International Journal of Science and Research (IJSR)
ISSN (Online): 2319-7064
Index Copernicus Value (2015): 78.96 | Impact Factor (2015): 6.391

Virtual Lab to Run Logarithmic Damping Decrement Experiments

Javier Montenegro Joo

Virtual Dynamics: Science & Engineering Virtual Labs. Edo. Astete 574, Las Magnolias de Surco, Lima 33, Lima, Perú.
Director@VirtualDynamics.Org ---- www.VirtualDynamics.Org

Abstract: ***A university-level educational Virtual Lab that in order to detect the damping an oscillator is experimenting applies the technique of Logarithmic Damping Decrement has been created. After input parameters (mass, elastic constant and viscosity of the medium) are entered by the user of the simulation module, this displays the corresponding curve of displacement vs time, x(t), on computer screen. Next the module allows the user to manually click with the mouse on the peaks (or valleys) of the displayed curve and, once the user has clicked 10 of these, the module computes the logarithmic decrement and from this, the experimental damping of the oscillator. The user may repeat this stage by clicking 10 valleys (or peaks) of the x(t) curve. With the aim on obtaining reference theoretical results, right after input data has been entered the module automatically detects the extreme displacements of the x(t) curve and it applies the logarithmic decrement algorithm to these data and from this the damping of the system is calculated. At the end of the simulation the virtual lab shows the theoretical as well as the experimental results so that the user can compare them. In this way, if the user has correctly clicked the extremes (peaks and/or valleys) of x(t), his experimental results –as it is expected- are verified as being very close to the expected result calculated by the virtual lab.***

Keywords: Interactive learning, Virtual experiments, Virtual Lab, Damped oscillations, logarithmic decrement

1. Introduction

In a damped oscillating system [1], where the viscosity of the medium is not known, the Logarithmic Decrement may be used to experimentally find the damping the system is enduring [2]-[4], then from this damping the viscosity of the environment may be calculated. The logarithmic decrement is defined as the natural logarithm of the ratio of any two successive extreme displacements in a damped oscillation. Obviously these two maximum amplitudes x_{n+1} *and* x_n are separated by a certain time t, so that:

$$\frac{x_{n+1}}{x_n} = e^{-\lambda t}, \qquad t = t_{n+1} - t_n$$

Where λ is a constant to be experimentally determined.

The exponent is negative because in a damped system the amplitudes of the oscillations shrink. In the case with no damping, if the amplitudes of the oscillations would increase, the exponent would be positive and, if the amplitudes were constant, the exponent would be zero.

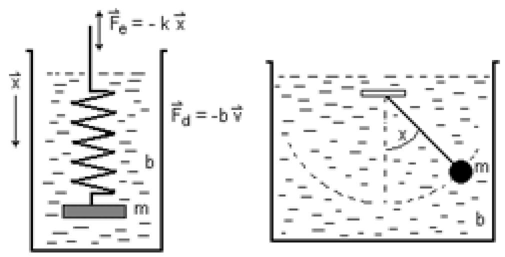

Figure 1: The oscillatory motion of the spring and the pendulum in the liquid is attenuated by the viscosity of the medium, whose viscosity constant is b and its damping force is F_d. The elastic force of the oscillating spring is F_e.

The equation above is valid provided the oscillations are uniform, this is, as long as the distance between orbit turns in State Space keeps constant. In Chaotic oscillators the displacements are far from being uniform and the State Space is literally chaotic, in the most common sense of the word [5], [6].

1.1 Damped oscillations

Damped oscillations (see Fig. 1) are characterized by the fact that the amplitudes of oscillation tend to reduce as time goes by. Obviously, the higher the damping, the quicker the oscillations shrinkage. Depending on the relationship between the natural frequency of the oscillator and the applied damping, damped oscillations are classified as Critical, Subcritical and Supercritical. In this paper the subcritical case is dealt with, this case is also known as that of underdamped oscillations. Figure 1 displays the most common models of oscillating systems preferred by physicists; these are the spring and the pendulum, and in this case both are immersed in liquid, which provides the viscosity the oscillators are experiencing.

1.2 The differential equation of the underdamped oscillator

From elementary university physics it is known that the differential equation of motion of a system oscillating in presence of a damping [1], [2] is

$$m\frac{d^2\vec{x}}{dt^2} = -k\vec{x} - b\vec{v} \qquad (1)$$

The first term on the right side of equation (1) is the reacting force of the spring (Hooke's Law) and the second term is the viscous damping, indicating that the faster the spring oscillates, the higher the resistance (minus sign) due to the viscosity b of the medium. Since the velocity is the first temporal derivative of the displacement, the scalar version of equation (1) is written as

$$\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$$

Here, two coefficients are identified:

$$\omega_o^2 = k/m \qquad G = b/2m \qquad (2)$$

Where ω_o is the natural frequency of the oscillator, this is the frequency of the free oscillator, and G is the damping, which depends on the viscosity b of the medium.

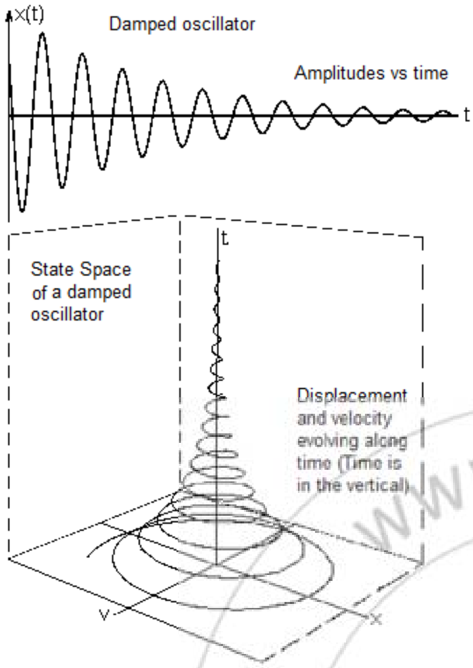

Figure 2: Top: time evolution x(t) of the oscillation for a damped oscillator. Bottom: the corresponding State Space, this is the 3D-plotting of displacement and velocity versus time. It can be seen in both graphs that the amplitudes of oscillation decrease as time elapses, until finally they stop. Notice that the distance between turns in the state space orbit is constant, which means that the oscillations are uniform. In chaotic oscillations these turns are messy

After inserting ω_o and G the differential equation becomes

$$\frac{d^2x}{dt^2} + 2G\frac{dx}{dt} + \omega_o^2 x = 0 \qquad (3)$$

In the underdamped case: $\omega_o > G$.

The solution of equation (3) is

$$x(t) = A\, e^{-Gt}\, Sin(\omega t + \alpha) \qquad (4)$$

After replacing eq. (4) in eq. (3) the following relationship between frequencies is obtained:

$$\omega^2 = \omega_o^2 - G^2 \qquad (5)$$

where ω is the theoretical angular frequency of the damped oscillator, and T is the period of the damped oscillations. As usual $T = 2\pi/\omega$. From equation (4) the velocity is

$$v(t) = \frac{dx}{dt} = Ae^{-Gt}[\omega\, Cos(\omega t + \alpha) - G\, Sin(\omega t + \alpha)]$$

$x(t)$ and $v(t)$ are used to construct the state space of the oscillations.

2. The Logarithmic Decrement

The logarithmic decrement [2]-[4] is based on the assumption that the shrinking in the maximum amplitude in an underdamped oscillation for any two successive oscillations, is given by

$$\frac{x_{n+1}}{x_n} = e^{-\lambda\, t} \qquad (6)$$

where it is assumed that the shrinking of the orbit in state space is constant. In this expression the values of x_n and x_{n+1} are the extreme displacements of any two successive oscillations, these are the peaks (or the valleys) in the x versus t plotting (see Figure 2) or the points where the (x, v) curve cuts the x-axis in the 2D version of the state space at angles 0 and π with the x-axis, respectively (see figure 3).

From elementary oscillation physics, the elapsed time between any two successive displacement extremes is a period T, hence eq. (6) must be rewritten as

$$\frac{x_{n+1}}{x_n} = e^{-\lambda\, T}$$

Then after applying logarithms to both sides:

$$\lambda = -\frac{1}{T}\ln\left[\frac{x_{n+1}}{x_n}\right] \qquad (7)$$

The period T in this equation is the period of the damped oscillations $T = 2\pi/\omega$. The value of λ in eq. (7) is obtained experimentally by averaging many cases of eq.(7).

On the other hand, applying eq.(4) to two successive amplitude peaks (separated in time by a period T):

$$\begin{cases} x_n = A\, e^{-G\,t}\, Sin(\omega t + \alpha) & (8) \\ x_{n+1} = A\, e^{-G(t+T)}\, Sin(\omega(t+T) + \alpha) & (9) \end{cases}$$

From eq. (9):

$$x_{n+1} = A\, e^{-G(t+T)}\, Sin(\omega t + \alpha + \omega T)$$

but $\omega = 2\pi f = 2\pi/T$, then:

$$x_{n+1} = A\, e^{-G(t+T)}\, Sin(\omega t + \alpha + 2\pi)$$

$$x_{n+1} = A\, e^{-G(t+T)}\, Sin(\omega t + \alpha) \qquad (10)$$

In this way from (8) and (10):

$$\frac{x_{n+1}}{x_n} = e^{-G\,T}$$

and taking logarithms to both sides of this equation:

$$G = -\frac{1}{T}\ln\left[\frac{x_{n+1}}{x_n}\right] \qquad (11)$$

Paper ID: ART2017747 DOI: 10.21275/ART2017747

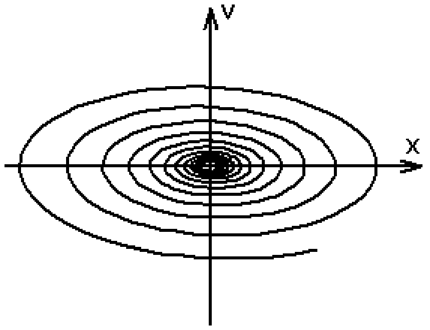

Figure 3: Projection of the (x, v, t) points of the 3D State Space over the XV-plane, for a damped oscillator. This is a 2D version of the State Space. In this research the values of extreme displacements, this is, the points intersecting the x-axis in this plotting are used to feed eq. (6). The peaks of the displacement are the intersections of the curve with the x-axis at 0^o and the valleys are those at 180^o.

From equations (11), (7) and (5):

$$\lambda = G = \sqrt{\omega_o^2 - \omega^2} \qquad (12)$$

In Chaos Theory [5], [6] the computational detection of peaks and valleys, this is, the extremes of the x(t) curve, used in this experiment is tantamount to extracting the Poincaré Maps at 0° and at 180° respectively (see figure 4). Chaos theory does not work precisely with the logarithmic decrement, but the algorithm to extract the Poincaré Map is similar to that used in the present work to extract the oscillation extremes. Hence for someone who has developed the computer programs to extract the Poincaré Map in a chaotic system, the logarithmic decrement is not so unfamiliar [7]-[9].

3. Some Reported Applications of the Logarithmic Decrement

Magalas [10], [11] reports the creation of a new algorithm to compute the logarithmic decrement with high precision. Magalas introduces the Optimization in Multiple Intervals algorithm (OMI), which is recommended for measurements when the damping is rather high.

Butterworth et al [12] present an application of the logarithmic decrement in civil engineering, specifically to assess the dynamic response of structures (buildings) to vibrations due to earthquakes.

Montenegro Joo [13] reported the creation of a Virtual Lab, which after generating the curve of displacement versus time x(t) based on user data, makes an automatic analysis of the curve to detect its frequency and, from this the damping the oscillator is experiencing.

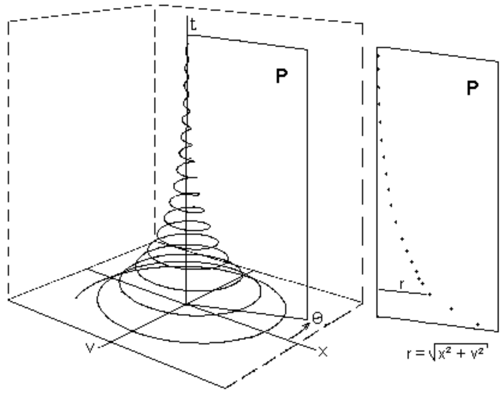

Figure 4: The Poincaré Plane P, may be seen as a tomographic cut along time in the 3D State Space. This plane P, is defined at some angle with the x-axis. The Poincaré Map is the set of all the intersections of the (x, v, t) curve with the plane P, at a predefined angle. It can be seen in this sketch that the peaks and valleys of the x(t) curve are those tomographic cuts at 0^o and at 180^o, respectively.

The virtual lab reported in the present paper is an improved and more advanced version of its predecessor [13] because now the module operates additionally on data supplied by the user by clicking on computer screen on the extremes of the x(t) curve. The module extracts information from these clicks to calculate the oscillation frequency and from this, the damping the oscillator is undergoing. Obviously this new version of the Virtual Lab fosters learning by intense interaction of its user with the computer simulation.

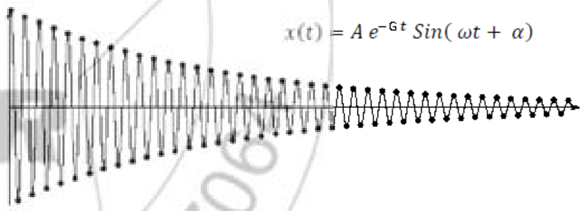

Figure5: Plotting of displacements versus time x(t) for the damped oscillator. This plotting has been generated by the Virtual Lab being reported. Peaks and valleys have been highlighted (black dots on the curve) after automatically extracting the Poincaré Maps at 0^o and at 180^o, respectively.

4. Executing the Experiment

The virtual lab being reported operates in two stages, automatic and manual, in the former stage –which the user is not aware of- theoretical results based on automatic detection of the extremes of x(t) are generated, while the latter stage is based on the user's interaction with the computer and generates experimental results. At the end of every simulation the user is enabled to compare theoretical with experimental results.

At the very beginning of an experiment, the virtual lab receives as input data the mass m of the oscillator, its elastic constant k as well as the damping b of the medium. Additionally the module reads the maximum amplitude "A" of the oscillations as well as the initial phase α. Also the time-steps of the simulation may be entered as part of the input data. In order to facilitate the input of user data, these are entered by means of scrollbars. The automatic process performed by the reported virtual lab may be appreciated in the flowchart in figure 6.

The maximum amplitude of oscillation, the initial phase and the time-steps are included so that the user of the virtual lab is enabled to appreciate the effects of changing them, entering these data is not expressly fundamental for execution of the virtual experiment, because they have been included by default, however, larger values of “A” generate larger amplitudes of oscillation, and these are easier to visualize when displayed on computer screen.

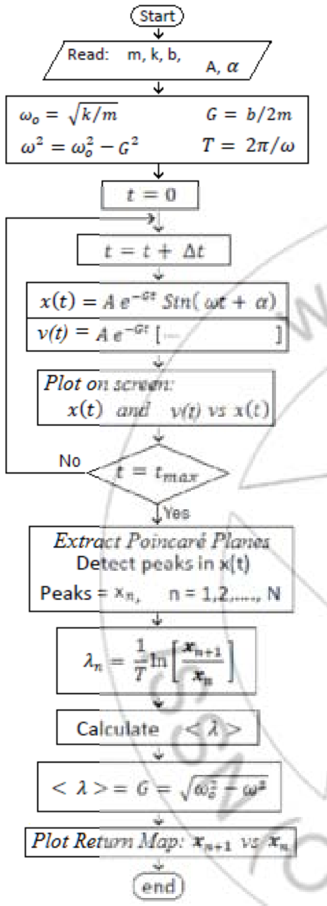

Figure 6: Flowchart of the automatic process performed by the Virtual Lab

Once the input data is entered the module automatically computes from eqs.(2) the theoretical angular frequency ω_o of the free oscillator and the theoretical damping G of the medium. Additionally from eq. (5) the theoretical value of the angular frequency of the damped oscillations ω is computed, as well as the period T of the damped oscillations.

Next the simulation of eq. (4) is executed during enough time for the oscillations to vanish. Notice that this is not indispensable at all, it would be enough running the experiment during a time equivalent to a few periods so that some values of x_n are available to evaluate eq. (7).

Once the simulation starts, the curve of displacements versus time x(t) as well as the state space are both simultaneously depicted on computer screen, as time elapses (see figures 3 and 5), and the maximum displacements (peaks and valleys) are automatically detected by extracting Poincaré maps at 0^o *and at* 180^o, respectively. Then equation (7) is evaluated for every two successive maximum amplitudes of oscillation andthe average λ is calculated and used together with eq. (12) to obtain the experimental value of the damping G of the system and the frequency ω of the damped oscillations.

The virtual lab automatically detects all the peaks and valleys in the curve of displacement vs time, x(t), however only a few peaks or valleys are necessary to this experiment; this may be appreciated next in the second stage of the experiment.

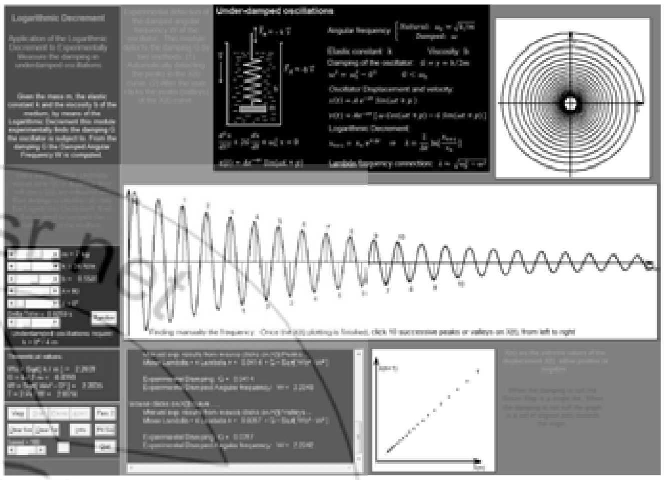

Figure 7: Screenshot of the logarithmic-decrement Virtual Lab. Besides the plotting of displacement vs time x(t), the module shows a 2D projection of the State Space and a Return Map. The numbered (from 1 to 10) peaks and valleys on the x(t) curve are those mouse-clicked by the user of the virtual lab.

In the manual stage, the module allows the user to identify with the naked eye 10 extremes (peaks) of the displacement vs. time -x(t)-curve displayed on screen. Right after the user clicks them with the mouse, the module computes the average logarithmic decrement λ and from this, the damping G the oscillator is subject to as well as the angular frequency ω of the damped oscillations. Usually these resulting data are close to their theoretical values calculated right at the start of the experiment. Next the user may click the valleys of the x(t) curve and obtain similar results. Once the user clicks on computer screen 10 peaks (valleys) of x(t) and the computer reports the corresponding experimental data, she can additionally click 10 valleys (peaks) and, another set of experimental data are calculated.

A final checking verifies that the experimental values of G and ω are very close (if not equal) to their theoretical values.

4.1 The Map of Return

The virtual lab displays also the Map of Return for each experiment, which is the plotting of extreme displacements x(n+1) vsx(n).In this plotting the user may see that in the damped oscillations this graph is a rather simple set of aligned dots going towards the origin.It is instructive to compare this plotting with the return map of a chaotic oscillator; it results evident that in the latter case the plotting is not so easy to comprehend [14].Many times the return map of a chaotic oscillator seems to have the self-similarity typical of fractal structures (see figure 9).

```
<><><>  Experiment N = 1     <><><>
Input data .....
    Mass  m = 7 kg
    Elastic constant  k = 34 N/m
    Viscosity  b =  0.5500
    Amplitude = 90 °
    Initial phase = 0 °   =  0.0000 rad
    Delta time =  0.0250 s
Theoretical results:
Damping of the oscillator  G =  0.0393
Natural Angular frequency  Wo =  2.2039 rad/s
Damped angular frequency   W =  2.2035 rad/s
Damped Oscillations period  T =  2.8514
N time-steps = 2500
Experiment duration =  62.5000 s

Mean Lambda = < Lambda > =  0.0393

Automatic experimental results:   < Lambda > = G
Experimental Damping:  G =  0.0393
Experimental Damped Angular freq.:  W =  2.2042

Mouse clicks on X(t) curve .......
Results from mouse clicks on X(t) Peaks ...
Mean Lambda = < Lambda > =  0.0414  = G ]

Experimental Damping:  G =  0.0414
Experimental Damped Angular freq.:  W =  2.2043

Mouse clicks on X(t) curve .......
Results from mouse clicks on X(t) Valleys ...
Mean Lambda = < Lambda > =  0.0397  = G ]

Experimental Damping:  G =  0.0397
Experimental Damped Angular freq.:  W =  2.2042
<><><><><><><><><><><><>
```

Figure 8: Simulation report generated by the virtual lab during an experiment. Right after input data are entered by the user, theoretical results are obtained. Then the module automatically detects the oscillation extremes and it calculates the logarithm decrement and from this, the damping of the system. Finally after the user clicks with the mouse the extremes of the x(t) curve, the value of the damping is calculated. In this way the user can easily and quickly compare theoretical and experimental computations.

5. Demonstrative execution of the Virtual Lab

In figure 8, a typical report generated by the simulator during an experiment is displayed. In this case the module has been fed with random input data. It may be seen that once input data are entered, theoretical results are computed. Then the module automatically detects the oscillation extremes and computes the logarithm decrement and from this, the damping of the system. In the manual part the user clicks with the mouse the extremes of the displacement curve and the value of the damping is obtained.

It can be seen that the experimental value of the damping G is in agreement with its theoretically calculated value.

6. Random input data

In order to enhance the performance of the virtual lab, this is enabled with the option of feeding it with random input data. In this way the user has to run the experiment and verify the results corresponding to data which are not his own. This random option avoids experimenting with any biased or subjective input data.

Random input data implies experimenting in a short time with a very large number of input-data combinations, which is rewarding to the user of the virtual lab, because she feels that the computer really makes it easy.

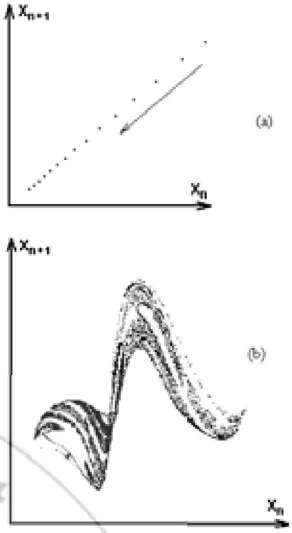

Figure 9: Maps of Return, the plotting of oscillation extremes x_{n+1} vs x_n (a) For the damped oscillator reported in this paper (b) For a chaotic oscillator, namely the nonlinear damped and forced oscillator. In the first case the aligned dots go towards the origin, because due to the damping the oscillation amplitudes continuously decrease. In the second case the plotting is rather complex and seems to be self-similar (fractal structure).The map of return in (a) includes only a few dots, while that in (b) includes several thousand dots.

7. The Advantage of Virtual experiments

The Logarithmic decrement module described in this paper is a good example of interactive learning via virtual experiments based on computer simulation.

In real life an experiment dealing with the logarithmic decrement would require a conventional laboratory with either a pendulum or a spring immersed in a liquid (see Fig.(1)), and enabled with the ability of changing the viscosity of the medium, which would require liquids of different densities. Obviously all this would be tedious, time demanding and limited to just a few viscosities. Additionally a traditional lab would require personnel to clean and to prepare the experiment room and to manage the equipment before and after the experiment.

8. Conclusions

A virtual lab to execute experiments dealing with the Logarithmic Decrement of the oscillation amplitudes for an underdamped oscillator has been developed. Giving the mass of the oscillator as well as its elastic constant and the viscosity of the medium, as input data, the developed module experimentally finds the damping of the oscillator by means of the logarithmic decrement method.

The simulation-based Virtual Lab reported in this paper sheds light on the fact that in real life situations, the damping of the oscillator and the viscosity of the medium in which the oscillator is vibrating, may be found by measuring a few successive extremes of the oscillation amplitudes and using the logarithm decrement, with no need of using computers.

The reported virtual module has been incorporated into the Physics Virtual Lab (PVL) [15], created some time ago by this author and which is a collection of university-level intuitively-easy-to-use physics simulators, in this way the PVL is continuously improved.

References

[1] Alonso M., Finn E. J., (1967) Fundamental University Physics, Addison-Wesley
[2] Balachandran Balakumar, Magrab E., (2004) Brooks-Cole Editors
[3] Thomson W., (2003) Theory of vibration with applications, Stanley Thormes Pub.
[4] Smith W. F., (2010) Waves and Oscillations, Oxford University Press.
[5] Montenegro Joo J., (2014) Multiplicity and Transitoriness of Chaotic Events, RIF-UNMSM 17(2)
[6] Montenegro Joo J., (2016) Variety and Finiteness of Chaotic Events in the NLDF Oscillator, Chaos and Complexity Letters, 10 (1).
[7] Montenegro Joo J., (2016) Velocity behaviour during chaotic events in the NLDF Oscillator, Chaos and Complexity Letters, 10 (2).
[8] Montenegro Joo J., (2016) Diversity and temporality of chaotic events, Industrial Data 19(1)
[9] Montenegro Joo J., (2016) The Higher the Damping, the Lower the Chaos Intensity, Chaos and Complexity Letters 10(3).
[10] Magalas L.B., (2006) Determination of the Logarithmic Decrement in Mechanical Spectroscopy, Solid State Phenomena Vol 115.
[11] Magalas L.B., Majewski M., (2008) Recent Advances in Determination of the Logarithmic Decrement and the Resonant Frequency in Low-Freq. Mech. Spectroscopy, Solid State Phenomena Vol. 137.
[12] Butterworth J., Jin Hee Lee, Davidson B., (2004) Experimental determination of modal damping from full scale testing, 13th World Conference on Earthquake Engineering, Vancouver, B.C., Canada August 1-6, 2004, Paper No. 310.
[13] Montenegro Joo J., (2016) Use of the Logarithmic Decrement to Assess the Damping in Oscillations, RIF-UNMSM 19(1).
[14] Montenegro Joo J., (2017) Viscosity levels and strength of chaos, International Journal of Science and Research (IJSR), 6(1).
[15] Montenegro Joo J., (1997) Physics Virtual Lab (PVL), VirtualDynamicsSoft.

Author Profile

Javier Montenegro Joo (JMJ), studied Physics and Computation (Computer Science) at San Marcos University in Lima, Perú. His graduate studies were in USA (Computational Physics) and in Brazil (Computer Vision). Currently JMJ is a university professor and he is founding Director of VirtualDynamics: Science & Engineering Virtual Labs. JMJ has developed virtual labs in the areas of Nonlinear Dynamics, Stochastic Aggregation, Cellular Automata, Percolation, Neural Networks, Genetic Algorithms, Digital Image Processing, Chaos, etc. JMJ's most successful brainchild is the Physics Virtual Lab (PVL), a collection of at least 191 intuitively-easy-to-use physics simulation modules, used by several educative institutions worldwide. JMJ has also created Imagery, a Digital Image Processing Virtual Lab. This author has been an Associate Member to the ICTP (Italy) along some years.In his spare time JMJ creates images of Algorithmic Art, which are based on mathematical and logical algorithms, devised by him.