

# Use of software and Extended Precision of Finney's Table Using Probit Analysis

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**Abstract:** In stochastic analyses, the use of quintile function is very common. For normal distributions which are observed in many of the phenomena in the nature or in laboratory setting, probit analysis is used for regression-based expression of data which show a binary response. In this context, the table of Finney is much used today. The present paper uses an approximation formula for a related function and modifies it resulting in a formula for extended precision of the Finney table values. Additionally, the author has created a software implementation of the same in the widely popular HTML5 format which can be used in almost any computing environment for greater computational access in a multitude of contexts which involves the table of Finney.

**Keywords:** probit analysis, stochastic analyses, table of Finney, HTML5

## 1. Introduction

In various experiments and succeeding analyses in science, various trends can be analyzed statistically but, cannot be predicted precisely. In these stochastic analyses, there is a need of proper characterization of the results from the experiment. One of the common methods of such characterization is the use of quintile function[1], apart from other means like the cumulative distribution function, characteristic function or the probability density function. The function is used as a part of the very popular set of "Monte-Carlo methods" whose usage transcends throughout numerous disciplines[3]. The quintile function calculates the value at which the probability of the random variable is lesser than or equal to the given probability, and has numerous uses in Biology and other branches of science. For normal distributions, the form of quintile function used is the 'probit' function. Specifically, in many contexts, the function is useful for specialized modeling of regression in variables which show a binary response.

An example of a common use of this kind of analysis is in the context of dose experiments in applied biology, but it is used in many other cases of probability analyses. However, the dose experiments are notable in the sense that the function was first widely used and popularized by the use of this method in estimating the effects of pesticides on pests[1]. Additionally, a major part of the modern use of this function is in toxicological experiments.

The further development of probit and its application in toxicological realms was made by Finney[2]. Among other developments, he notably used a table (called by his name in modern days) which is used in the laboratory for data analysis till this day. The tabulated values in the table do not stem from the use of the general probit function directly, but is a variation, with a modification of the function which was done to prevent negative values[2]. Unless this addition is taken into account, the values of the probit function will not match and the uses of the method by Finney using the table would not work.

In this context, two basic things are addressed. One is the creation of the formula for the tabular values from the table by Finney with extended precision using elementary functions. The second is the software implementation of the same in a cross-platform standard that can be accessed by a majority of devices worldwide, in the widely popular HTML5 standard.

The probit function is a non-elementary function and does not have a closed mathematical form. This requires the user to compute the function by carefully using the numerical methods, and approaches differ depending on the precision required and the computing power available. The function is stated as:

$$\text{Probit}(p) = \sqrt{2} * \text{erf}^{-1}(2p-1)$$

Here, difficulties lie in the calculation of the inverse error function due to its non-elementary nature. The function is available in numerous software, including Microsoft Excel (which is a commercial software) and programming languages such as R (can be difficult to implement by non-specialists requiring a Graphical User Interface(GUI)). It can be calculated by forming a non-linear ordinary differential equation by the method of Steinbrecher et al [4], but although it can be very precise, with precision as much as needed being computable, those methods have the drawbacks of being computationally taxing and algorithmically intensive. For example, the raw series expansion of the inverse error function is –

$$\text{erf}^{-1}(z) = \frac{1}{2}\sqrt{\pi} \left( z + \frac{\pi}{12} z^3 + \frac{7\pi^2}{480} z^5 + \frac{127\pi^3}{40320} z^7 + \frac{4369\pi^4}{5806080} z^9 + \frac{34807\pi^5}{182476800} z^{11} + \dots \right).$$

The above expansion can be computationally taxing if large sets of data are used. As a viable alternative, we can approach the above function with suitable approximations, while still maintaining higher levels of precision than what can be achievable by using data from the Finney's table alone. This will be useful for, among others, toxicological experiments where the table is used often. Being in a cross-platform language, the calculation of the probit function can be done with the majority of computers and mobile devices.

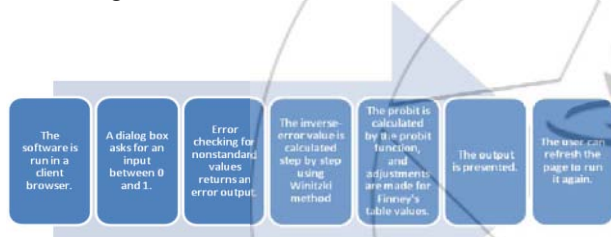
The latter can be particularly effective when an out-of-lab experiment is done, or for extended stays in research areas which can be less connected to the usual conveniences of a laboratory.

The approximation function used here is from a paper by Sergey Winitzki [5]. In that paper, the error function is put to a form that is easily inverted analytically, forming the below form of the inverse error function-

$$\operatorname{erf}^{-1} x \approx \left[ -\frac{2}{\pi a} - \frac{\ln(1-x^2)}{2} + \sqrt{\left( \frac{2}{\pi a} + \frac{\ln(1-x^2)}{2} \right)^2 - \frac{1}{a} \ln(1-x^2)} \right]^{1/2}$$

From this form, the adjustments are made for the match of the values with the Finney's table which avoids any negative values. The precision achieved by this method is better than  $4 \cdot 10^{-3}$ , and thus correlates with the data from the Finney's table very well. Additionally, the software allows values for multiple decimal points, thus improving over the precision that can be achieved by using the table alone. The final form of the equation is put in an user interface shell so that the procedure can be run with people not familiar with using the programming languages by calling on the functions itself. Thus, the program has a GUI for easy use.

The basic algorithm of the software is as follows:



**Figure 1:** A brief algorithm of the process in the software

The software is available in the link below, where it runs in the browser. The requirements in the browser include the compulsory support for HTML5 canvases. The software has been tested in multiple desktop, laptop and mobile device and found to run successfully in most of them, all of which included the HTML5 support in the browser of choice. The use of dedicated graphical elements in the GUI is avoided to conserve resources and to ensure cross platform compatibility. Using the language of HTML5, the final software uses graphical elements of the client browser, improving the speed and reliability considerably. Variants of the program, which is not demonstrated here, can batch process data and can improve the time advantage drastically.  
<https://bishnu.itch.io/probit-finneys-table-extended-precision-using-elementary-functions>

## 2. Conclusion

Thus we can have a better precision of data analysis through the Finneys table in probit analyses using the form of the inverse error function by Winitzki with modifications for use in the probit function. Additionally, the universal format of the software helps in using almost any computing devices to get the values required in the probit analysis. Thus the

method can be applied to numerous contexts including research labs, commercial applications and even by non-experts for stochastic analyses in day to day problems.

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## Author Profile



**Bishnu Goswami** received the B.Sc Honours degree with first class from Burdwan Raj College under The University of Burdwan in 2015. He qualified UGC-NET JRF having Rank 61 in India in the year 2016. He is also going to obtain M.Sc degree (with Aquaculture and Fisheries specialization) from the Department of Zoology, the university of Burdwan shortly. He published: Books:2 nos and Papers:4



**Jagatpati Tah** rendered his service as PG teacher for 22 years, visited countries - USA, Canada, France including SAARC countries Bangladesh, Nepal and Bhutan. Published 352 papers and 12 books in Bengali and English, including two textbooks. He obtained his degrees-B.Sc, M.Sc and PhD from the University of Burdwan. Received more than 30 awards including Mother India, FLS, FISCA, Science Flame etc. He has participated in 31 international and 72 national seminar conferences till date serving as chairman and co-chairman in various national and international conferences.  
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