Some New Bi-Measures of Fuzzy Entropy

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Abstract: A bi-measure of fuzzy entropy is the sum of two functions, at least one of which must be a measure of entropy. Some new parametric measures of fuzzy entropy have been deduced.

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1. Introduction

In 1965, Zadeh [5] introduced fuzzy sets. A fuzzy set $\mathcal{A}$ is represented as $\mathcal{A} = \{x_i / \mu_{\mathcal{A}}(x_i); i = 1,2,\ldots,n\}$ where $\mu_{\mathcal{A}}(x_i)$ is a membership function.

A measure of fuzzy entropy $F(\mu_{\mathcal{A}}(x_i))$ in a fuzzy set $\mathcal{A}$ should satisfy at least the following properties.

1. It should be defined for all $\mu_{\mathcal{A}}(x_i)$ in the range of $0 \leq \mu_{\mathcal{A}}(x_i) \leq 1$, $i = 1,2,\ldots,n$.
2. $F(\mu_{\mathcal{A}}(x_i))$ should be continuous in this region.
3. $F(\mu_{\mathcal{A}}(x_i)) = 0$ when $\mu_{\mathcal{A}}(x_i) = 0$ or 1.
4. $F(\mu_{\mathcal{A}}'(x_i)) = F(1 - \mu_{\mathcal{A}}'(x_i))$.
5. $F(\mu_{\mathcal{A}}(x_i))$ should be maximum when $\mu_{\mathcal{A}}(x_i) = \frac{1}{2}$, $i = 1,2,\ldots,n$.
6. It should be increasing function of $\mu_{\mathcal{A}}(x_i)$ when $0 \leq \mu_{\mathcal{A}}(x_i) \leq \frac{1}{2}$ and other variables are kept fixed and it should be decreasing function of $\mu_{\mathcal{A}}(x_i)$ when $\frac{1}{2} \leq \mu_{\mathcal{A}}(x_i) \leq 1$ and other variables are kept fixed.

Fuzzy entropy corresponding to Shannon’s [4] measure of entropy is:

Fuzzy entropy corresponding to Kapur’s [2] measure of entropy is:

Fuzzy entropy corresponding to Ranyi’s [3] measure of entropy is:

Fuzzy entropy corresponding to Havrda and Charvat’s [1] measure of entropy is:

$F_2(\mathcal{A}) = \frac{1}{1-\alpha} \left( \sum_{i=1}^{n} \left( \mu_{\mathcal{A}}^\alpha(x_i) + (1 - \mu_{\mathcal{A}}(x_i))^\alpha \right) \right) - 1$; $\alpha \neq 1, \alpha > 0$.

$F_3(\mathcal{A}) = \frac{1}{1-\alpha} \left( \sum_{i=1}^{n} \ln \left( \mu_{\mathcal{A}}^\alpha(x_i) + (1 - \mu_{\mathcal{A}}(x_i))^\alpha \right) \right) - 1$; $\alpha \neq 1, \alpha > 0$.

$F_4(\mathcal{A}) = - \sum_{i=1}^{n} \left( \mu_{\mathcal{A}}(x_i) \ln \mu_{\mathcal{A}}(x_i) + (1 - \mu_{\mathcal{A}}(x_i)) \ln(1 - \mu_{\mathcal{A}}(x_i)) \right)$

$+ \frac{1}{a^2} \sum_{i=1}^{n} \left( (1 + a \mu_{\mathcal{A}}(x_i)) \ln (1 + a \mu_{\mathcal{A}}(x_i)) + (1 + a (1 - \mu_{\mathcal{A}}(x_i)) \ln(1 + a (1 - \mu_{\mathcal{A}}(x_i))) \right)$

$- 2 \frac{1}{a} \ln (1 + a)$, $a \geq -1$.

$F_5(\mathcal{A}) = - \sum_{i=1}^{n} \left( \mu_{\mathcal{A}}(x_i) \ln \mu_{\mathcal{A}}(x_i) + (1 - \mu_{\mathcal{A}}(x_i)) \ln(1 - \mu_{\mathcal{A}}(x_i)) \right)$

$+ \frac{1}{a^2} \sum_{i=1}^{n} \left( (1 + a \mu_{\mathcal{A}}(x_i)) \ln (1 + a \mu_{\mathcal{A}}(x_i)) + (1 + a (1 - \mu_{\mathcal{A}}(x_i)) \ln(1 + a (1 - \mu_{\mathcal{A}}(x_i))) \right)$

$- 2 \frac{1}{a^2} \ln (1 + a)$, $a \geq -1$.
2. Some new Bi-Measures of fuzzy entropy

Consider the measure

$$F_{a,b,k}(A) = -\sum_{i=1}^{n} (\mu_{A}(x_i) \ln \mu_{A}(x_i) + (1-\mu_{A}(x_i) ) \ln(1-\mu_{A}(x_i) ) +$$

$$\frac{b}{a^k} \sum_{i=1}^{n} ((1+a\mu_{A}(x_i)) \ln(1+a\mu_{A}(x_i)) + (1+(a-1)\mu_{A}(x_i)) \ln(1+a(1-\mu_{A}(x_i))) )$$

$$- 2 \frac{b}{a^k} (1+a) \ln(1+a)$$

where $a,b,k$ are parameters.

Here $F_{a,b,k}(A)$ is defined for all $\mu_{A}(x_i)$, in the range $0 \leq \mu_{A}(x_i) \leq 1$, $i=1,2,...,n$ and it should be continuous in this range.

Now $F_{a,b,k}(A)$ can be written as $\sum_{i=1}^{n} f(x_i)$, where

$$f(x) = -x \ln x + (1-x) \ln(1-x) + \frac{b}{a^k} ((1+a)x \ln(1+ax) + (1+(a-1)x) \ln(1+a(1-x)) )$$

$$- 2 \frac{b}{a^k} (1+a) \ln(1+a)$$

Then

$$f'(x) = -\ln x + \ln(1-x) + \frac{b}{a^k} (a \ln(1+ax) - a \ln(1+a(1-x)) )$$

$$f''(x) = \frac{1+(a-c)x}{x(1+ax)} - \frac{1+(a-c)(1-x)}{(1-x)(1-a(1-x))} ; \quad c = \frac{b}{a^k}.$$

Hence $F_{a,b,k}(A)$ will be concave, if $(k-2) \ln a \geq 0$ or $(k-1) \ln a \geq 0$. ..........(2)

$$1+(a-c)x \geq 0 \quad \text{and} \quad 1+(a-c)(1-x) \geq 0$$

But it is true when

$$1 \geq c \text{ or } a \geq c \quad \text{i.e. when } a^{k-2} \geq b \text{ or } a^{k-1} \geq b.$$ ..........(1)

3. Special Cases:

1) When $a^{k-2} \geq b$ or $a^{k-1} \geq b$, $F_{a,b,k}(A)$ represent a three parametric bi-measure of fuzzy entropy.

When $b=1, k=1,$

$F_{a,b,k}(A) = F_4(A).$

2) When $b=1, k=2,$

$F_{a,b,k}(A) = F_5(A).$

3) When $0 < b \leq 1$, then (1) will be hold if

4) When $0 < b \leq 1$, then (1) will be hold if

5) When $b > 1$, we can find suitable value of $a$, $k$ using (2). Thus in any case, the family of bi-measure of fuzzy entropy is much larger than $F_4(A)$ and $F_5(A)$.

References


