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Subclass of Meromorphically Uniformly Convex Functions Defined by Linear Operator

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Abstract: ***In this paper, we introduced a new subclass of meromorphically uniformly convex functions with positive coefficients and obtain coefficient estimates, growth and distortion theorems, extreme points, closure theorems and radius of starlikeness and convexity for the new subclass*** $\Sigma_w^*(\alpha,k)$.

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1. Introduction

Let A be the class of functions $f(z)$ which are analytic in the open unit disk

$$U=\{z:|z|<1\} \tag{1.1}$$

As usual, we denote by S the subclass of A, consisting of functions which are also univalent in *U*.

Let *w* be fixed point in *U* and $A(w)=\{f\in H(U): f(w)=f'(w)-1=0\}$ In [19], Kanas and Ronning introduced the following classes $S_w=\{f\in A(w): f$ is univalent in U$\}$

$$ST_w=\left\{f\in A(w):\operatorname{Re}\left(\frac{(z-w)f'(z)}{f(z)}\right)>0, z\in U\right\}$$

$$CV_w=\left\{f\in A(w):1+\operatorname{Re}\left(\frac{(z-w)f''(z)}{f'(z)}\right)>0, z\in U\right\} \tag{1.2}$$

Later Acu and Owa [1] studied the classes extensively.

The class ST_w is defined by geometric property that the image of any circular arc centered at *w* is starlike with respect to *f(w)*, and the corresponding class CV_w is defined by the property that the image of any circular arc centered at *w* is convex. We observed that the definitions are somewhat similar to the ones introduced by Goodman in [7],[8] for uniformly starlike and convex function except that , in this case, the point *w* is fixed. Let Σ_w denote the subclass of *A(w)* consisting of the function of the form

$$f(z)=\frac{1}{z-w}+\sum_{n=1}^{\infty}a_n(z-w)^n \tag{1.3}$$

The functions *f(z),* in Σ_w are said to be starlike functions of order α if and only if

$$\operatorname{Re}\left\{-\frac{(z-w)f'(z)}{f(z)}\right\}>\alpha,(z-w)\in U. \tag{1.4}$$

for some α $(0\le\alpha<1)$. we denote by $ST_w^*(\alpha)$ the class of all starlike function of order α.

Similarly, a function f in Σ_w is said to be convex of order α if and only if

$$\operatorname{Re}\left\{-1-\frac{(z-w)f''(z)}{f'(z)}\right\}>\alpha,\ (z-w)\in U. \tag{1.5}$$

for some α $(0\le\alpha<1)$. We denote by $C_w(\alpha)$ the class of all convex functions of order α.

For the function $f\in\Sigma_w$, we define

$$I_\lambda^0 f(z)=f(z),$$

$$I_\lambda^1 f(z)=(z-w)f'(z)+\frac{2}{z-w},$$

$$I_\lambda^2 f(z)=(z-w)\left(I^1 f(z)\right)'+\frac{2}{z-w}, \tag{1.6}$$

and for k =1,2,3…. we can write

$$I_\lambda^k f(z)=(z-w)(I^{k-1}f(z))'+\frac{2}{z-w}$$

$$=\frac{1}{z-w}+\sum_{n=1}^{\infty}\left[1+\lambda(n-1)\right]^k a_n(z-w)^n, \tag{1.7}$$

where $\lambda\ge1, k\ge0$ and $((z-w)\in U)$.

The differential operator I_1^k is studied by Ghanim and Darus [9],[10] and Ghanim et al., [11].

Let us define the function $\overline{\phi}(a,c;z)$

$$\overline{\phi}(a,c:z)=\frac{1}{z-w}+\sum_{n=0}^{\infty}\left|\frac{(a)_{n+1}}{(c)_{n+1}}\right|a_n(z-w)^n \quad (1.8)$$

for $c\neq 0,-1,-2,\ldots,$, and $a\in \mathbb{C}\,/\{0\}$,

where $(\lambda)_n=\lambda(\lambda+1)_{n+1}$ is the Pochammer symbol. We note that

$$\overline{\phi}(a,c;z)=\frac{1}{z-w}\,{}_2F_1(1,a,c;z) \quad (1.9)$$

where

$${}_2F_1(b,a,c;z)=\sum_{n=0}^{\infty}\frac{(b)_n(a)_n}{(c)_n}\frac{(z-w)^n}{n!} \quad (1.10)$$

is the well-known Gaussian hypergeometric function. Corresponding to the function $\overline{\phi}(a,c;z)$ using the Hadamard product for $f\in\Sigma$, we define a new linear operator $L_w^*(a,c)$ on Σ by

$$L_w^*(a,c)f(z)=\overline{\phi}(a,c;z)*f(z)=\frac{1}{z-w}+\sum_{n=1}^{\infty}\left|\frac{(a)_{n+1}}{(c)_{n+1}}\right|a_n(z-w)^n, \quad (1.11)$$

The meromorphic functions with the generalized hypergeometric functions were considered recently by Dziok and Srivastava [4],[5], Liu [15] and Srivastava [16],[17],[18], Cho and Kim [2,3].

For a function $f\in L_w^*(a,c)f(z)$ we define

$$I^0(L_w^*(a,c)f(z))=L_w^*(a,c)f(z)$$

and for k= 1,2,3,………..

$$I^k(L_w^*(a,c)f(z))=z(I^{k-1}(L_w^*(a,c)f(z))'+\frac{2}{z-w}$$

$$=\frac{1}{z-w}+\sum_{n=1}^{\infty}n^k\left|\frac{(a)_{n+1}}{(c)_{n+1}}\right|a_n(z-w)^n \quad (1.12)$$

We note that $I^k(L_w^*(a,a)f(z))$ studied by Frasin and Darus [9,10] and Ghanian et al [11]. Now, for $\alpha(-1\le\alpha<1)$, we let $\Sigma_w^*(\alpha,k)$ be the subclass of A consisting of the form (1.3) and satisfying the analytic criterion

$$\mathrm{Re}\left\{\frac{I^{k+1}L_w^*(a,c)f(z)}{I^kL_w^*(a,c)f(z)}-\alpha\right\}>\left|\frac{I^{k+1}L_w^*(a,c)f(z)}{I^kL_w^*(a,c)f(z)}-1\right|, z\in U$$

(1.13)

where $L_w^*(a,c)f(z)$ is given by (1.11).

The main objective of this paper is to obtain necessary and sufficient conditions for the functions $f\in\Sigma_w^*(\alpha,k)$. Furthermore, we obtain extreme points, growth and distortion bounds and closure properties for the class $\Sigma_w^*(\alpha,k)$.

2. Coefficient Estimate

In this section we obtain necessary and sufficient conditions for functions f in the class $\Sigma_w^*(\alpha,k)$.

Theorem 1. A function f of the form (1.3) is in $\Sigma_w^*(\alpha,k)$ if

$$\sum_{n=1}^{\infty}n^k[2n-1-\alpha]\left|\frac{(a)_{n+1}}{(c)_{n+1}}\right||a_n|\le 1-\alpha \quad (2.1)$$

$-1\le\alpha<1$.

Proof. It suffices to show that

$$\left|\frac{I^{k+1}L_w^*(a,c)f(z)}{I^kL_w^*(a,c)f(z)}-1\right|-\mathrm{Re}\left|\frac{I^{k+1}L_w^*(a,c)f(z)}{I^kL_w^*(a,c)f(z)}-1\right|\le 1-\alpha$$

We have

$$\left|\frac{I^{k+1}L_w^*(a,c)f(z)}{I^kL_w^*(a,c)f(z)}-1\right|-\mathrm{Re}\left|\frac{I^{k+1}L_w^*(a,c)f(z)}{I^kL_w^*(a,c)f(z)}-1\right|$$

$$\le 2\left|\frac{I^{k+1}L_w^*(a,c)f(z)}{I^kL_w^*(a,c)f(z)}-1\right|\le\frac{2\sum_{n=1}^{\infty}n^k(n-1)\left|\frac{(a)_{n+1}}{(c)_{n+1}}\right||a_n||z-w|^n}{\frac{1}{|z-w|}-\sum_{n=1}^{\infty}n^k\left|\frac{(a)_{n+1}}{(c)_{n+1}}\right||a_n||z-w|^n}$$

Letting $(z-w)\to 1$ along the real axis, we obtain

$$\le\frac{2\sum_{n=1}^{\infty}n^k(n-1)\left|\frac{(a)_{n+1}}{(c)_{n+1}}\right||a_n|}{1-\sum_{n=1}^{\infty}n^k\left|\frac{(a)_{n+1}}{(c)_{n+1}}\right||a_n|}$$

This last expression is bounded above by $(1-\alpha)$ if

$$\sum_{n=1}^{\infty}n^k[2n-1-\alpha]\left|\frac{(a)_{n+1}}{(c)_{n+1}}\right||a_n|\le 1-\alpha.$$

Hence the theorem.

Our assertion in Theorem 1 is sharp for functions of the form

$$f_n(z)=\frac{1}{z-w}+\sum_{n=1}^{\infty}\frac{(1-\alpha)|(c)_{n+1}|}{n^k[2n-1-\alpha]|(a)_{n+1}|}(z-w)^n,$$

(2.2)

$(n\ge 1; k\in N_0)$.

Corollary 1. Let the functions f be defined by (1.11) and let $f\in A$ then

$$a_n\le\frac{(1-\alpha)|(c)_{n+1}|}{n^k[2n-1-\alpha)]|(a)_{n+1}|} \quad (2.3)$$

$(n \geq 1; k \in N_0)$.

Theorem 2. Let f define by (1.3) and

$g(z) = \frac{1}{z-w} + \sum_{n=1}^{\infty} b_n (z-w)^n$ be in the class $\Sigma_w^*(\alpha, k)$

Then the function h defined by

$$h(z) = (1-\lambda) f(z) + \lambda g(z) = \frac{1}{z-w} + \sum_{z}^{\infty} q_n (z-w)^n \quad (2.4)$$

where $q_n = (1-\lambda)a_n + \lambda b_n, 0 \leq \lambda < 1$ is also in the class $\Sigma_w^*(\alpha, k)$.

3. Growth and Distortion Theorem

Theorem 3: Let the function f defined by (1.11) be in the $\Sigma_w^*(\alpha, k)$ then

$$\frac{1}{r} - r \leq |f(z)| \leq \frac{1}{r} + r \quad \textbf{(3.1)}$$

Equality holds for the function

$$f(z) = \frac{1}{z-w} + z - w \quad (3.2)$$

Proof. Since $\Sigma_w^*(\alpha, k)$, by Theorem 1,

$$\sum_{n=1}^{\infty} n^k [2n-1-\alpha] \frac{|(a)_{n+1}|}{|(c)_{n+1}|} |a_n| \leq 1-\alpha.$$

Now

$$(1-\alpha) \sum_{n=1}^{\infty} \frac{|(a)_{n+1}|}{|(c)_{n+1}|} a_n = \sum_{n=1}^{\infty} (1-\alpha) \frac{|(a)_{n+1}|}{|(c)_{n+1}|} a_n \leq$$

$$\sum_{n=1}^{\infty} n^k [2n-1-\alpha] \frac{|(a)_{n+1}|}{|(c)_{n+1}|} |a_n| \leq 1-\alpha.$$

and therefore,

$$\sum_{n=1}^{\infty} \frac{|(a)_{n+1}|}{|(c)_{n+1}|} a_n \leq 1$$

Since

$$f(z) = \frac{1}{z-w} + \sum_{n=1}^{\infty} \frac{|(a)_{n+1}|}{|(c)_{n+1}|} a_n (z-w)^n$$

$$|f(z)| = \left| \frac{1}{z-w} + \sum_{n=1}^{\infty} \frac{|(a)_{n+1}|}{|(c)_{n+1}|} a_n (z-w)^n \right|$$

$$\leq \frac{1}{|z-w|} + \sum_{n=1}^{\infty} \frac{|(a)_{n+1}|}{|(c)_{n+1}|} a_n |z-w|^n$$

$$\leq \frac{1}{r} + r \sum_{n=1}^{\infty} \frac{|(a)_{n+1}|}{|(c)_{n+1}|} a_n \leq \frac{1}{r} + r$$

and

$$|f(z)| = \left| \frac{1}{z-w} + \sum_{n=1}^{\infty} \frac{|(a)_{n+1}|}{|(c)_{n+1}|} a_n (z-w)^n \right|$$

$$\geq \frac{1}{|z-w|} - \sum_{n=1}^{\infty} \frac{|(a)_{n+1}|}{|(c)_{n+1}|} a_n |z-w|^n$$

$$\geq \frac{1}{r} - r \sum_{n=1}^{\infty} \frac{|(a)_{n+1}|}{|(c)_{n+1}|} a_n \geq \frac{1}{r} - r$$

which yields the theorem.

Theorem 4. Let the function f defined by (1.11) be in the class $\Sigma_w^*(\alpha, k)$

Then

$$\frac{1}{r^2} - 1 \leq |f'(z)| \leq \frac{1}{r^2} + 1 \quad (3.4)$$

Equality holds for the function $f(z) = \frac{1}{z-w} + z - w.$

Proof. We have

$$|f'(z)| = \left| \frac{-1}{(z-w)^2} + \sum_{n=1}^{\infty} n \frac{(a)_{n+1}}{(c)_{n+1}} a_n (z-w)^{n-1} \right|$$

$$\leq \frac{1}{|z|^2} + \sum_{n=1}^{\infty} n \left| \frac{(a)_{n+1}}{(c)_{n+1}} \right| a_n |(z-w)|^{n-1}$$

$$\leq \frac{1}{r^2} + \sum_{n=1}^{\infty} \frac{|(a)_{n+1}|}{|(c)_{n+1}|} n a_n \quad (3.5)$$

Since $f(z) \in \Sigma^*_w(\alpha, k)$ we have

$$(1-\alpha) \sum_{n=1}^{\infty} \frac{|(a)_{n+1}|}{|(c)_{n+1}|} n a_n \leq \sum_{n=1}^{\infty} n^{k-1} [2n-1-\alpha] \frac{|(a)_{n+1}|}{|(c)_{n+1}|} n a_n$$

$$\leq 1-\alpha.$$

Hence,

$$\sum_{n=1}^{\infty} \frac{|(a)_{n+1}|}{|(c)_{n+1}|} n a_n \leq 1 \quad (3.6)$$

Substituting (3.6) in (3.5), we get

$$|f'(z)| \leq \frac{1}{r^2} + 1$$

$$|f'(z)| \geq \frac{1}{r^2} - 1$$

This completes the proof.

4. Radii of Starlikeness and Convexity

The radii of starlikeness and convexity for the class for the class $\Sigma_w^*(\alpha, k)$ is given by the following theorems.

Theorem 5. If the function *f* be defined by (1.11) is in the class $\Sigma_w^*(\alpha,k)$ then *f* is meromorphically starlike of order $\delta(0\le\delta<1)$ in $(z-w)<r_1$,

where

$$r_1=r_1(\alpha,\beta,k)=\inf_{n\ge1}\left\{\frac{n^k(1-\delta)[2n-1-\alpha]}{(n+2-\delta)(1-\alpha)}\right\}^{\frac{1}{n+1}} \quad (4.1)$$

The result is sharp for the function *f* given by (2.2).

Proof. It suffices to prove that

$$\left|\frac{(z-w)(I^kf(z))'}{I^kf(z)}+1\right|<1-\delta. \quad (4.2)$$

For $(z-w)<r_1$ the left hand side we have

$$\left|\frac{z\left(I^kf(z)\right)'}{I^kf(z)}+1\right|=\left|\frac{\sum_{n=1}^{\infty}n^k(n+1)\frac{|(a)_{n+1}|}{|(c)_{n+1}|}a_n|z-w|^n}{\frac{1}{z-w}+\sum_{n=1}^{\infty}n^k\frac{|(a)_{n+1}|}{|(c)_{n+1}|}a_n(z-w)^n}\right|$$

$$\le\frac{\sum_{n=1}^{\infty}n^k(n+1)\frac{|(a)_{n+1}|}{|(c)_{n+1}|}a_n|z-w|^n}{\left|\frac{1}{z-w}\right|-\sum_{n=1}^{\infty}n^k\frac{|(a)_{n+1}|}{|(c)_{n+1}|}a_n|z-w|^n} \quad (4.3)$$

The last expression is less than $1-\delta$ if

$$\sum_{n=1}^{\infty}n^k(n+1)\frac{|(a)_{n+1}|}{|(c)_{n+1}|}a_n|z-w|^n\le \quad (4.4)$$

$$(1-\delta)\left(\frac{1}{|z-w|}-\sum_{n=1}^{\infty}n^k\frac{|(a)_{n+1}|}{|(c)_{n+1}|}a_n|z-w|^n\right)$$

or

$$\sum_{n=1}^{\infty}n^k\frac{(n+2-\delta)}{1-\delta}\frac{|(a)_{n+1}|}{|(c)_{n+1}|}|z-w|^{n+1}\le1. \quad (4.5)$$

With the aid of (2.1) and (4.5) is true if

$$\sum_{n=1}^{\infty}n^k\frac{(n+2-\delta)}{(1-\delta)}|z-w|^{n+1}\le\frac{n^k[2n-1-\alpha]}{(1-\alpha)} \quad (4.6)$$

$n\ge1.$

Solving (4.6) for $|z-w|$, we obtain

$$|z-w|<\left\{\frac{n^k(1-\delta)[2n-1-\alpha]}{(n+2-\delta)(1-\alpha)}\frac{(a)_{n+1}}{(c)_{n+1}}\right\}^{\frac{1}{n+1}}$$

This completes the proof of Theorem 5.

Theorem 6. If the function *f* be defined by (1.11) is in the class $\Sigma_w^*(\alpha,k)$ then *f(z)* is meromorphically convex of order $\delta(0\le\delta<1)$ in $(z-w)<r_2$ where

$$r_2=r_2(\alpha,k)=$$

$$\inf_{n\ge1}\left\{\frac{n^k(1-\delta)[2n-1-\alpha]}{(n+2-\delta)(1-\alpha)}\right\}^{\frac{1}{n+1}} \quad (4.7)$$

The result is sharp for the function *f* given by (2.2)

Proof: By using the technique employed in the proof of Theorem, we can show that

$$\left|\frac{(z-w)f''(z)}{f'(z)}+2\right|\le(1-\delta) \quad (4.8)$$

for $|z-w|<r_2$, with the aid of Theorem 1. Thus we have the assertion of Theorem 6.

5. Convex Linear Combinations

Our next result involves linear combinations of several functions of the type (2.2).

Theorem 7. Let

$$f_0(z)=\frac{1}{z-w} \quad (5.1)$$

and

$$f_0(z)=\frac{1}{z-w}+\frac{(1-\alpha)|(c)_{n+1}|}{n^k[2n-1-\alpha]|(a)_{n+1}|}(z-w)^n \quad (5.2)$$

$n\ge1,-1<\alpha\le1$ and $k\ge0$.

Then $f(z)\in\Sigma^*_w(\alpha,k)$ if and only if it can be expressed in the form

$$f(z)=\sum_{n=0}^{\infty}\lambda_nf_n(z)$$

$$\text{where } \lambda_n\ge0 \text{ and } \sum_{n=0}^{\infty}\lambda_n=1 \quad (5.3)$$

Proof. From (5.1), (5.2) and (5.3), it is easily seen that

$$f(z)=\sum_{n=0}^{\infty}\lambda_nf_n(z)=\frac{\lambda_0}{z-w}+\sum_{n=1}^{\infty}\frac{\lambda_n(1-\alpha)|(c)_{n+1}|}{n^k[2n(-1-\alpha)]|(a)_{n+1}|}(z-w)^n$$

Since

$$\sum_{n=1}^{\infty}\frac{n^k[2n-1-\alpha)]|(a)_{n+1}|}{(1-\alpha)|(c)_{n+1}|}\frac{\lambda_n(1-\alpha)|(c)_{n+1}|}{n^k[2n-1-\alpha)]|(a)_{n+1}|}$$

$$=\sum_{n=1}^{\infty}\lambda_n=1-\lambda_0\le1$$

It follows from Theorem 1 that $f(z)\in\Sigma^*_w(\alpha,k)$.

Conversely, let us suppose that $f\in\Sigma^*_w(\alpha,k)$. Since

$$a_n\le\frac{(1-\alpha)|(c)_{n+1}|}{n^k[2n-1-\alpha)]|(a)_{n+1}|}$$

$n\ge1,-1<\alpha\le1$ and $k\ge0$.

Setting $\lambda_n = \dfrac{n^k[2n-1-\alpha)]|(a)_{n+1}|}{(1-\alpha)|(c)_{n+1}|}, n \geq 1, k \geq 0$ and

$$\lambda_0 = 1 - \sum_{n=1}^{\infty} \lambda_n$$

It follows that $f(z) = \sum_{n=0}^{\infty} \lambda_n f_n(z)$. This completes the proof of the theorem.

Finally, we prove the following:

Theorem 8: The class $\Sigma_w^*(\alpha, k)$ is closed under convex linear combinations.

Proof: Suppose that the function $f_1(z)$ and $f_2(z)$ defined by

$$f_j(z) = \frac{1}{z-w} + \sum_{n=1}^{\infty} \left| \frac{(a)_{n+1}}{(c)_{n+1}} \right| a_{n,j} (z-w)^n$$

$$(j = 1, 2; z - w \in U) \qquad (5.4)$$

are in the class $\Sigma_w^*(\alpha, k)$. Setting

$$f(z) = \mu f_1(z) + (1-\mu) f_2(z), \ (0 \leq \mu < 1) \qquad (5.5)$$

We find from (5.4) that

$$f(z) = \frac{1}{z-w} + \sum_{n=1}^{\infty} \left| \frac{(a)_{n+1}}{(c)_{n+1}} \right| \{\mu a_{n,1} + (1-\mu) a_{n,2}\} (z-w)^n \qquad (5.6)$$

$(0 \leq \mu < 1, z - w) \in U)$

In view of Theorem 1, we have

$$\sum_{n=0}^{\infty} n^k [2n-1-\alpha)] \left| \frac{(a)_{n+1}}{(c)_{n+1}} \right| \{\mu a_{n,1} + (1-\mu) a_{n,2}\}$$

$$\mu \sum_{n=0}^{\infty} n^k [2n-1-\alpha)] \left| \frac{(a)_{n+1}}{(c)_{n+1}} \right| a_{n,1}$$

$$+(1-\mu) \sum_{n=0}^{\infty} n^k [2n-1-\alpha)] \left| \frac{(a)_{n+1}}{(c)_{n+1}} \right| a_{n,2}$$

$$\leq \mu(1-\alpha) + (1-\mu)(1-\alpha) = (1-\alpha)$$

which shows that $f(z) \in \Sigma^*_w(\alpha, k)$, Hence the Theorem.

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