	$\{\varpi_2\}$	$\{\varpi_2\}$	$\{\varpi_2, \varpi_5\}$
$\{\varpi_3, \varpi_4\}$	$\{\varpi_3, \varpi_4\}$	$\{\varpi_3, \varpi_4\}$	$\{\varpi_3, \varpi_4\}$	$\{\varpi_3, \varpi_4\}$	$\{\varpi_3, \varpi_4\}$
$\{\varpi_3, \varpi_5\}$	ϕ	ϕ	$\{\varpi_3\}$	$\{\varpi_3\}$	$\{\varpi_3, \varpi_5\}$
$\{\varpi_4, \varpi_5\}$	ϕ	ϕ	$\{\varpi_4\}$	$\{\varpi_4\}$	$\{\varpi_4, \varpi_5\}$
$\{\varpi_1, \varpi_2, \varpi_3\}$	$\{\varpi_1, \varpi_2\}$	$\{\varpi_1, \varpi_2\}$	$\{\varpi_1, \varpi_2, \varpi_3\}$	$\{\varpi_1, \varpi_2, \varpi_3\}$	$\{\varpi_1, \varpi_2, \varpi_3\}$
$\{\varpi_1, \varpi_2, \varpi_4\}$	$\{\varpi_1, \varpi_2\}$	$\{\varpi_1, \varpi_2\}$	$\{\varpi_1, \varpi_2, \varpi_4\}$	$\{\varpi_1, \varpi_2, \varpi_4\}$	$\{\varpi_1, \varpi_2, \varpi_4\}$
$\{\varpi_1, \varpi_2, \varpi_5\}$	$\{\varpi_1, \varpi_2\}$	$\{\varpi_1, \varpi_2, \varpi_5\}$	$\{\varpi_1, \varpi_2\}$	$\{\varpi_1, \varpi_2, \varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_5\}$
$\{\varpi_1, \varpi_3, \varpi_4\}$	$\{\varpi_3, \varpi_4\}$	$\{\varpi_3, \varpi_4\}$	$\{\varpi_1, \varpi_3, \varpi_4\}$	$\{\varpi_1, \varpi_3, \varpi_4\}$	$\{\varpi_1, \varpi_3, \varpi_4\}$
$\{\varpi_1, \varpi_3, \varpi_5\}$	ϕ	ϕ	$\{\varpi_1, \varpi_3, \varpi_5\}$	$\{\varpi_1, \varpi_3, \varpi_5\}$	$\{\varpi_1, \varpi_3, \varpi_5\}$
$\{\varpi_1, \varpi_4, \varpi_5\}$	ϕ	ϕ	$\{\varpi_1, \varpi_4, \varpi_5\}$	$\{\varpi_1, \varpi_4, \varpi_5\}$	$\{\varpi_1, \varpi_4, \varpi_5\}$
$\{\varpi_2, \varpi_3, \varpi_4\}$	$\{\varpi_3, \varpi_4\}$	$\{\varpi_3, \varpi_4\}$	$\{\varpi_2, \varpi_3, \varpi_4\}$	$\{\varpi_2, \varpi_3, \varpi_4\}$	$\{\varpi_2, \varpi_3, \varpi_4\}$
$\{\varpi_2, \varpi_3, \varpi_5\}$	ϕ	ϕ	$\{\varpi_2, \varpi_3, \varpi_5\}$	$\{\varpi_2, \varpi_3, \varpi_5\}$	$\{\varpi_2, \varpi_3, \varpi_5\}$
$\{\varpi_2, \varpi_4, \varpi_5\}$	ϕ	ϕ	$\{\varpi_2, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_4, \varpi_5\}$
$\{\varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_3, \varpi_4\}$	$\{\varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_3, \varpi_4\}$	$\{\varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_3, \varpi_4, \varpi_5\}$
$\{\varpi_1, \varpi_2, \varpi_3, \varpi_4\}$	$\{\varpi_1, \varpi_2, \varpi_3, \varpi_4\}$	$\{\varpi_1, \varpi_2, \varpi_3, \varpi_4\}$	$\{\varpi_1, \varpi_2, \varpi_3, \varpi_4\}$	$\{\varpi_1, \varpi_2, \varpi_3, \varpi_4\}$	$\{\varpi_1, \varpi_2, \varpi_3, \varpi_4\}$
$\{\varpi_1, \varpi_2, \varpi_3, \varpi_5\}$	$\{\varpi_1, \varpi_2\}$	$\{\varpi_1, \varpi_2, \varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_3, \varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_3, \varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_3, \varpi_5\}$
$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	$\{\varpi_1, \varpi_2\}$	$\{\varpi_1, \varpi_2, \varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$
$\{\varpi_1, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_3, \varpi_4\}$	$\{\varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_1, \varpi_3 \varpi_4, \varpi_5\}$	$\{\varpi_1, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_1, \varpi_3, \varpi_4, \varpi_5\}$
$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_3, \varpi_4\}$	$\{\varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$
$V(D)$	$V(D)$	$V(D)$	$V(D)$	$V(D)$	$V(D)$
ϕ	ϕ	ϕ	ϕ	ϕ	ϕ

Table 3.5: According to Example (3.5), $Cl_m(V(H))$ and $Cl_{jm}(V(H))$, where $j \in \{s, p, \gamma, \beta\}$, for all $H \subseteq D$.

$V(H)$	$Cl_m(V(H))$	$Cl_{sm}(V(H))$	$Cl_{pm}(V(H))$	$Cl_{\gamma m}(V(H))$	$Cl_{\beta m}(V(H))$
$\{\varpi_1\}$	$\{\varpi_1, \varpi_2, \varpi_5\}$	$\{\varpi_1, \varpi_2\}$	$\{\varpi_1\}$	$\{\varpi_1\}$	$\{\varpi_1\}$
$\{\varpi_2\}$	$\{\varpi_1, \varpi_2, \varpi_5\}$	$\{\varpi_1, \varpi_2\}$	$\{\varpi_2\}$	$\{\varpi_2\}$	$\{\varpi_2\}$
$\{\varpi_3\}$	$\{\varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_3, \varpi_4\}$	$\{\varpi_3\}$	$\{\varpi_3\}$	$\{\varpi_3\}$
$\{\varpi_4\}$	$\{\varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_3, \varpi_4\}$	$\{\varpi_4\}$	$\{\varpi_4\}$	$\{\varpi_4\}$
$\{\varpi_5\}$	$\{\varpi_5\}$	$\{\varpi_5\}$	$\{\varpi_5\}$	$\{\varpi_5\}$	$\{\varpi_5\}$
$\{\varpi_1, \varpi_2\}$	$\{\varpi_1, \varpi_2, \varpi_5\}$	$\{\varpi_1, \varpi_2\}$	$\{\varpi_1, \varpi_2, \varpi_5\}$	$\{\varpi_1, \varpi_2\}$	$\{\varpi_1, \varpi_2\}$
$\{\varpi_1, \varpi_3\}$	$V(D)$	$V(D)$	$\{\varpi_1, \varpi_3\}$	$\{\varpi_1, \varpi_3\}$	$\{\varpi_1, \varpi_3\}$
$\{\varpi_1, \varpi_4\}$	$V(D)$	$V(D)$	$\{\varpi_1, \varpi_4\}$	$\{\varpi_1, \varpi_4\}$	$\{\varpi_1, \varpi_4\}$
$\{\varpi_1, \varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_5\}$	$\{\varpi_1, \varpi_5\}$	$\{\varpi_1, \varpi_5\}$	$\{\varpi_1, \varpi_5\}$
$\{\varpi_2, \varpi_3\}$	$V(D)$	$V(D)$	$\{\varpi_2, \varpi_3\}$	$\{\varpi_2, \varpi_3\}$	$\{\varpi_2, \varpi_3\}$
$\{\varpi_2, \varpi_4\}$	$V(D)$	$V(D)$	$\{\varpi_2, \varpi_4\}$	$\{\varpi_2, \varpi_4\}$	$\{\varpi_2, \varpi_4\}$
$\{\varpi_2, \varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_5\}$	$\{\varpi_2, \varpi_5\}$	$\{\varpi_2, \varpi_5\}$	$\{\varpi_2, \varpi_5\}$
$\{\varpi_3, \varpi_4\}$	$\{\varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_3, \varpi_4\}$	$\{\varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_3, \varpi_4\}$	$\{\varpi_3, \varpi_4\}$
$\{\varpi_3, \varpi_5\}$	$\{\varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_3, \varpi_5\}$	$\{\varpi_3, \varpi_5\}$	$\{\varpi_3, \varpi_5\}$
$\{\varpi_4, \varpi_5\}$	$\{\varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_4, \varpi_5\}$	$\{\varpi_4, \varpi_5\}$	$\{\varpi_4, \varpi_5\}$
$\{\varpi_1, \varpi_2, \varpi_3\}$	$V(D)$	$V(D)$	$\{\varpi_1, \varpi_2, \varpi_3, \varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_3, \varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_3\}$
$\{\varpi_1, \varpi_2, \varpi_4\}$	$V(D)$	$V(D)$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_4\}$
$\{\varpi_1, \varpi_2, \varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_5\}$
$\{\varpi_1, \varpi_3, \varpi_4\}$	$V(D)$	$V(D)$	$\{\varpi_1, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_1, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_1, \varpi_3, \varpi_4\}$
$\{\varpi_1, \varpi_3, \varpi_5\}$	$V(D)$	$V(D)$	$\{\varpi_1, \varpi_3, \varpi_5\}$	$\{\varpi_1, \varpi_3, \varpi_5\}$	$\{\varpi_1, \varpi_3, \varpi_5\}$
$\{\varpi_1, \varpi_4, \varpi_5\}$	$V(D)$	$V(D)$	$\{\varpi_1, \varpi_4, \varpi_5\}$	$\{\varpi_1, \varpi_4, \varpi_5\}$	$\{\varpi_1, \varpi_4, \varpi_5\}$
$\{\varpi_2, \varpi_3, \varpi_4\}$	$V(D)$	$V(D)$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$
$\{\varpi_2, \varpi_3, \varpi_5\}$	$V(D)$	$V(D)$	$\{\varpi_2, \varpi_3, \varpi_5\}$	$\{\varpi_2, \varpi_3, \varpi_5\}$	$\{\varpi_2, \varpi_3, \varpi_5\}$
$\{\varpi_2, \varpi_4, \varpi_5\}$	$V(D)$	$V(D)$	$\{\varpi_2, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_4, \varpi_5\}$
$\{\varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_3, \varpi_4, \varpi_5\}$
$\{\varpi_1, \varpi_2, \varpi_3, \varpi_4\}$	$V(D)$	$V(D)$	$V(D)$	$V(D)$	$V(D)$
$\{\varpi_1, \varpi_2, \varpi_3, \varpi_5\}$	$V(D)$	$V(D)$	$\{\varpi_1, \varpi_2, \varpi_3, \varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_3, \varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_3, \varpi_5\}$

$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	$V(D)$	$V(D)$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_4 \varpi_5\}$
$\{\varpi_1, \varpi_3, \varpi_4, \varpi_5\}$	$V(D)$	$V(D)$	$\{\varpi_1, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_1, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_1, \varpi_3, \varpi_4, \varpi_5\}$
$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$	$V(D)$	$V(D)$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_1, \varpi_3, \varpi_4, \varpi_5\}$
$V(D)$	$V(D)$	$V(D)$	$V(D)$	$V(D)$	$V(D)$
ϕ	ϕ	ϕ	ϕ	ϕ	ϕ

Table 3.6: According to Example (3.5),$Bd_m(V(H))$ and $Bd_{jm}(V(H))$, where $j \in \{s, p, \gamma, \beta\}$, for all $H \subseteq D$.

$V(H)$	$Bd_m(V(H))$	$Bd_{sm}(V(H))$	$Bd_{pm}(V(H))$	$Bd_{\gamma m}(V(H))$	$Bd_{\beta m}(V(H))$
$\{\varpi_1\}$	$\{\varpi_1, \varpi_2, \varpi_5\}$	$\{\varpi_1, \varpi_2\}$	ϕ	ϕ	ϕ
$\{\varpi_2\}$	$\{\varpi_1, \varpi_2, \varpi_5\}$	$\{\varpi_1, \varpi_2\}$	ϕ	ϕ	ϕ
$\{\varpi_3\}$	$\{\varpi_1, \varpi_2, \varpi_5\}$	$\{\varpi_3, \varpi_4\}$	ϕ	ϕ	ϕ
$\{\varpi_4\}$	$\{\varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_3, \varpi_4\}$	ϕ	ϕ	ϕ
$\{\varpi_5\}$	$\{\varpi_5\}$	$\{\varpi_5\}$	$\{\varpi_5\}$	$\{\varpi_5\}$	$\{\varpi_5\}$
$\{\varpi_1, \varpi_2\}$	$\{\varpi_5\}$	ϕ	$\{\varpi_5\}$	ϕ	ϕ
$\{\varpi_1, \varpi_3\}$	$V(D)$	$V(D)$	ϕ	ϕ	ϕ
$\{\varpi_1, \varpi_4\}$	$V(D)$	$V(D)$	ϕ	ϕ	ϕ
$\{\varpi_1, \varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_5\}$	$\{\varpi_5\}$	$\{\varpi_5\}$	ϕ
$\{\varpi_2, \varpi_3\}$	$V(D)$	$V(D)$	ϕ	ϕ	ϕ
$\{\varpi_2, \varpi_4\}$	$V(D)$	$V(D)$	ϕ	ϕ	ϕ
$\{\varpi_2, \varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_5\}$	$\{\varpi_5\}$	$\{\varpi_5\}$	ϕ
$\{\varpi_3, \varpi_4\}$	$\{\varpi_5\}$	ϕ	$\{\varpi_5\}$	ϕ	ϕ
$\{\varpi_3, \varpi_5\}$	$\{\varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_5\}$	$\{\varpi_5\}$	ϕ
$\{\varpi_4, \varpi_5\}$	$\{\varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_5\}$	$\{\varpi_5\}$	ϕ
$\{\varpi_1, \varpi_2, \varpi_3\}$	$\{\varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_5\}$	$\{\varpi_5\}$	ϕ
$\{\varpi_1, \varpi_2, \varpi_4\}$	$\{\varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_5\}$	$\{\varpi_5\}$	ϕ
$\{\varpi_1, \varpi_2, \varpi_5\}$	$\{\varpi_5\}$	ϕ	$\{\varpi_5\}$	$\{\varpi_5\}$	ϕ
$\{\varpi_1, \varpi_3, \varpi_4\}$	$\{\varpi_1, \varpi_2, \varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_5\}$	$\{\varpi_5\}$	$\{\varpi_5\}$	ϕ
$\{\varpi_1, \varpi_3, \varpi_5\}$	$V(D)$	$V(D)$	ϕ	ϕ	ϕ
$\{\varpi_1, \varpi_4, \varpi_5\}$	$V(D)$	$V(D)$	ϕ	ϕ	ϕ
$\{\varpi_2, \varpi_3, \varpi_4\}$	$\{\varpi_1, \varpi_2, \varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_5\}$	$\{\varpi_5\}$	$\{\varpi_5\}$	ϕ
$\{\varpi_2, \varpi_3, \varpi_5\}$	$V(D)$	$V(D)$	ϕ	ϕ	ϕ
$\{\varpi_2, \varpi_4, \varpi_5\}$	$V(D)$	$V(D)$	ϕ	ϕ	ϕ
$\{\varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_5\}$	ϕ	$\{\varpi_5\}$	ϕ	ϕ
$\{\varpi_1, \varpi_2, \varpi_3, \varpi_4\}$	$\{\varpi_5\}$	$\{\varpi_5\}$	$\{\varpi_5\}$	$\{\varpi_5\}$	$\{\varpi_5\}$
$\{\varpi_1, \varpi_2, \varpi_3, \varpi_5\}$	$\{\varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_3, \varpi_4\}$	ϕ	ϕ	ϕ
$\{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}$	$\{\varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_3, \varpi_4\}$	ϕ	ϕ	ϕ
$\{\varpi_1, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_5\}$	$\{\varpi_1, \varpi_2\}$	ϕ	ϕ	ϕ
$\{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$	$\{\varpi_1, \varpi_2, \varpi_5\}$	$\{\varpi_1, \varpi_2\}$	ϕ	ϕ	ϕ
$V(D)$	ϕ	ϕ	ϕ	ϕ	ϕ
ϕ	ϕ	ϕ	ϕ	ϕ	ϕ

Proposition 3.14 Let (D, Ω) be a supratopological space and $H \subseteq D$, then $Int_u(V(H)) \subseteq Int_{ju}(V(H)) \subseteq V(H) \subseteq Cl_{ju}(V(H)) \subseteq Cl_u(V(H))$ for all $j = \alpha, s, p, \gamma, \beta$and $j \neq r$.

Proof. The proof of the five case are similar, so we will only prove the case $j = s$.

$Int_u(V(H)) = \cup \{V(O) \in \Omega;\ V(O) \subseteq V(H)\}$

$\subseteq \cup \{V(O) \in S\Omega;\ V(O) \subseteq V(H)\}$ since $\Omega \subseteq S\Omega$

$= Int_{su}(V(H)) \subseteq V(H),$ (1)

On the other hand, we have

$V(H) \subseteq Cl_{su}(V(H)) = \cap \{V(F);\ V(F) \in SF$ and $V(H) \subseteq V(F)\}$

$\subseteq \cap \{V(F);\ V(F) \in F$ and $V(H) \subseteq V(F)\}$ since $F \subseteq SF$

$= Cl_u(V(H))$. (2)

From (1) and (2) we get $Int_u(V(H)) \subseteq Int_{ju}(V(H)) \subseteq V(H) \subseteq Cl_{ju}(V(H)) \subseteq Cl_u(V(H))$

Remark 3.15 The above proposition is not necessarily true in the case of $j = r$ as shown in the next example.

Example 3.16 According to Example (3.4).

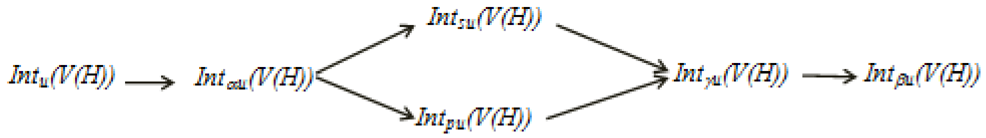
$F_{\xi m} = \{V(D), \phi, \{\varpi_5\}, \{\varpi_1, \varpi_2\}, \{\varpi_1, \varpi_2, \varpi_5\}, \{\varpi_1, \varpi_2, \varpi_3, \varpi_4\}\}$,

$\Omega_{\xi m} = \{V(D), \phi, \{\varpi_5\}, \{\varpi_3, \varpi_4\}, \{\varpi_3, \varpi_4, \varpi_5\}, \{\varpi_1, \varpi_2, \varpi_3, \varpi_4\}\}$.

Hence, $R\Omega_{\xi m} = \{V(D), \phi, \{\varpi_5\}, \{\varpi_1, \varpi_2, \varpi_3, \varpi_4\}\}$ and $RF_{\xi m} = \{V(D), \phi, \{\varpi_5\}, \{\varpi_1, \varpi_3, \varpi_4, \varpi_5\}\}$.

If $H = (V(H), E(H))$: $V(H) = \{\varpi_2, \varpi_3, \varpi_4\}$, $E(H) = \{(\varpi_3, \varpi_4), (\varpi_4, \varpi_3)\}$, then $Int_m(V(H)) = \{\varpi_3, \varpi_4\}$, $Cl_m(V(H)) = \{\varpi_1, \varpi_2, \varpi_3, \varpi_4\}$, $Int_{rm}(V(H)) = \phi$, $Cl_{rm}(V(H)) = \{\varpi_1, \varpi_2, \varpi_3, \varpi_4\}$. Obviously, $Int_m(V(H)) \nsubseteq Int_{rm}(V(H))$ and$Cl_m(V(H)) = Cl_{rm}(V(H))$.

Proposition 3.17 Let(D, Ω) be a supratopological space. Then the relations between suprainterior and near suprainterior of $H \subseteq D$ are given by the following diagram:



Proof. By using Proposition (3.14), $Int_u(V(H)) \subseteq Int_{\alpha u}(V(H))$. We shall prove $Int_{\alpha u}(V(H)) \subseteq Int_{su}(V(H))$.

$Int_{\alpha u}(V(H)) = \bigcup \{V(O) \in \alpha\Omega;\ V(O) \subseteq V(H)\}$
$\subseteq \bigcup \{V(O) \in S\Omega;\ V(O) \subseteq V(H)\} = Int_{su}(V(H))$, since $\alpha\Omega \subseteq S\Omega$.
Thus, $Int_{\alpha u}(V(H)) \subseteq Int_{su}(V(H))$. Similarly can prove the other cases.

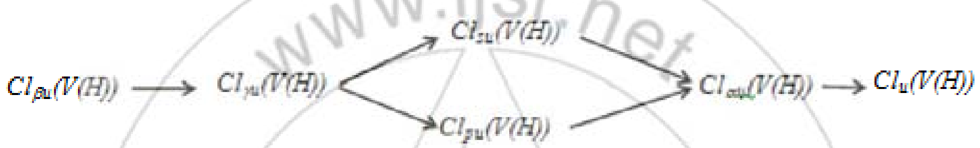
Remark 3.18 Let (D, Ω) be a supratopological space. Then the following statement are not true in general for every $H \subseteq D$.

(a) $Int_u(V(H)) = Int_{\alpha u}(V(H)) = Int_{su}(V(H)) = Int_{\gamma u}(V(H)) = Int_{\beta u}(V(H))$,
(b) $Int_u(V(H)) = Int_{\alpha u}(V(H)) = Int_{pu}(V(H)) = Int_{\gamma u}(V(H)) = Int_{\beta u}(V(H))$.

Example 3.19 According to Example (3.5).

(a) Let $H = (V(H), E(H))$: $V(H) = \{\varpi_3, \varpi_5\}$, $E(H) = \{(\varpi_5, \varpi_3)\}$, then $Int_m(V(H)) = \phi$, $Int_{\alpha m}(V(H)) = \phi$, $Int_{sm}(V(H)) = \phi$, $Int_{\gamma m}(V(H)) = \{\varpi_3\}$ and $Int_{\beta m}(V(H)) = \{\varpi_3, \varpi_5\}$.
(b) Let $H = (V(H), E(H))$: $V(H) = \{\varpi_3, \varpi_4, \varpi_5\}$, $E(H) = \{(\varpi_3, \varpi_4), (\varpi_4, \varpi_3), (\varpi_4, \varpi_5), (\varpi_5, \varpi_3)\}$, then $Int_m(V(H)) = \{\varpi_3, \varpi_4\}$, $Int_{\alpha m}(V(H)) = \{\varpi_3, \varpi_4\}$, $Int_{pm}(V(H)) = \{\varpi_3, \varpi_4\}$, $Int_{\gamma m}(V(H)) = \{\varpi_3, \varpi_4, \varpi_5\}$ and $Int_{\beta m}(V(H)) = \{\varpi_3, \varpi_4, \varpi_5\}$.

Proposition 3.20 Let(D, Ω) be a supratopological space. Then the relations between supraclosure and near supraclosure of $H \subseteq D$ are given by the following diagram:



Proof. By using Proposition (3.14), $Cl_{\alpha u}(V(H)) \subseteq Cl_u(V(H))$. We shall prove $Cl_{pu}(V(H)) \subseteq Cl_{\alpha u}(V(H))$.
$Cl_{pu}(V(H)) = \bigcap \{V(F);\ V(F) \in PF$ and $V(H) \subseteq V(F)\}$
$\subseteq \bigcap \{V(F);\ V(F) \in \alpha F$ and $V(H) \subseteq V(F)\} = Cl_{\alpha u}(V(H))$, since $\alpha F \subseteq PF$.
Thus, $Cl_{pu}(V(H)) \subseteq Cl_{\alpha u}(V(H))$. Similarly we can prove the other cases.

Remark 3.21 Let (D, Ω) be a supratopological space. Then the following statements are not true in general for every $H \subseteq D$.

(a) $Cl_u(V(H)) = Cl_{\alpha u}(V(H)) = Cl_{su}(V(H)) = Cl_{\gamma u}(V(H)) = Cl_{\beta u}(V(H))$,
(b) $Cl_u(V(H)) = Cl_{\alpha u}(V(H)) = Cl_{pu}(V(H)) = Cl_{\gamma u}(V(H)) = Cl_{\beta u}(V(H))$.

Example 3.22 According to Example (3.5).

(a) Let $H = (V(H), E(H))$: $V(H) = \{\varpi_2, \varpi_3, \varpi_4\}$, $E(H) = \{(\varpi_3, \varpi_4), (\varpi_4, \varpi_3)\}$, then $Cl_m(V(H)) = V(D)$, $Cl_{\alpha m}(V(H)) = V(D)$, $Cl_{sm}(V(H)) = V(D)$, $Cl_{\gamma m}(V(H)) = \{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}$ and $Cl_{\beta m}(V(H)) = \{\varpi_2, \varpi_3, \varpi_4\}$.
(b) Let $H = (V(H), E(H))$: $V(H) = \{\varpi_1, \varpi_3, \varpi_4\}$, $E(H) = \{(\varpi_3, \varpi_4), (\varpi_4, \varpi_3)\}$, then $Cl_m(V(H)) = V(D)$, $Cl_{\alpha m}(V(H)) = V(D)$, $Cl_{pm}(V(H)) = \{\varpi_1, \varpi_3, \varpi_4, \varpi_5\}$, $Cl_{\gamma m}(V(H)) = \{\varpi_1, \varpi_3, \varpi_4, \varpi_5\}$ and $Cl_{\beta m}(V(H)) = \{\varpi_1, \varpi_3, \varpi_4\}$.

Remark 3.23 The$Int_{su}(V(H))$ (resp. $Cl_{su}(V(H))$) and the $Int_{pu}(V(H))$ (resp. $Cl_{pu}(V(H))$) are not comparable as shown in Example (3.24).

Example 3.24 According to Example (3.5). Let $H = (V(H), E(H))$: $V(H) = \{\varpi_1, \varpi_2, \varpi_3, \varpi_5\}$, $E(H) = \{(\varpi_1, \varpi_2), (\varpi_2, \varpi_1), (\varpi_2, \varpi_5), (\varpi_5, \varpi_1), (\varpi_5, \varpi_3)\}$. $Int_{sm}(V(H)) = \{\varpi_1, \varpi_2, \varpi_5\}$, $Int_{pm}(V(H)) = \{\varpi_1, \varpi_2, \varpi_3, \varpi_5\}$.

Proposition 3.25 Let (D, Ω)be a supratopological space and $H \subseteq D$. then $Bd_{ju}V(H) \subseteq Bd_uV(H)$ for all $j = \alpha, s, p, \gamma, \beta$and $j \neq r$
Proof. By using Proposition (3.14), the proof is obvious.

Remark 3.26 According to Example (3.4). Let $H = (V(H), E(H))$: $V(H) = \{\varpi_1, \varpi_2\}$, $E(H) = \{(\varpi_1, \varpi_2), (\varpi_2, \varpi_1)\}$, then $Bd_m(V(H)) = Cl_m(V(H)) - Int_m(V(H)) = \{\varpi_1, \varpi_2\} - \phi = \{\varpi_1, \varpi_2\}$, $Bd_{rm}(V(H)) = Cl_{rm}(V(H)) - Int_{rm}(V(H)) = \{\varpi_1, \varpi_2, \varpi_3, \varpi_4\} - \phi = \{\varpi_1, \varpi_2, \varpi_3, \varpi_4\}$. So, $Bd_{rm}(V(H)) \nsubseteq Bd_m(V(H))$.

4. Near Supra-Lower and Near Supra-Upper Approximations

In this section we study the approximation space $D = (V(D), E(D))$ from a supratopologicalview. By generating the M-space from the approximation space $D = (V(D), E(D))$, we can obtain Ω_{ξ_m}. We proved that Ω_{ξ_m} is a unique class of subd. g. 's of D which forms a supratopology on D.

Definition 4.1 Let $D = (V(D), E(D))$ be an approximation space and Ω be a supratopology on $V(D)$ induced from D by any method. Then $D = (V(D), E(D), \Omega)$ is said to be a supra-approximation space.

Definition 4.2 Let $D = (V(D), E(D), \Omega)$ be a supra-approximation space. The supra-Lower (resp. supra-upper) approximations of $H \subseteq D$ are denoted by $L_u(V(H))$ (resp.$U_u(V(H))$) and defined by:

$$L_u(V(H)) = Int_u(V(H)),\ U_u(V(H)) = Cl_u(V(H)),$$

Supra-boundary (resp. supra-positive and supra-negative) regions of H are denoted by $Bd_u(V(H))$ (resp. $POS_u(V(H))$ and $NEG_u(V(H))$) and defined by:

$$Bd_u(V(H)) = U_u(V(H)) - L_u(V(H)), POS_u(V(H)) = L_u(V(H)),$$
$$NEG_u(V(H)) = V(D) - U_u(V(H)).$$

Definition 4.3 Let $D = (V(D), E(D), \Omega)$ be a supra-approximation space. The near supra-lower (resp. near supra-upper) approximations of $H \subseteq D$ are denoted by $L_{ju}(V(H))$ (resp. $U_{ju}(V(H))$) and defined by:

$$L_{ju}(V(H)) = Int_{ju}(V(H)),\ U_{ju}(V(H)) = Cl_{ju}(V(H)),$$

where $j = r, \alpha, s, p, \gamma, \beta$.

Near supra-boundary (resp. near supra-positive and near supra-negative) regions of H are denoted by $Bd_{ju}(V(H))$ (resp. $POS_{ju}(V(H))$ and $NEG_{ju}(V(H))$) and defined by:

$$Bd_{ju}(V(H)) = U_{ju}(V(H)) - L_{ju}(V(H)), POS_{ju}(V(H)) = L_{ju}(V(H)),$$
$$NEG_{ju}(V(H)) = V(D) - U_{ju}(V(H)),$$

where $j = r, \alpha, s, p, \gamma, \beta$.

Proposition 4.4 Let $D = (V(D), E(D), \Omega)$ be a supra-approximation space and $H \subseteq D$. Then $L_u(V(H)) \subseteq L_{ju}(V(H)) \subseteq V(H) \subseteq U_{ju}(V(H)) \subseteq U_u(V(H))$ for all $j = \alpha, s, p, \gamma, \beta$ and $j \neq r$.

Proof. The proof is immediately derived from Proposition (3.2.14).

Remark 4.5 Proposition (4.4) is not necessarily true in the case of $j = r$ as the following example illustrates.

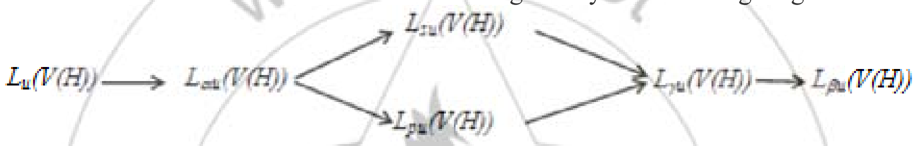
Example 4.6 Let $D = (V(D), E(D), \Omega)$ be a supra-approximation space which is give in Example (3.11) in [19].

$F = \{V(D), \phi, \{\varpi_1\}, \{\varpi_2\}, \{\varpi_3\}, \{\varpi_4\}, \{\varpi_5\}, \{\varpi_1, \varpi_4\}, \{\varpi_1, \varpi_5\}, \{\varpi_3, \varpi_4\}, \{\varpi_1, \varpi_3, \varpi_4\}, \{\varpi_2, \varpi_3, \varpi_5\}, \{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}\}$,

$\Omega = \{V(D), \phi, \{\varpi_1\}, \{\varpi_1, \varpi_4\}, \{\varpi_2, \varpi_5\}, \{\varpi_1, \varpi_2, \varpi_5\}, \{\varpi_2, \varpi_3, \varpi_4\}, \{\varpi_2, \varpi_3, \varpi_5\}, \{\varpi_1, \varpi_2, \varpi_3, \varpi_4\}, \{\varpi_1, \varpi_2, \varpi_3, \varpi_5\}, \{\varpi_1, \varpi_2, \varpi_4, \varpi_5\}, \{\varpi_1, \varpi_3, \varpi_4, \varpi_5\}, \{\varpi_2, \varpi_3, \varpi_4, \varpi_5\}\}$.

If $H = (V(H), E(H))$: $V(H) = \{\varpi_1, \varpi_3\}$, $E(H) = \{(\varpi_1, \varpi_1)\}$, then $L_m(V(H)) = \{\varpi_1\}$, $U_m(V(H)) = \{\varpi_1, \varpi_3, \varpi_4\}$, $L_{rm}(V(H)) = \{\varpi_1\}$, $U_{rm}(V(H)) = V(D)$. Obviously, $L_{rm}(V(H)) = L_m(V(H))$ and $U_{rm}(V(H)) \nsubseteq U_m(V(H))$.

Proposition 4.7 Let $D = (V(D), E(D), \Omega)$ be a supra-approximation space. Then the relations between supra-lower approximation and near supra-lower approximations of $H \subseteq D$ are given by the following diagram:



Proof. The proof is immediately derived from Proposition (4.4).

Proposition 4.8 Let $D = (V(D), E(D), \Omega)$ be a supra-approximation space. Then the relations between supra-upper approximation and near supra-upper approximations of $H \subseteq D$ are given by the following diagram:

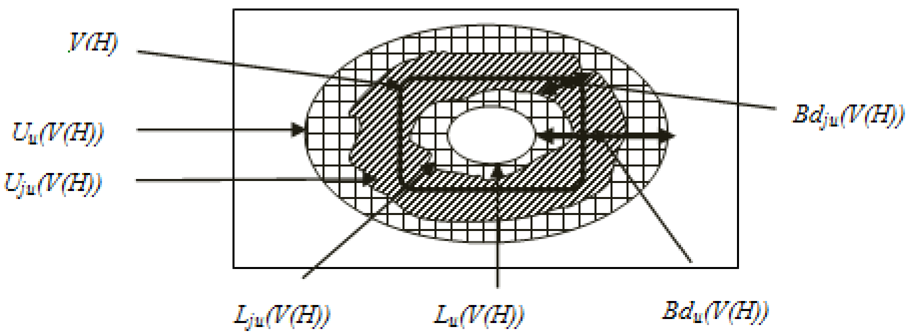
Proof. By using Proposition (4.4), the proof is obvious.

Proposition 4.9 Let $D = (V(D), E(D), \Omega)$ be a supra-approximation space, and let $H \subseteq D$. Then $Bd_{ju}(V(H)) \subseteq Bd_u(V(H))$ for all $j = \alpha, s, p, \gamma, \beta$ and $j \neq r$.

Proof. By using Proposition (4.4), the proof is obvious.

Remark 4.10 In general, the above proposition is not true in the case of $j = r$. Since in Example (3.11) in [19], if $H = (V(H), E(H))$: $V(H) = \{\varpi_2\}$, $E(H) = \phi$, then

$$Bd_m(V(H)) = U_m(V(H)) - L_m(V(H)) = \{\varpi_2\} - \phi = \{\varpi_2\},$$
$$Bd_{rm}(V(H)) = U_{rm}(V(H)) - L_{rm}(V(H)) = \{\varpi_2, \varpi_5\} - \phi = \{\varpi_2, \varpi_5\}.$$ So, $Bd_{rm}(V(H)) \nsubseteq Bd_m(V(H))$



Paper ID: ART2017834 DOI: 10.21275/ART2017834

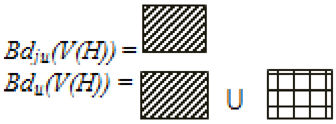
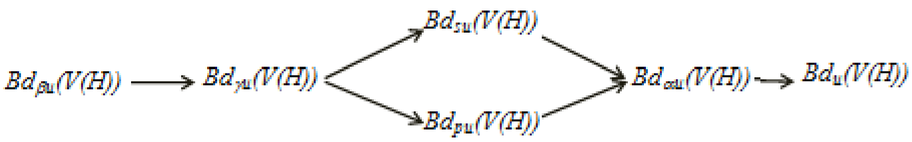


Figure 4.1: illustrates the relation between $Bd_u(V(H))$ and $Bd_{ju}(V(H))$ of $H \subseteq D$ in a supra-approximation space $D = (V(D), E(D), \Omega)$

Proposition 4.12 Let $D = (V(D), E(D), \Omega)$ be a supra-approximation space. Then the relations between supra-boundary approximation and near supra-boundary approximations of $H \subseteq D$ are given by the following diagram:



Proof. By using Propositions (4.7) and (4.8), the proof is obvio

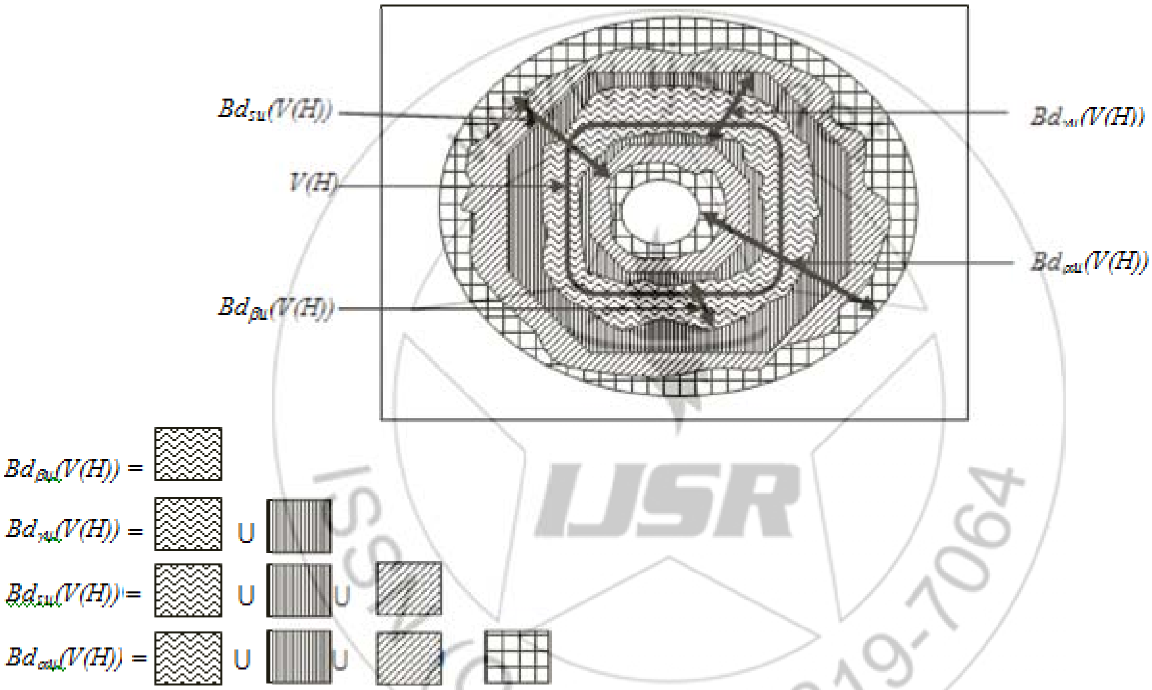
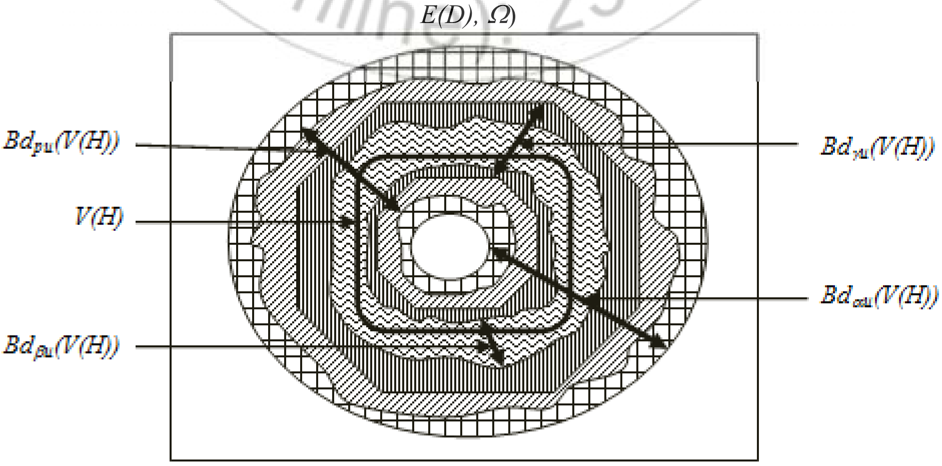


Figure 4.2: Illustrates the relation between $Bd_{ju}(V(H))$, for all $j = \alpha, s, \gamma, \beta$ of $H \subseteq D$ in a supra-approximation space $D = (V(D), E(D), \Omega)$



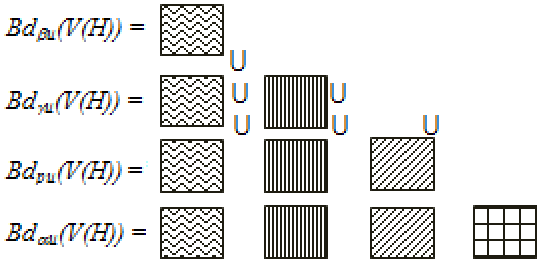


Figure 4.3: illustrates the relation between $Bd_{ju}(V(H))$ for all $j = \alpha, p, \gamma, \beta$ of $H \subseteq D$ in a supra-approximation space $D = (V(D), E(D), \Omega)$.

Definition 4.13 Let $D = (V(D), E(D), \Omega)$ be a supra-approximation space. The supra-accuracy of the approximations of $H \subseteq D$ is denoted by $\eta_u(V(H))$ and is defined by:

$$\eta_u(V(H)) = 1 - \frac{|Bd_u(V(H))|}{|V(D)|}.$$

Definition 4.14 Let $D = (V(D), E(D), \Omega)$ be a supra-approximation space. The near supra-accuracy of the approximations of $H \subseteq D$ is denoted by $\eta_{ju}(V(H))$ and is defined by:

$$\eta_{ju}(V(H)) = 1 - \frac{|Bd_{ju}(V(H))|}{|V(D)|} \text{ for all } j = r, \alpha, s, p, \gamma, \beta.$$

Proposition 4.15 Let $D = (V(D), E(D), \Omega)$ be a supra-approximation space and let $H \subseteq D$. Then $\eta_u(V(H)) \leq \eta_{ju}(V(H))$ for all $j = \alpha, s, p, \gamma, \beta$ and $j \neq r$.

Proof: By using Propositions (4.7) and (4.8), the proof is obvious.

Proof. By using Proposition (4.4), the proof is obvious.

In general, this proposition is not true in the case of $j = r$ as the following example illustrates.

Example 4.16 Let $D = (V(D), E(D), \Omega)$ be a supra-approximation space which is give in Example (3.11) in [19]. If $H = (V(H), E(H))$: $V(H) = \{\varpi_1, \varpi_2, \varpi_5\}$, $E(H) = \{(\varpi_1, \varpi_1), (\varpi_5, \varpi_2), (\varpi_5, \varpi_5)\}$, then $\eta_m(V(H)) = 3/5$ and $\eta_{rm}(V(H)) = 2/5$. Thus, $\eta_m(V(H)) \nleq \eta_{rm}(V(H))$.

Proposition 4.17 Let $D = (V(D), E(D), \Omega)$ be a supra-approximation space. Then the relations between supra-accuracy approximation and near supra-accuracy approximations of $H \subseteq D$ are given as follows:

References

[1] M. E. Abd El-Monsef, S. N. El-Deeb and R. A. Mahmoud,"β-Open Sets and β-Continuous Mappings," Bull. Fac. Sc. Assuit Univ., Vol.12, pp. 77-90, 1983.
[2] M. E. Abd El-Monsef, M. Shokry and Y. Y. Yousif, "Near Approximations in G_m-Closure Spaces," (ISRN) Applied Mathematics, Article ID 240315, 23 pages, doi:10.5402/2012/240315, Hindawi Publishing Corporation, USA, 2012.
[3] D. Andrijevic, "On b-Open Sets," Mat. Vesnik, Vol.48, pp.59-64, 1996.
[4] D. Andrijevic, "Semi-preopen Sets," ibid. Vol.38, pp.24-32, 1986.
[5] A. A. El-Atik, "A Study of Some Types of Mappings on Topological Spaces," M. Sc. Thesis, Tanta Univ., Egypt, 1997.
[6] N. Levine, "Semi-open Sets and Semi-Continuity in Topological Spaces," Amer. Math. Monthly, Vol.70, pp.36-41, 1963.
[7] A. S. Mashhour, M. E. Abd El-Monsef and S. N. El-Deeb, "On Pre Continuous and Weak Pre Continuous Mappings," Proc. Math. Phys. Soc. Egypt, Vol.53, pp.47-53, 1982.
[8] S. Mashhour, A. A. Allam, F. S. Mahmoud and F. H. Khedr, "On Supratopological Spaces," Indian J. Pure and Appl. Math. no.4, 18, 322-329, 1987.
[9] O. Njastad, "On Some Classes of Nearly Open Sets,"Pacific J. Math., Vol.15, pp.961-970, 1965.
[10] A. E. Radwan and Y. Y. Yousif, "Near Rough and Near Exact Subgraphs in G_m-Closure Spaces," International Journal of Computer Science Issues (IJCSI), Mauritius, Vol.9, Issue 2, No.3, pp.131-140, March 2012.
[11] M. Shokry and Y. Y. Yousif, "Closure Operators on Graphs," Australian Journal of Basic and Applied Sciences, Australia, Vol.5, No.11, pp.1856-1864, 2011.
[12] M. Shokry and Y. Y. Yousif, "Connectedness in Graphs and G_m-Closure Spaces," Journal of Computer Sciences, International Centre For Advance Studies, India, Vol.22, No.3, pp.77-86, 2011.
[13] M. Shokry and Y. Y. Yousif, "G_m-Closure Spaces on Digraphs and Near Boundary Region and Near Accuracy in G_m-closure Approximation Spaces," International Journal of Intelligent Information Processing, Korea, 2012. (Accepted, waiting publication).
[14] M. Shokry and Y. Y. Yousif, "Pre-Topology Generated by the Shortest Path Problem," International Journal of Contemporary Mathematical Sciences, Hikari Ltd, Bulgaria, Vol.7, pp.805-820, 2012.
[15] M. Stone, "Application of the Theory of Boolean Rings to General Topology," Trans. Amer. Math. Soc., Vol.41, pp.374-481, 1937.

[16] R. J. Wilson, "Introduction to Graph Theory," Fourth, 1996.
[17] Y. Y. Yousif, "Topological Generalizations of Rough Concepts," International Journal of Advanced Scientific and Technical Research, R S. Publication, Issue 5, Vol. 3, pp.265-272, May-June 2015.
[18] Y. Y. Yousif and Ahmed Issa Abdul-naby, "Rough and Near Rough Probability in G_m-closure Spaces," International Journal of Mathematics Trends and Technology, Seventh Sense Research Group, Vol. 30, No. 2, pp. 68-78, February 2016.
[19] Y. Y. Yousif and S. S. Obaid, "Topological Structures Using Mixed Degree Systems in Graph Theory," International Journal of Applied Mathematics & Statistical Sciences, International Academy of Sciences, Vol. 5, Issue 2, pp. 51-72, Feb-Mar 2016.