New Optics Solutions for the Nonlinear (2+1)-Dimensional Generalization of Complex Nonlinear Schrödinger Equation

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Abstract: ***In this paper, new optic solutions of (2+1)-dimensional generalization of complex nonlinear Schrödinger equations are obtained via the powerful Extended Jacobian Elliptic functions expansion method. When the modulus $m \to 1$ or $m \to 0$, the doubly periodic solutions degenerate soliton solutions including bright or dark solitons.***

Keywords: Extended Jacobian Elliptic function, Complex Schrodinger evolution equations, Solitary wave solution, Soliton solutions.

1. Introduction

Exact solutions may describe not only the propagation of nonlinear waves but also spatially localized structure of permanent shape that may be of interest to experiments [10]. Since the inverse scattering transformation (IST) was presented [1], there has been increasing interesting in searching for new soliton equations and the related issue of the construction of exact solutions to a wide class of nonlinear soliton equations in soliton theory. Up to now, many powerful methods have been developed such as inverse scattering transformation (IST) [10], [1]. Bücklund transformation [10], [21], Cole-Hopf transformation [35], Hirota's bilinear method [26], Tanh method [27], Extended Sech function method [12], [13], the Weierstrass elliptic function method [8]. The Jacobi elliptic function expansion metod and the extended Jacobi elliptic function expansion metod [3], [23], [24], [25] and so on. Special exact solutions of evolution equations may be found by using direct ansätz methods. To construct the proper ansätz, a clue may be given from Painléve analysis, which is based on seeking solutions whose movable critical points are poles only. Thus, the use of elliptic function in the ansätz is rather natural because they are the most general functions having such singular points and has the relations with nonlinear equations. Up to now, in the ansätz, four theta functions [2], three Jacobian elliptic functions [2], four Jacobian elliptic functions [3], and one Weierstrass elliptic function [8] has been used. In fact, there are twelve Jacobian elliptic functions including Jacobian elliptic sine function, Jacobian elliptic cosine function, Jacobian elliptic and function of the third as well as nine Jacobian elliptic functions defined by Glaisher [4], [5].

In this paper, we present an extended Jacobian elliptic function method [2] ,[8], [23], [24] and its algorithm with symbolic computation to construct new doubly-periodic solutions for the (2+1)-dimensional generalization of coupled nonlinear Schrödinger equations which was presented by Maccari [8], [21]:

$$iu_t + u_{xx} - uv = 0$$
$$iw_t + w_{xx} - wv = 0 \qquad (1)$$
$$v_y - \left(|u|^2 + \delta |w|^2\right)_x = 0$$

The paper is organized to discuss the extended Jacobian elliptic function method considered and its algorithm in section 2. A full analysis of the method considered applied to (1) is given in section 3. Finally, Illustrations of special cases of some solutions are plotted with chosen parameters.

2. The extended Jacobian elliptic function algorithm

For a given system of nonlinear partial differential equation (NLPDE) of the form:

$$\mathbf{P}(\mathbf{u}(\mathbf{x}), \mathbf{u}'(\mathbf{x}), \mathbf{u}''(\mathbf{x}), \cdots) = \mathbf{0} \qquad (2)$$

where the components of the dependent variable $\mathbf{u}$ are $u, v, w, \cdots$ and the components of the independent variable $\mathbf{x}$ are x, y, z, t .

Step 1:
We look for travelling wave solution in the form:

$$u_i(\mathbf{x}) = U_i(\xi), \quad \xi = kx + ly + \lambda t \qquad (3)$$

where k, l and λ are arbitrary constants. Substituting (3) into (2) gives rise to a system of nonlinear ordinary differential equations (NLODEs):

$$\mathbf{F}(U_i, \frac{dU_i}{d\xi}, \frac{d^2U_i}{d\xi^2}, \cdots) = 0 \qquad (4)$$

which are integrated as long as all terms contain derivatives. If applicable the constant of integration is not set to zero.

Step 2:
We seek the doubly periodic solution of (4) expressed in the form:

$$U_i(\xi)=a_{i0}+\sum_{j=1}^{n} f^{j-1}(\xi)\left[a_{ij} f(\xi)+b_{ij} g(\xi)+c_{ij} h(\xi)\right] \quad (5)$$

where $f(\xi), g(\xi)$ and $h(\xi)$ are the Jacobian elliptic sine , cosine and the Jacobian elliptic function of the third kind, respectively (see [4], [5]). The considered Jacobian elliptic functions posses the following properties:

$$\mathrm{sn}^2\xi+\mathrm{cn}^2\xi=1, \quad \mathrm{dn}^2\xi+m^2\mathrm{sn}^2\xi=1 \quad (6)$$

$$(\mathrm{sn}\xi)'=\mathrm{cn}\xi\mathrm{dn}\xi, \quad (\mathrm{cn}\xi)'=-\mathrm{sn}\xi\mathrm{dn}\xi, \quad (\mathrm{dn}\xi)'=-m^2\mathrm{sn}\xi\mathrm{cn}\xi, \quad (7)$$

where $(\ )'=\frac{d}{d\xi}$.

We determine the parameter n if we balance the highest order derivative with the highest nonlinear term in (4). Defining the degree of U as $D[U]=n$ gives rise to the degree of other expressions as:

$$D\left[\frac{d^qU}{d\xi^q}\right]=n+q, \quad D\left[U^p\left(\frac{d^qU}{d\xi^q}\right)^s\right]=np+s(n+q) \quad (8)$$

Step 3:

Substituting (5) into (4) and setting coefficients of $f^k g^j h^i$ (for $i,j=0,1,\ k=0,1,2,...$) to zero, will generate an algebraic system of equations from which the unknowns $a_{i0}, a_{ij}, b_{ij}, c_{ij}$ and the parameters k,l,λ can be obtained.

Step 4:

Building the doubly periodic solutions by substituting the obtained results in step 3 into (5) and reverse step 1we obtain the explicit solutions in the original variables.

Remark:

It is well known that [34], the modulus m ranging between 0 and 1. As for $m\to 1$ the Jacobian elliptic functions $\mathrm{sn}(\xi), \mathrm{cn}(\xi)$ and $\mathrm{dn}(\xi)$ degenerate as $\tanh(\xi), \mathrm{sech}(\xi)$ and $\mathrm{sech}(\xi)$, respectively. While, $\mathrm{sn}(\xi), \mathrm{cn}(\xi)$ and $\mathrm{dn}(\xi)$ degenerate as $\sin(\xi), \cos(\xi)$ and 1, respectively. Therefore (5) degenerate the solutions in the form:

$$U_i(\xi)=a_{i0}+\sum_{j=1}^{n}\tanh^{j-1}(\xi)\left[a_{ij}\tanh(\xi)+d_{ij}\,\mathrm{sech}(\xi)\right] \quad (9)$$

3. The (2+1)-Dimensional Generalization of cou-Pled Nonlinear Schrödinger Equation

According to the algorithm described in the previous section, we apply the transformations:

$$u(x,y,t)=\Psi_1(\xi)\exp(i\eta), \quad w(x,y,t)=\Psi_2(\xi)\exp(i\eta), \quad v(x,y,t)=\Phi(\xi), \quad \xi=kx+ly+\lambda t, \quad \eta=\alpha x+\beta y+\gamma t \quad (10)$$

where k,l,λ,α,β and γ are nonzero arbitrary constants to be determined later. Consequently, the (2+1)-dimensional generalized coupled nonlinear Schrödinger equations (1) are reduced to the following nonlinear system of ordinary differential equations (NLODEs):

$$\begin{aligned}
&k^2\Psi_1(\xi)''-(\alpha^2+\gamma)\Psi_1(\xi)-\Psi_1(\xi)\Phi(\xi)=0\\
&k^2\Psi_2(\xi)''-(\alpha^2+\gamma)\Psi_2(\xi)-\Psi_2(\xi)\Phi(\xi)=0\\
&l\Phi(\xi)'-k\left(2\Psi_1(\xi)\Psi_1(\xi)'+2\delta\Psi_2(\xi)\Psi_2(\xi)'\right)=0
\end{aligned} \quad (11)$$

Providing that $\lambda=-2\alpha k$. Integrating the third equation of (11), we obtain:

$$\Phi(\xi)=\frac{k}{l}\left(\Psi_1(\xi)^2+\delta\Psi_2(\xi)^2\right)+C_1 \quad (12)$$

Where C_1 is constant of integration. Returning the value of $\Phi(\xi)$ into (11) and rearranging them to give coupled non-linear equations:

$$\begin{aligned}
&\Psi_1(\xi)''-r_1\Psi_1(\xi)-r_2\left(\Psi_1(\xi)^3+\delta\Psi_1(\xi)\Psi_2(\xi)^2\right)=0\\
&\Psi_2(\xi)''-r_1\Psi_2(\xi)-r_2\left(\Psi_1(\xi)^2\Psi_2(\xi)+\delta\Psi_2(\xi)^3\right)=0
\end{aligned} \quad (13)$$

where $r_1=\frac{1}{k^2}\left(\alpha^2+\gamma+C_1\right), r_2=\frac{1}{kl}$. Balancing the highest order derivative with the highest nonlinear term we obtain $n_1=n_2=1$ and the ansätz:

$$\begin{aligned}
&\Psi_1(\xi)=a_0+a_1\,\mathrm{sn}(\xi)+a_2\,\mathrm{cn}(\xi)+a_3\,\mathrm{dn}(\xi)\\
&\Psi_2(\xi)=b_0+b_1\,\mathrm{sn}(\xi)+b_2\,\mathrm{cn}(\xi)+b_3\,\mathrm{dn}(\xi)
\end{aligned} \quad (14)$$

Substituting (14) into (13), an algebraic system of the unknowns a_i, b_j ($i,j=0,1,2,3$) is built by setting the coefficients of $\mathrm{sn}^k\,\mathrm{cn}^j\,\mathrm{dn}^i$ for ($i,j=0,1,\ k=0,1,2,3$) to zero as follows:

$$
\begin{aligned}
&-6a_0a_1a_2r_2 - 2\delta a_2b_0b_1r_2 - 2\delta a_1b_0b_2r_2 - 2\delta a_0b_1b_2r_2 = 0,\\
&-2a_1a_2b_0r_2 - 2a_0a_2b_1r_2 - 2a_0a_1b_2r_2 - 6\delta b_0b_1b_2r_2 = 0,\\
&-6a_0a_1a_3r_2 - 2\delta a_3b_0b_1r_2 - 2\delta a_1b_0b_3r_2 - 2\delta a_0b_1b_3r_2 = 0,\\
&-2a_1a_3b_0r_2 - 2a_0a_3b_1r_2 - 2a_0a_1b_3r_2 - 6\delta b_0b_1b_3r_2 = 0,\\
&-6a_0a_2a_3r_2 - 2\delta a_3b_0b_2r_2 - 2\delta a_2b_0b_3r_2 - 2\delta a_0b_2b_3r_2 = 0,\\
&-6a_1a_2a_3r_2 - 2\delta a_3b_1b_2r_2 - 2\delta a_2b_1b_3r_2 - 2\delta a_1b_2b_3r_2 = 0,\\
&-2a_2a_3b_0r_2 - 2a_0a_3b_2r_2 - 2a_0a_2b_3r_2 - 6\delta b_0b_2b_3r_2 = 0,\\
&-2a_2a_3b_1r_2 - 2a_1a_3b_2r_2 - 2a_1a_2b_3r_2 - 6\delta b_1b_2b_3r_2 = 0,\\
&-a_0r_1 - a_0^3r_2 - 3a_0a_2^2r_2 - 3a_0a_3^2r_2 - \delta a_0b_0^2r_2 - 2\delta a_2b_0b_2r_2\\
&\quad -\delta a_0b_2^2r_2 - 2\delta a_3b_0b_3r_2 - \delta a_0b_3^2r_2 = 0,\\
&-3a_0a_1^2r_2 + 3a_0a_2^2r_2 + 3m^2a_0a_3^2r_2 - 2\delta a_1b_0b_1r_2 - \delta a_0b_1^2r_2\\
&\quad +2\delta a_2b_0b_2r_2 + \delta a_0b_2^2r_2 + 2m^2\delta a_3b_0b_3r_2 + m^2\delta a_0b_3^2r_2 = 0,\\
&-a_1 - m^2a_1 - a_1r_1 - 3a_0^2a_1r_2 - 3a_1a_2^2r_2 - 3a_1a_3^2r_2 - \delta a_1b_0^2r_2\\
&\quad -2\delta a_0b_0b_1r_2 - 2\delta a_2b_1b_2r_2 - \delta a_1b_2^2r_2 - 2\delta a_3b_1b_3r_2 - \delta a_1b_3^2r_2 = 0,\\
&2m^2a_1 - a_1^3r_2 + 3a_1a_2^2r_2 + 3m^2a_1a_3^2r_2 - \delta a_1b_1^2r_2 + 2\delta a_2b_1b_2r_2\\
&\quad +\delta a_1b_2^2r_2 + 2m^2\delta a_3b_1b_3r_2 + m^2\delta a_1b_3^2r_2 = 0,\\
&-a_2 - a_2r_1 - 3a_0^2a_2r_2 - a_2^3r_2 - 3a_2a_3^2r_2 - \delta a_2b_0^2r_2 - 2\delta a_0b_0b_2r_2\\
&\quad -\delta a_2b_2^2r_2 - 2\delta a_3b_2b_3r_2 - \delta a_2b_3^2r_2 = 0,\\
&2m^2a_2 - 3a_1^2a_2r_2 + a_2^3r_2 + 3m^2a_2a_3^2r_2 - \delta a_2b_1^2r_2 - 2\delta a_1b_1b_2r_2\\
&\quad +\delta a_2b_2^2r_2 + 2m^2\delta a_3b_2b_3r_2 + m^2\delta a_2b_3^2r_2 = 0,\\
&-m^2a_3 - a_3r_1 - 3a_0^2a_3r_2 - 3a_2^2a_3r_2 - a_3^3r_2 - \delta a_3b_0^2r_2 - \delta a_3b_2^2r_2\\
&\quad -2\delta a_0b_0b_3r_2 - 2\delta a_2b_2b_3r_2 - \delta a_3b_3^2r_2 = 0,\\
&2m^2a_3 - 3a_1^2a_3r_2 + 3a_2^2a_3r_2 + m^2a_3^3r_2 - \delta a_3b_1^2r_2 + \delta a_3b_2^2r_2\\
&\quad -2\delta a_1b_1b_3r_2 + 2\delta a_2b_2b_3r_2 + m^2\delta a_3b_3^2r_2 = 0,\\
&-b_0r_1 - a_0^2b_0r_2 - a_2^2b_0r_2 - a_3^2b_0r_2 - \delta b_0^3r_2 - 2a_0a_2b_2r_2\\
&\quad -3\delta b_0b_2^2r_2 - 2a_0a_3b_3r_2 - 3\delta b_0b_3^2r_2 = 0,\\
&-a_1^2b_0r_2 + a_2^2b_0r_2 + m^2a_3^2b_0r_2 - 2a_0a_1b_1r_2 - 3\delta b_0b_1^2r_2\\
&\quad +2a_0a_2b_2r_2 + 3\delta b_0b_2^2r_2 + 2m^2a_0a_3b_3r_2 + 3m^2\delta b_0b_3^2r_2 = 0,\\
&-b_1 - m^2b_1 - b_1r_1 - 2a_0a_1b_0r_2 - a_0^2b_1r_2 - a_2^2b_1r_2 - a_3^2b_1r_2\\
&\quad -3\delta b_0^2b_1r_2 - 2a_1a_2b_2r_2 - 3\delta b_1b_2^2r_2 - 2a_1a_3b_3r_2 - 3\delta b_1b_3^2r_2 = 0,\\
&2m^2b_1 - a_1^2b_1r_2 + a_2^2b_1r_2 + m^2a_3^2b_1r_2 - \delta b_1^3r_2 + 2a_1a_2b_2r_2\\
&\quad +3\delta b_1b_2^2r_2 + 2m^2a_1a_3b_3r_2 + 3m^2\delta b_1b_3^2r_2 = 0,\\
&-b_2 - b_2r_1 - 2a_0a_2b_0r_2 - a_0^2b_2r_2 - a_2^2b_2r_2 - a_3^2b_2r_2\\
&\quad -3\delta b_0^2b_2r_2 - \delta b_2^3r_2 - 2a_2a_3b_3r_2 - 3\delta b_2b_3^2r_2 = 0,\\
&2m^2b_2 - 2a_1a_2b_1r_2 - a_1^2b_2r_2 + a_2^2b_2r_2 + m^2a_3^2b_2r_2 - 3\delta b_1^2b_2r_2\\
&\quad +\delta b_2^3r_2 + 2m^2a_2a_3b_3r_2 + 3m^2\delta b_2b_3^2r_2 = 0,\\
&-m^2b_3 - b_3r_1 - 2a_0a_3b_0r_2 - 2a_2a_3b_2r_2 - a_0^2b_3r_2 - a_2^2b_3r_2 - a_3^2b_3r_2\\
&\quad -3\delta b_0^2b_3r_2 - 3\delta b_2^2b_3r_2 - \delta b_3^3r_2 = 0,\\
&2m^2b_3 - 2a_1a_3b_1r_2 + 2a_2a_3b_2r_2 - a_1^2b_3r_2 + a_2^2b_3r_2 + m^2a_3^2b_3r_2\\
&\quad -3\delta b_1^2b_3r_2 + 3\delta b_2^2b_3r_2 + m^2\delta b_3^3r_2 = 0,
\end{aligned} \tag{15}
$$

By the aid of Mathematica we solve the above algebraic system and obtain the following new solutions families:

Family 1.

$$b_3 = \mp\sqrt{-\frac{2+a_3^2r_2}{\delta r_2}},\ r_1 = 2-m^2,$$

$$a_0 = a_1 = a_2 = b_0 = b_1 = b_2 = 0 \tag{16}$$

where a_3 is an arbitrary constant and $r_1 = \frac{1}{k^2}\left(\alpha^2+\gamma+C_1\right)$. , $r_2 = \frac{1}{kl}$ Thus the solution of (1) will have the form:

$$u(x,y,t) = a_3\,\mathrm{dn}[kx+ly-2\alpha kt]\exp\left(i\left(\alpha x+\beta y+\gamma t\right)\right)$$

$$w(x,y,t) = -\sqrt{-\frac{2+a_3^2r_2}{\delta r_2}}\,\mathrm{dn}[kx+ly-2\alpha kt]$$
$$\exp\left(i\left(\alpha x+\beta y+\gamma t\right)\right)$$

$$v(x,y,t) = -\frac{2k}{lr_2}\mathrm{dn}[kx+ly-2\alpha kt]^2 \tag{17}$$

Family 2.

$$b_2 = \mp\sqrt{-\frac{2m^2+a_2^2r_2}{\delta r_2}},\ r_1 = -1+2m^2,$$

$$a_0 = a_1 = a_3 = b_0 = b_1 = b_3 = 0 \tag{18}$$

where a_2 is an arbitrary constant and $r_1 = \frac{1}{k^2}\left(\alpha^2+\gamma+C_1\right)$, $r_2 = \frac{1}{kl}$. Thus the solution of (1) will have the form:

$$u(x,y,t) = a_2\,\mathrm{cn}[kx+ly-2\alpha kt]\exp\left(i\left(\alpha x+\beta y+\gamma t\right)\right)$$

$$w(x,y,t) = -\sqrt{-\frac{2m^2+a_2^2r_2}{\delta r_2}}\,\mathrm{cn}[kx+ly-2\alpha kt]$$
$$\exp\left(i\left(\alpha x+\beta y+\gamma t\right)\right)$$

$$v(x,y,t) = -\frac{2km^2}{lr_2}\mathrm{cn}[kx+ly-2\alpha kt]^2 \tag{19}$$

Family 3.

$$b_1 = \mp\frac{\text{ä}\sqrt{-2m^2+a_1^2r_2}}{\sqrt{\delta r_2}},\ r_1 \to -1-m^2,$$

$$a_0 = a_2 = a_3 = b_0 = b_2 = b_3 = 0 \tag{20}$$

where a_1 is an arbitrary constant and $r_1 = \frac{1}{k^2}\left(\alpha^2+\gamma+C_1\right)$. , $r_2 = \frac{1}{kl}$. Thus the solution of (1) will have the form:

$$u(x,y,t) = a_1\,\mathrm{sn}[kx+ly-2\alpha kt]\exp\left(i\left(\alpha x+\beta y+\gamma t\right)\right)$$

$$w(x,y,t) = -\frac{\text{ä}\sqrt{-2m^2+a_1^2r_2}}{\sqrt{\delta r_2}}\mathrm{sn}[kx+ly-2\alpha kt]$$
$$\exp\left(i\left(\alpha x+\beta y+\gamma t\right)\right)$$

$$v(x,y,t) = \frac{2km^2}{lr_2}\mathrm{sn}[kx+ly-2\alpha kt]^2$$

, (21)

Family 4.

$$a_2 = \mp \ddot{a} a_1,\ b_1 = \mp \frac{\ddot{a}}{\sqrt{2}}\sqrt{\frac{1}{\delta}\left(2a_1^2 - \frac{m^2}{r_2}\right)},$$
$$b_2 = \pm\frac{1}{\sqrt{2}}\sqrt{\frac{1}{\delta}\left(2a_1^2 - \frac{m}{r_2}\right)},\ r_1 = \frac{1}{2}(-2+m^2)$$
$$a_0 = a_3 = b_0 = b_3 = 0, \qquad (22)$$

where a_1 is an arbitrary constant and $r_1 = \frac{1}{k^2}\left(\alpha^2+\gamma+C_1\right)$, $r_2 = \frac{1}{kl}$. Thus the solution of (1) will have the form:

$$u(x,y,t) = a_1(-\ddot{a}\,\mathrm{cn}[kx+ly-2\alpha kt] + \mathrm{sn}[kx+ly-2\alpha kt])\exp\left(i\left(\alpha x+\beta y+\gamma t\right)\right)$$
$$w(x,y,t) = -\frac{\sqrt{-2m^2+4a_1^2 r_2}}{2\sqrt{\delta r_2}}(\mathrm{cn}[kx+ly-2\alpha kt] + \ddot{a}\,\mathrm{sn}[kx+ly-2\alpha kt])\exp\left(i\left(\alpha x+\beta y+\gamma t\right)\right)$$
$$v(x,y,t) = -\frac{km^2\,\mathrm{cn}[kx+ly-2\alpha kt]^2}{2lr_2} - \frac{\ddot{a}km^2\,\mathrm{cn}[kx+ly-2\alpha kt]\,\mathrm{sn}[kx+ly-2\alpha kt]}{lr_2} + \frac{km^2\,\mathrm{sn}[kx+ly-2\alpha kt]^2}{2lr_2} \qquad (23)$$

Family 5.

$$a_3 = \mp\frac{a_2}{m},\ b_2 = \mp\frac{1}{\sqrt{2}}\sqrt{-\frac{1}{\delta}\left(2a_2^2 + \frac{m^2}{r_2}\right)},$$
$$b_3 = \mp\frac{1}{\sqrt{2}}\sqrt{-\frac{1}{\delta}\left(\frac{2a_2^2}{m^2} + \frac{1}{r_2}\right)},\ r_1 = \frac{1}{2}(1+m^2)$$
$$a_0 = a_1 = b_0 = b_1 = 0, \qquad (24)$$

where a_2 is an arbitrary constant and $r_1 = \frac{1}{k^2}\left(\alpha^2+\gamma+C_1\right)$. $r_2 = \frac{1}{kl}$. Thus the solution of (1) will have the form:

$$u(x,y,t) = \frac{a_2}{m}(\mp m\,\mathrm{cn}[kx+ly-2\alpha kt] \mp \mathrm{dn}[kx+ly-2\alpha kt])\exp\left(i\left(\alpha x+\beta y+\gamma t\right)\right)$$
$$w(x,y,t) = \frac{1}{\sqrt{2}m}\sqrt{-\frac{1}{\delta}\left(2a_2^2+\frac{m^2}{r_2}\right)}(\mp m\,\mathrm{cn}[kx+ly-2\alpha kt] \mp \mathrm{dn}[kx+ly-2\alpha kt])\exp\left(i\left(\alpha x+\beta y+\gamma t\right)\right)$$
$$v(x,y,t) = -\frac{km^2\,\mathrm{cn}[kx+ly-2\alpha kt]^2}{2lr_2} + \frac{km\,\mathrm{cn}[kx+ly-2\alpha kt]\,\mathrm{dn}[kx+ly-2\alpha kt]}{lr_2} - \frac{k\,\mathrm{dn}[kx+ly-2\alpha kt]^2}{2lr_2} \qquad (25)$$

Family 6.

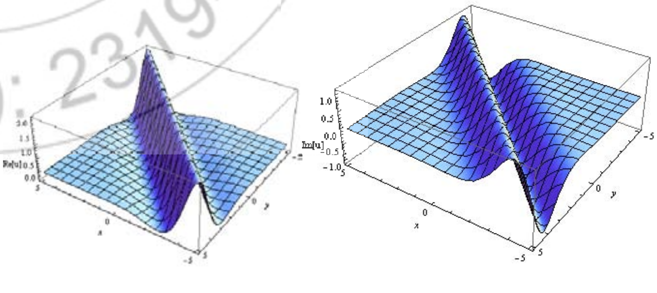
$$a_3 = \mp\frac{\ddot{a}a_1}{m},\ b_1 = \mp\frac{\ddot{a}}{\sqrt{2}}\sqrt{\frac{1}{\delta}\left(2a_1^2 - \frac{m^2}{r_2}\right)},$$
$$b_3 = \mp\frac{1}{\sqrt{2}}\sqrt{-\frac{1}{\delta}\left(-\frac{2a_1^2}{m^2}+\frac{1}{r_2}\right)},\ r_1 = \frac{1}{2} - m$$
$$a_0 = a_2 = b_0 = b_2 = 0 \qquad (26)$$

where a_1 is an arbitrary constant and $r_1 = \frac{1}{k^2}\left(\alpha^2+\gamma+C_1\right)$. $r_2 = \frac{1}{kl}$. Thus the solution of (1) will have the form:

$$u(x,y,t) = \frac{a_1}{m}(-\ddot{a}\,\mathrm{dn}[kx+ly-2\alpha kt] + m\,\mathrm{sn}[kx+ly-2\alpha kt])\exp\left(i\left(\alpha x+\beta y+\gamma t\right)\right)$$
$$w(x,y,t) = -\frac{\sqrt{-2m^2+4a_1^2 r_2}}{2m\sqrt{\delta r_2}}(\mathrm{dn}[kx+ly-2\alpha kt] + \ddot{a}m\,\mathrm{sn}[kx+ly-2\alpha kt])\exp\left(i\left(\alpha x+\beta y+\gamma t\right)\right)$$
$$v(x,y,t) = -\frac{k\,\mathrm{dn}[kx+ly-2\alpha kt]^2}{2lr_2} - \frac{\ddot{a}km\,\mathrm{dn}[kx+ly-2\alpha kt]\,\mathrm{sn}[kx+ly-2\alpha kt]}{lr_2} + \frac{km^2\,\mathrm{sn}[kx+ly-2\alpha kt]^2}{2lr_2} \qquad (27)$$

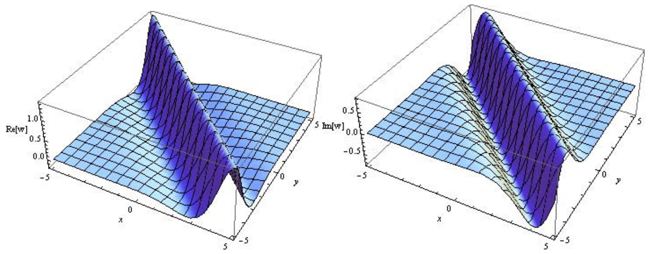
Finally, as our special case, we may represent the graphs to have the corresponding solitary wave solutions of the first solution of (25) as m approaches 1. The chosen values of the parameters are $k = l = \alpha = \beta = \gamma = 1 = a_2 = 1, \delta = r_2 = -1$. Where the corresponding surfaces are:

$$u(x,y,t) = 2\,\mathrm{sech}[-x-y+t]\exp\left(i\left(x+y+t\right)\right),$$
$$w(x,y,t) = \sqrt{2}\,\mathrm{sech}[-x-y+t]\exp\left(i\left(x+y+t\right)\right),$$
$$v(x,y,t) = 2\,\mathrm{sech}[-x-y+t]^2$$



(a) (b)

Figure 1: Exact surface of (25) when $t = 10^{-2}$ $x \in [-5,5]$ and $y \in [-5,5]$: (a) $\mathrm{Re}[u(x,y,t)]$ (b) $\mathrm{Im}[u(x,y,t)]$.



(a) (b)

Figure 2: Exact surface of (25)when $t = 10^{-2}$ $x \in [-5,5]$and $y \in [-5,5]$: (a) $\mathrm{Re}[w(x,y,t)]$ (b) $\mathrm{Im}[w(x,y,t)]$

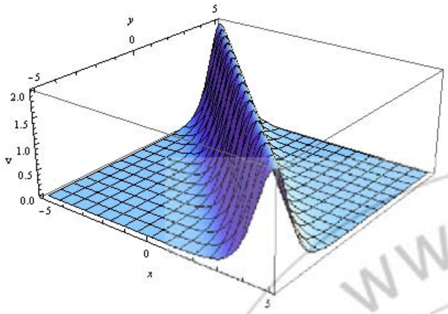


Figure 3: Exact surface of (25) when $t = 10^{-2}$ $x \in [-5,5]$and $y \in [-5,5]$.

4. Conclusion

In summary, we have presented the extended Jacobian elliptic function expansion and its algorithm based on three Jacobian elliptic functions. The (2+1)-dimensional generalization of coupled nonlinear Schrödinger equations of the form is chosen to illustrate our algorithm such that six families of new exact doubly periodic solutions are obtained. When the modulus $m \to 1$ or $m \to 0$, the obtained solutions degenerate as solitary wave solutions including bright solitons, dark solitons, new solitonic solutions as well as trigonometric function solutions. Mathematica software is used in computations.

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