Fibrewise Soft Topolgical Spaces

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Abstract: ***In this work we define and study new concept of fibrewise topological spaces, namely fibrewise soft topological spaces, Also, we introduce the concepts of fibrewise closed soft topological spaces, fibrewise open soft topological spaces, fibrewise soft near compact spaces and fibrewise locally soft near compact spaces.***

Keywords: Soft set, soft continuous, fibrewise soft topological spaces, fibrewise closed soft topological spaces, fibrewise open soft topological spaces, fibrewise soft near compact spaces and fibrewise locally soft near compact space.

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1. Introduction and Preliminaries

To begin with we work in the category of fibrewise sets over a given set, called the base set. If the base set is denoted by B then a fibrewise set over B consists of a set H together with a function $P: H \to B$, called the projection. For each point b *of* B the fibre over b is the subset $H_b = P^{-1}(b)$ of H; fibres may be empty since we do not require P to be surjective, also for each subset B^* of B we regard $H_{B^*} = P^{-1}(B^*)$ as a fibrewise set over B^* with the projection determined by P. Molodtsov [16] generalized with the introduction of soft sets the traditional concept of a set in the classical researches. With the introduction of the applications of soft sets [15], the soft set theory has been the research topic and have received attention gradually [5, 13, 17, 18]. The applications of the soft sets are redetected so as to develop and consolidate this theory, utilizing these new applications;a uni-int decision-making method was established [8]. Numerous notions of general topology were involved in soft sets and then authors developed theories about soft topological spaces. Shabir and Naz [21] mentioned this term to define soft topological space. After that definition, I. Zorlutuna et al. [25], A ygunoglu et al. [7] and Hussain et al. [11] continued to search the properties of soft topological space. They obtained a lot of vital conclusion in soft topological spaces. We studied the connected between fibrewise topological spaces and soft topological space also some related concepts such as fibrewise soft open, fibrewise soft closed, fibrewise soft near compact and fibrewise locally soft near compact. The purpose of this paper is introduced a new class of fibrewise topology called fibrewise soft topological space are introduced and few of their properties are investigated, we built on some of the result in [1, 19, 22, 23].

Definition 1.1. [12] Let H and K are fibrewise sets over B, with projections $P_H: H \to B$ and $P_K: K \to B$, respectively, a function $\phi: H \to K$ is said to be fibrewise if $P_K \circ \phi = P_H$, in other words if $\phi(H_b) \subset K_b$ for each point b of B.

Note that a fibrewise function $\phi: H \to K$ over B determines, by restriction, a fibrewise function $\phi_{B^*}: H_{B^*} \to K_{B^*}$ over B^* for each subset B^* of B .

Definition 1.2. [16] Let U be an initial universe and E be a set of parameters. Let $P(U)$denote the power set of U and A be a non-empty subset of E. A pair (F, A) is called a soft set over H, where F is a mapping given by $F: A \to P(U)$. In other words, a soft set over U is a parameterized family of subset of the universe U. For $\varepsilon \epsilon A$, $F(\varepsilon)$ may be considered as the set of ε_approximate elements of the soft set (F, A).

Note that the set of all soft sets over U will be denoted by $S(U)$.

Example 1.3. Suppose that there are six houses in the universe $U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ under consideration, and that $E = \{e_1, e_2, e_3, e_4, e_5\}$ is a set of decision parameters. The e_i $(i = 1, 2, 3, 4, 5)$ stand for the parameters ''expensive'', ''beautiful'', ''wooden'', ''cheap'', and ''in green surroundings'', respectively. Consider the mapping F_A given by ''houses (.)''; (.) is to be filled in by one of the parameters $h_i \in E$.For instance, $F_A(e_1)$ means''houses (expensive)'', and its functional value is the set $\{u \in U : u$ is an expensive house$\}$. Suppose that $A = \{e_1, e_3, e_4\} \subseteq E$ and $F_A(e_1) = \{u_1, u_4\}$, $F_A(e_3) = U$, and $F_A(e_4) = \{u_1, u_3, u_5\}$. Then, we can view the soft set F_A as consisting of the following collection of approximations: $F_A = \{(e_1, \{u_2, u_4\}), (e_3, U), (e_4, \{u_1, u_3, u_5\})\}$.

Definition 1.4. [9] Let $F_A \in S(U)$. A soft topology on F_A, denoted by $\tilde{\tau}$, is a collection of soft subsets of F_A having following properties:

(a) $F_A, F_\Phi \in \tilde{\tau}$

(b) $\{F_{Ai} \tilde{\subseteq} F_A : i \in I \subseteq N\} \subseteq \tilde{\tau} \Rightarrow \tilde{\bigcup}_{i \in I} F_{Ai} \in \tilde{\tau}$

(c) $\{F_{Ai} \tilde{\subseteq} F_A : 1 \leq i \leq n, n \in N\} \subseteq \tilde{\tau} \Rightarrow \tilde{\bigcap}_{i=1}^{n} F_{Ai} \in \tilde{\tau}$.

The pair $(F_A, \tilde{\tau})$ is called a soft topological space.

Example 1.5. [9] Let $U = \{u_1, u_2, u_3\}$, $E = \{e_1, e_2, e_3\}$, $A = \{e_1, e_2\} \subseteq E$ and $F_A = \{(e_1, \{u_1, u_2\}), (e_2, \{u_2, u_3\})\}$. Then $\tilde{\tau}_1 = \{F_\Phi, F_A\}$, $\tilde{\tau}_2 = \tilde{P}(F_A)$ and $\tilde{\tau}_3 = \{F_\Phi, F_A, \{(e_1, \{u_2\})\}, \{$

$(e_1,\{u_2\}),(e_2,\{u_3\})\}$, $\{(e_1, \{u_1,u_2\}),(e_2,\{u_2\})\}$are soft topologies on F_A.

Definition 1.6. [9]

a) Let $(F_A,\tilde{\tau})$ be a soft topological space. Then, every element of $\tilde{\tau}$ is called a soft open set. Clearly, F_Φ and F_A are soft open sets.

b) Let $(F_A,\tilde{\tau})$ be a soft topological space and $F_B \tilde{\subseteq} F_A$. Then, the soft closure of F_B, denoted $\overline{F_B}$, is defined as the soft intersection of all soft closed supersets of F_B. Note that $\overline{F_B}$ is the smallest soft closed set that containing F_B.

c) Let $(F_A,\tilde{\tau})$ be a soft topological space and $\alpha \in F_A$. If there is a soft open set F_B such that $\alpha \in F_B$, then F_B is called a soft open neighborhood (or soft neighborhood) of α. The set of all soft neighborhoods of α, denoted $\tilde{V}(\alpha)$, is called the family of soft neighborhoods of α; that is; $\tilde{V}(\alpha) = \{F_B : F_B \in \tilde{\tau}, \alpha \in F_B\}$.

d) Let $(F_A,\tilde{\tau})$ be a soft topological space and $F_B \tilde{\subseteq} F_A$. Then, F_B is said to be soft closed if the soft set F_B^c is soft open.

e) Let $(F_A,\tilde{\tau})$ be a soft topological space and $F_B \tilde{\subseteq} F_A$. Then, the collection $\tilde{\tau}_{F_B}=\{F_{Ai} \tilde{\cap} F_B : F_{Ai} \in \tilde{\tau}, i \in I \subseteq N\}$is called a soft subspace topology on F_B. Hence, $(F_B, \tilde{\tau}_{F_B})$ is called a soft topological subspace of $(F_A,\tilde{\tau})$.

Definition 1.7. A soft set (F,A) in a soft topological space $(F_A,\tilde{\tau})$ is called

(a) Soft α-open set [2] if $(F,A) \tilde{\subseteq} int(cl(int(F,A)))$.

(b) Soft pre-open (briefly soft P-open) set [6] if $(F,A) \tilde{\subseteq} int(cl(F,A))(cl(int(F,A))) \tilde{\subseteq} (F,A))$.

(c) Soft sime-open (briefly soft S-open) set [10] if $(F,A) \tilde{\subseteq} (cl(int(F,A))(int(cl(F,A)) \tilde{\subseteq} (F,A))$.

(d) Soft b–open set [4] if $(F,A) \tilde{\subseteq} int(cl((F,A))) \tilde{\cup} cl(int((F,A)))$.

(e) Soft β-open set [6] if $(F,A) \tilde{\subseteq} cl(int(cl(F,A)))$.

The complement of a soft α-open (resp. Soft S-open, soft P-open, soft b-open and soft β-open) set is called soft α-closed (resp. Soft S-closed, soft P-clo-sed, soft b-closed and soft β-closed) set. The family of all soft α-open (resp. Soft S-open, soft P-open, soft b- open and soft β-open) sets of $(F_A,\tilde{\tau})$ are larger than $\tilde{\tau}$ and closed under forming arbitrary union. We will call these families soft near topology (briefly S. j-topology), where $j \in \{\alpha,S,P,b,\beta\}$.

Definition 1.8. [20] Let H and K be two non-empty sets and E be the parameter set. Let $\{f_e : H \to K, e \in E\}$ be a collection of functions. Then a mapping $f: SE(H,E) \to SE(K,E)$ defined by $f(e_h) = e_{f_e(h)}$ is called a soft mapping, where SE(H,E) and SE(K,E) are sets of all soft elements of the soft sets $(\tilde{H},E)$ and $(\tilde{K},E)$ respectively.

Definition 1.9. [2, 4, 14, 24] A soft mapping $\phi : (H,\tau,E) \to (K,\sigma,L)$ is said to be soft near continuous (briefly S. j-continuous) if the inverse image of each soft open set of K is a soft j-open set in H where $j \in \{\alpha,S,P,b,\beta\}$.

Example 1.10. Let $H=\{h_1,h_2,h_3\}$, $E = \{e_1, e_2\}$ and $\tilde{\tau}_1 = \{\phi, \tilde{H}, (F\_1, E), (F\_2, E)\}$, $\tilde{\tau}\_2=\{\phi, \tilde{H}, (G_1,E),(G_2,E)\}$ be two soft topologies defined on H; $(F_1,E),(F_2,E)$, (G_1,E) and (G_2,E) are soft sets over H, defined as : following: $F_1(e_1)=\{h_1,h_2\}$, $F_1(e_2)=\{h_3\}$, $F_2(e_1)=H$, $F_2(e_2) =\{h_3\}$ and $G_1(e_1)=\{h_1\}$, $G_1(e_2) = \{h_3\}$, $G_2(e_1) = \{h_1,h_3\}$, $G_2(e_2)=\{h_2,h_3\}$. If we get the mapping $f : (H,\tilde{\tau}_1) \to (H,\tilde{\tau}_2)$ defined as $f(h_1)=f(h_2)=h_1$, $f(h_3)=h_3$ then since $f^{-1}(G_1,E)=(F_1,E)$ and $f^{-1}(G_2,E)=(F_2,E)$, f is a soft continuous mapping.

Definition 1.11. [2, 14, 4, 3] A mapping $\phi : (H,\tau,E) \to (K,\sigma,L)$ is said to be

(a) Soft near-open (briefly, S. j-open) map if the image of every soft open set in H is S.j-open set in K, where $j \in \{\alpha,S,P,b,\beta\}$.

(b) Soft near-closed (briefly, S. j-closed) map if the image of every soft closed set in $H$is S. j-closed set in K, where $j \in \{\alpha,S,P,b,\beta\}$.

Definition 1.12. [14, 4, 24] Let $\phi : (H,\tau,E) \to (K,\sigma,L)$ be a function. ϕ is called soft near irresolute(briefly, S. j-irresolute) if the inverse image of soft j-open set in K is soft j-open in H, where $j \in \{\alpha,S,P,b,\beta\}$.

Proposition 1.13. [2] If $\phi : H \to K$ is a soft pre-continuous and soft semi-continuous, then ϕ is soft α-continuous.

2. Fibrewise Soft Topological Spaces

In this section, we give a definition of fibrewise soft topology and its related properties.

Definition 2.1. Assume that (B,Ω,G) is a soft topology space the fibrewise soft near topology space (briefly, F.W.S. j-topological space) on a fibrewise set H over B meany any Soft j-topology space on H for which the projection P is soft near continuous(briefly, S. j-continuous) where $j \in \{\alpha,S,P,b,\beta\}$.

Remark 2.2. In F.W.S. topological space we work over at soft topological base space B, say. When B is a point-space the theory reduces to that of ordinary soft topology. A F.W.S. topological (resp., S. j-topological) space over B is just a soft topological (resp., S. j-topological) space H together with a soft continuous (resp., S. j-continuous) projection $P_{fu} :(H,\tau,E) \to (B,\Omega,G)$. So the implication between F.W.S. topological spaces and the families of F.W.S. j-topological spaces are given in the following diagram where j$\in \{\alpha,S,P,b,\beta\}$.

F.W.S. topological space
⇓
F.W.S. α-topological space ⇒ F.W.S. S-topological space
⇓ ⇓
F.W.S.P-topological space ⇒ F.W.S.b-topological space
⇓
F.W.S.β-topological space

Example 2.3. Let $H = B = \{a, b, c, d\}$, $E = \{e_1, e_2, e_3\}$, $G = \{g_1, g_2, g_3\}$, (H, τ, E) and let (B, Ω, K) be a F.W.S. topological space. Define $f : H \to B$ and $u : E \to G$ as $f(a) = \{b\}$, $f(b) = \{d\}$, $f(c) = \{a\}, f(d) = \{c\}$, $u(e_1) = \{g_2\}, u(e_2) = \{g_1\}, u(e_3) = \{g_3\}$.
$\tau = \{\phi, H, (F_1, E), (F_2, E), (F_3, E), \ldots, (F_{15}, E)\}$;$(F_1, E)$, (F_2, E), (F_3, E),.., $(F_{15}, E)$are soft sets over (H, τ, E), defined as follows:
$(F_1, E) = \{(e_1, \{a, b\}), (e_2, \{c\}), (e_3, \{a, c\})\}$,
$(F_2, E) = \{(e_1, \{b\}), (e_2, \{a, b\}), (e_3, \{a, b\})$
$(F_3, E) = \{(e_1, \{b\}), (e_3, \{a\})\}$,
$(F_4, E) = \{(e_1, \{a, b\}), (e_2, H), (e_3, H)\}$,
$(F_5, E) = \{(e_1, \{c\}), (e_2\{a, c\}), (e_3, \{b\})\}$,
$(F_6, E) = \{(e_2, \{c\})\}$,
$(F_7, E) = \{(e_1, H), (e_2, \{a, c\}), (e_3, H)\}$,
$(F_8, E) = \{(e_2, \{a\}), (e_3, \{b\})\}$,
$(F_9, E) = \{(e_1, \{b, c\}), (e_2, H), (e_3, \{a, b\})\}$,
$(F_{10}, E) = \{(e_1, \{b, c\}), (e_2, \{a, c\}), (e_3, \{a, b\})\}$,
$(F_{11}, E) = \{(e_1, \{a, c\}), (e_2, \{b\})\}$,
$(F_{12}, E) = \{(e_1, \{b\}), (e_2, H), (e_3, \{a, b\})\}$,
$(F_{13}, E) = \{(e_1, \{b\}), (e_2, \{a\}), (e_3, \{a, c\})\}$,
$(F_{14}, E) = \{(e_1, \{a, b\}), (e_2, \{a, c\}), (e_3, H)\}$,
$(F_{15}, E)= \{(e_1, \{b\}), (e_2, \{c\}), (e_3, \{a\})\}$.

$\Omega=\{\phi, B, (F, G)\}$ and $(F, G)=$ $\{(g_1, \{a, c, d\}), (g_2, \{a, b, d\}), (g_3, \{b, d\})\}$ and let projection $P_{fu} : (H, \tau, E) \to (B, \Omega, G)$ be a soft mapping. Then (F, G) is a soft open in (B, Ω, G) and $P_{fu}^{-1}((F, K)) = \{(e_1, \{a, b, c\}), (e_2, \{b, c, d\}), (e_3, \{a, b\})\}$ is a soft α-open but not soft open in (H, τ, E). Therefore, P_{fu} is a soft α-continuous but not soft continuous Thus, (H, τ, E) is F.W.S. α-topological space but not F.W.S. topological space.

Example 2.4. Let$H =B =\{a, b, c, d\}$, $E = \{e_1, e_2, e_3\}$, $G = \{g_1, g_2, g_3\}$, (H, τ, E) and let (B, Ω, G) be a F.W.S. topological space. Define $f : H \to B$ and $u : E \to G$ as $f(a) = \{b\}, f(b)=\{d\}$, $f(c)=\{a\}, f(d)=\{c\}$, $f(b) = \{d\}$, $u(e_1) =\{g_2\}, u(e_2)=\{g_1\}, u(e_3)=\{g_3\}$. Let us consider the F.W.S. topological space (H, τ, E) over (B, Ω, G) given in Example (2.3): that is, $\tau = \{\phi, H, (F_1, E), (F_2, E), (F_3, E), \ldots, (F_{15}, E)\}$, $\Omega = \{\phi, B, (M, G)\}$, and $(M, G) = \{(g_3, \{d\})\}$ and let projection $P_{fu} : (H, \tau, E) \to (B, \Omega, G)$ be a soft mapping. Then $(M, G)$is a soft open in (B, Ω, G) and $P_{fu}^{-1}((M, G))=\{(e_3, \{b\})\}$ is a soft p-open but not soft α- open in (H, τ, E). Therefore, $P_{fu}$is a soft p-continuous but not softα-continuous. Thus, (H, τ, E) is F.W.S. p-topological space but not F.W.S.α-topological space.

Example 2.5. Let $H = B = \{a, b, c, d\}$, $E = \{e_1, e_2, e_3\}$, and $G = \{g_1, g_2, g_3\}$ and (H, τ, E) and let (B, Ω, K) be a F.W.S. topological space. Define $f : H \to B$and $u : E \to G$ as

$$f(a) = \{d\}, f(b) = \{d\}, f(c) = \{a\}, f(d) = \{c\}, \ u(e_1) = \{g_2\}, u(e_2) = \{g_1\}, u(e_3) = \{g_3\}$$

Let us consider the F.W.S. topological space (H, τ, E) over (B, Ω, G) given in Example (2.3);that is,$\tau =\{\phi, H,$ $(F_1, E), (F_2, E), (F_3, E), \ldots, (F_{15}, E)\}, \Omega=\{\phi, B, (J, G)\}$ and $(J, G) = \{(g_1, \{a\}),$ $(g_2, \{d\}), (g_3, \{b, d\})\}$ and let projection $P_{fu} : (H, \tau, E) \to (B, \Omega, G)$ be a soft mapping. Then (J, G) is a soft open in (B, Ω, G) and $P_{fu}^{-1}((J, G)) = \{(e_1, \{b\}), (e_2, \{c\}), (e_3, \{a, c\})\}$ is a soft α-open but not soft open in (H, τ, E). Therefore, P_{fu} is a soft s-continuous but not soft α-continuous. Thus, (H, τ, E) is F.W.S. s-topological pace but not F.W.S. α- topological space.

Example 2.6. Let $H = B = \{a, b, c, d\}$, $E = \{e_1, e_2, e_3\}$, and $G = \{g_1, g_2, g_3\}$ and $(H, \tau, E)$and let (B, Ω, G) be a F.W.S. topological space. Define $f : H \to B$ and $u : E \to G$ as $f(a) = \{b\}, f(b) = \{d\}, f(c) = \{a\}, f(d) = \{c\}, u(e_1) = \{g_2\}, u(e_2) = \{g_1\}, u(e_3) = \{g_3\}$. Let us consider the F.W.S. topological space (H, τ, E) over (B, Ω, G) given in Example (2.3); that is, $\tau = \{\phi, H, (F_1, E), (F_2, E), (F_3, E), \ldots, (F_{15}, E)\}$, $\Omega = \{\phi, B, (O, G)\}$, and $(O, G) = \{(g_1, \{a, b, c\}), (g_2, \{d\}),$ $(g_3, \{c, d\})\}$and let the projection $P_{fu}: (H, \tau, E) \to (B, \Omega, G)$be a soft mapping. Then (O, G) is a soft open in (B, Ω, G) and $P_{fu}^{-1}((O, G))=\{(e_2, \{c, a, d\}), (e_1, \{b\}),$ $(e_3, \{d, b\})\}$ is a soft b-open but not soft S- open in (H, τ, E). Therefore, P_{fu} is a soft b-continuous but not soft s-continuous. Thus, (H, τ, E) is F.W.S. b-topological space but not F.W.S. S- topological space.

Example 2.7. Let $H = B = \{a, b, c, d\}$, $E = \{e_1, e_2, e_3\}$, $G = \{g_1, g_2, g_3\}$, (H, τ, E) and let (B, Ω, G)be a F.W.S .topological space. Define $f : H \to B$ and $u : E \to G$ as $f(a) = \{b\}, f(b) = \{d\}, f(c) = \{a\}, f(d) = \{c\}$, $u(e_1) = \{g_2\}$, $u(e_2) = \{g_1\}$, $u(e_3) = \{g_3\}$. Let us consider the F.W.S. topological space (H, τ, E) over (B, Ω, G) given in Example(2.3);that is

$\tau = \{\phi, H, (F_1, E), (F_2, E), (F_3, E), \ldots, (F_{15}, E)\}$,
$\Omega = \{\phi, B, (N, G)\}$, and $(N, G) = \{(g_1, \{a, b\}), (g_2, \{c, d\}),$

$(g_3, \{a, b, d\})\}$and let the projection $P_{fu}: (H, \tau, E) \to (B, \Omega, G)$ be a soft mapping. Then (N, G) is a soft open in (B, Ω, G) and $P_{fu}^{-1}((N, G))$= {(e_2, {a, c}), (e_1, {d, b}), (e_3, {c, a, b})}. is a soft b-open but not soft p- open in (H, τ, E). Therefore, P_{fu} is a soft b-continuous but not soft p-continuous. Thus, (H, τ, E) is F.W.S. b-topological space but not F.W.S. p-topological space.

Example 2.8. Let $H = B = \{a, b, c, d\}$, $E = \{e_1, e_2, e_3\}$, $G = \{g_1, g_2, g_3\}$, $(H, \tau, E)$and let (B, Ω, G)be a F.W.S topological space. Define $f: H \to B$ and $u: E \to G$ as $f(a) = \{b\}, f(b) = \{d\}, f(c) = \{a\}, f(d) = \{c\}, u(e_1) = \{g_2\}$, $u(e_2)$= $\{g_1\}$, $u(e_3) = \{g_3\}$. Let us consider the F.W.S. topological space (H, τ, E) over (B, Ω, G) given in Example (2.3); that is, τ={$\phi, H, (F_1, E), (F_2, E), (F_3, E), \ldots, (F_{15}, E)$}, Ω={$\phi, B, (L, G)$}, and (L, G)= {(g_1, {b, d}), (g_2, {a, c}), (g_3, {a, b, d})} and let the projection $P_{fu}: (H, \tau, E) \to (B, \Omega, G)$ be a soft mapping. Then (L, G) is a soft open in (B, Ω, G) and $P_{fu}^{-1}((L, G))$= $\{(e_1, \{a, b\}), (e_2, \{c, d\}), (e_3, \{a, c, d\})\}$ is a soft β-open but not soft b-open in (H, τ, E). Therefore, P_{fu} is a soft β-continuous but not soft b-continuous. Thus, (H, τ, E) is F.W.S. β-topological space but not F.W.S. b-topological space.

Proposition 2.9. F.W.S. s-topological space and F.W.S. p-topological space iff F.W.S. α-topological space.

Proof . ($\Longrightarrow$) Let (H, τ, E) be a F.W.S. s-topological space over (B, Ω, G), and be a F.W.S. p- topological space over (B, Ω, G) then the projection $P_{fu}: (H, \tau, E) \to (B, \Omega, G)$ exists. To show that P_{fu} is softα-continuous. Since (H, τ, E) is F.W.S.s-topological space over (B, Ω, G) and (H, τ, E) be a F.W.S. p- topological space over (B, Ω, G), then P_{fu} is soft s-continuous and soft p-continuous then P_{fu} is soft α-continuous by proposition (1.15). Thus, (H, τ, E) is F.W.S. α-topological space over (B, Ω, G).
($\Longleftarrow$) It obvious.

Let $\phi: (H) \to (K, \sigma, L)$ be a fibrewise soft function, (H) is a fibrewise set and (K, σ, L) is a fibrewise topological space over (B, Ω, G). We can give (H, τ, E) the induced (resp. j-induced) soft topology, in the ordinary sense, and this is necessarily a F.W.S. topology (resp. j-topology). We may refer to it, as the induced (resp. j-induced) F.W.S. topology, where $j \in \{\alpha, S, P, b, \beta\}$, and note the following characterizations.

Proposition 2.10. Let $\phi: (H, \tau, E) \to (K, \sigma, L)$ be a fibrewise soft function, (K, σ, L) a F.W.S. topological space over (B, Ω, G) and (H, τ, E) has the induced F.W.S. topology. Then for each F.W.S. topological space (Z, γ, M), a fibrewise soft function $\psi: (Z, \gamma, M) \to (H, \tau, E)$ is soft j-continuous iff the composition $\phi \circ \psi: (Z, \gamma, M) \to (K, \sigma, L)$ is soft j-continuous, where $j \in \{\alpha, S, P, b, \beta\}$.

Proof. ($\Longrightarrow$) Suppose that ψ is soft j-continuous. Let $z \in Z_b$, where $b \in B$ and (N, L) soft open set of $(\varphi \circ \psi)(z) = k \in K_b$ in (K, σ, L). Since ϕ is soft continuous, $\phi^{-1}(N, L)$is a soft open set containing $\psi(z)$=$h \in H_b$in (H, τ, E). Since ψ is soft j-continuous, then $\Psi^{-1}(\phi^{-1}(N, L))$ is a soft j-open set containing $z \in Z_b$ in (Z, γ, M) and $\psi^{-1}(\phi^{-1}(N, L)$=$(\phi \circ \psi)^{-1}(N, L)$ is a soft j-open set containing $z \in Z_b$ in (Z, γ, M), where $j \in \{\alpha, S, P, b, \beta\}$.
($\Longleftarrow$) Suppose that $\phi \circ \psi$ is soft j-continuous. Let $z \in Z_b$, where $b \in B$ and (F, E) soft open set of $\psi(z) = h \in H_b$ in (H, τ, E). Since ϕ is open, $\phi(F, E)$ is a soft open set containing $\phi(h) = \phi(\psi(z)) = k \in K_b$in (K, σ, L). Since $\phi \circ \psi$ is soft j-continuous, then $(\phi \circ \psi)^{-1}(\phi(F, E)) = \Psi^{-1}(F, E)$ is a soft j-open set containing $z \in Z_b$ in (Z, γ, M), where $j \in \{\alpha, S, P, b, \beta\}$.

Proposition 2.11. Let $\phi: (H, \tau, E) \to (K, \sigma, L)$ be a fibrewise soft function, (K, σ, L) a fibrewise soft topological space over (B, Ω, G) and (H, τ, E) has the j-induced fibrewise soft topology. If for each F.W.S. topological space (Z, γ, M), a fibrewise soft function $\psi: (Z, \gamma, M) \to (H, \tau, E)$ is soft j-irresolute iff the composition $\phi \circ \psi: (Z, \gamma, M) \to (K, \sigma, L)$ is soft j-continuous, where $j \in \{\alpha, S, P, b, \beta\}$.

Proof . The proof is similar to the proof of Proposition (2.10).

Proposition 2.12. Let $\phi: (H, \tau, E) \to (K, \sigma, L)$ be a fibrewise soft function, (K, σ, L) a fibrewise soft topological space over (B, Ω, G) and (H, τ, E) has the induced fibrewise soft topology. If for each fibrewise soft topological space (Z, γ, M), a fibrewise soft function $\psi: (Z, \gamma, M) \to (H, \tau, E)$ is soft open, surjective iff the composition $\phi \circ \psi: (Z, \gamma, M) \to (K, \sigma, L)$ is soft open.

Proof . The proof is similar to the proof of Proposition (2.10).

3. Fibrewise Soft Near Closed and Soft Near Open Topological Spaces

In this section, we introduce the concepts of fibrewise soft near closed, soft near open topological spaces. Several topological properties on the obtained concepts are studied.

Definition 3.1. A F.W.S. topological space (H, τ, E) over (B, Ω, G) is called fibrewise soft j-closed (briefly, F.W.S. j-closed) if the projection $P_{fu}$is soft j-closed, where $j \in \{\alpha, S, P, b, \beta\}$.

Proposition 3.2. Let $\phi: (H, \tau, E) \to (K, \sigma, L)$ be a closed fibrewise soft function, where (H, τ, E) and (K, σ, L) are F.W.S. topological spaces over (B, Ω, G). If (K, σ, L) is F.W.S. j-closed, then (H, τ, E) is F.W.S. j-closed, where $j \in \{\alpha, S, P, b, \beta\}$.

Proof. Suppose that $\phi : (H,\tau,E) \to (K,\sigma,L)$ is closed fibrewise soft function and (K,σ,L) is F.W.S. j-closed i.e., the projection $P_{K(qd)} : (K,\sigma,L) \to (B,\Omega,G)$ is soft j-closed. To show that (H,τ,E) is F.W.S. j-closed i.e., the projection $P_{H(fu)} : (H,\tau,E) \to (B,\Omega,K)$ is soft j-closed. Now let (F,C) be a soft closed subset of H_b, where $b \in B$, since ϕ is soft closed, then $\phi(F,C)$ is closed subset of K_b. Since $P_{K(fu)}$ is soft j-closed, then $P_{K(qd)}(\phi(F,C))$ is soft j-closed in (B,Ω,G), but $P_{K(qd)}(\phi(F,C)) = (P_{K(qd)} \circ \phi)(F,C) = P_{H(fu)}(F,C)$ is soft j-closed in (B,Ω,G). Thus, $P_{H(fu)}$ is soft j-closed and (H,τ,E) is F.W.S. j-closed, where j$\in \{\alpha,S,P,b,\beta\}$.

Proposition 3.3. Let (H,τ,E) be a F.W.S topological space over (B,Ω,G). Suppose that (H_i,E_i) is F.W.S. j-closed for each member (H_i,E_i) of a finite covering of (H,τ,E). Then (H,τ,E) is F.W.S. j- closed.

Proof. Let (H,τ,E) be a F.W.S. topological space over (B,Ω,G), then the projection $P_{fu}: (H,\tau,E) \to (B,\Omega,G)$ exists. To show that P_{fu} is soft j-closed. Now, since (H_i,E_i) is F.W.S. j-closed, then the projection $P_{i(fu)} : (H_i,E_i) \to (B,G)$is soft j-closed for each member (H_i,E_i) of a finite covering of (H,τ,E). Let (F,C) be a soft j-closed subset of (H,τ,E), then $P_{fu}(F,C) = \cup\left((H_i,E_i)\cap(F,C)\right)$ which is a finite union of soft closed sets and hence P_{fu} is soft j-closed. Thus, (H,τ,E) is F.W.S. j-closed, where $j \in \{\alpha,S,P,b,\beta\}$.

Proposition 3.4. Let (H,τ,E) be a F.W.S. topological space over (B,Ω,G). Then (H,τ,E) is F.W.S. j-closed iff for each fibre soft (H_b,E_b) of (H,τ,E) and each soft open set (F,E) of (H_b,E_b) in (H,τ,E), there exists a soft j-open set (F,G) of b such that $(H_{(F,G)},E_{(F,G)}) \subset (F,E)$, where $j \in \{\alpha,S,P,b,\beta\}$.

Proof : $(\Longrightarrow)$ Suppose that (H,τ,E) is F.W.S. j-closed i.e., the projection $P_{fu} : (H,\tau,E) \to (B,\Omega,G)$ is soft j-closed. Now, let $b \in B$ and (F,E) soft open set of (H_b,E_b) in (H,τ,E), then $(H,\tau,E)\_(F,E)$ is soft closed in (H,τ,E), this implies $P_{fu}((H,\tau,E) - (F,E))$ is soft j-closed in (B,Ω,E), let $(F,G) = (B,\Omega,G)$-$P_{fu}((H,\tau,E) - (F,E))$, then$(F,G)$ a soft j-open set of b in (B, Ω, G) and $(H_{(F,E)}, \tau_{(F,E)}, E_{(F,E)})$ $P_{fu}^{-1}(F,G)(H,\tau,E) - P_{fu}^{-1}($ $P_{fu}((H,\tau,E) - (F,E))) \subset (F,E)$, where $j \in \{\alpha,S,P,b,\beta\}$.

$(\Longleftarrow)$ Suppose that the assumption hold and $P_{fu} : (H,\tau,E) \to (B,\Omega,G)$. Now, let (F,C) be a soft closed subset of (H,τ,E) and $b \in B - P(F,C)$ and each soft open set (F,E) of fibre soft (H_b,E_b) in (H,τ,E). By assumption there exists a soft j-open (F,G) of b such that $(H_{(F,G)},E_{(F,G)}) \subset (F,E)$. It is easy to show that $(F,G) \subset (B,\Omega,G) - P_{fu}(F,C)$, hence $(B,\Omega,G) - P(F,L)$ is soft j-open in (B,Ω,G) and this implies $P(F,L)$ is soft j-closed in (B,Ω,G) and P_{fu} is soft j-closed. Thus, (H,τ,E) is F.W.S. j-closed, where $j \in \{\alpha,S,P,b,\beta\}$.

Definition 3.5. A F.W.S (H,τ,E) over (B,Ω,G) is called fibrewise soft near open (briefly, F.W.S. j-open) if the projection P_{fu} is soft j-open where j$\in \{\alpha,S,P,b,\beta\}$.

Proposition 3.6. Let $\phi : (H,\tau,E) \to (K,\sigma,L)$ be a soft open fibrewise function, where (H,τ,E) and $(K,\sigma,L)$are F.W.S.topological spaces over (B,Ω,G). If (K,L) is F.W.S. j-open, then (H,τ,E) is F.W.S. j-open, where $j \in \{\alpha,S,P,b,\beta\}$.

Proof. Suppose that $\phi : (H,\tau,E) \to (K,\sigma,L)$ is open fibrewise soft function and (K,σ,L) is F.W.S. j-open i.e., the projection $P_{K(qd)} : (K,\sigma,L) \to (B,\Omega,G)$ is soft j-open. To show that (H,τ,E) is F.W.S. j-open i.e., the projection $P_{H(fu)} : (H,\tau,E) \to (B,\Omega,G)$ is soft j-open. Now let (F,E) is soft open subset of H_b, where $b \in B$, since ϕ is soft open, then $\phi(F,E)$ is soft open subset of K_b, since $P_{K(qd)}$ is soft j-open, then $P_{K(qd)}(\phi(F,E))$is soft j-open in (B,Ω,G), but $P_{K(fu)}(\phi(F,E)) = (P_{K(qd)} \circ \phi)(F,E)$ is soft j-open in (B,Ω,G). Thus, $P_{H(fu)}$ is soft j-open and (H,τ,E) is F.W.S. j-open, where j$\in \{\alpha,S,P,b,\beta\}$.

Proposition 3.7. Let (H_r,τ_r,E_r) be a finite family of F.W.S. j-open spaces over (B,Ω,G). Then the F.W.S. topological product $(H,\tau,E) = \prod_B(H_r,\tau_r,E_r)$ is also F.W.S. j-open, where $j\in\{\alpha,S,P,b,\beta\}$.

Proof. Let (H_r,τ_r,E_r) be a finite family of F.W.S. j-open. Suppose that $(H,\tau,E)=\prod_B(H_r,\tau_r,E_r)$ is a F.W.S over (B,Ω,G), then $P_{fu} : (H,\tau,E)=\prod_B(H_r,\tau_r,E_r) \to (B,\Omega,G)$ is exists. To show that P_{fu} is soft j-open. Now, since (H_r,τ_r,E_r) be a finite family of F.W.S. j-open spaces over (B,Ω,G), then the projection $P_{r(fu)}:(H_r,\tau_r,E_r) \to (B,\Omega,G)$ is soft j-open for each r. Let (F,E) be a soft open subset of (H,τ,E), then $P_{fu}(F,E)=$ $P_{fu}(\prod_B((H_r,\tau_r,E_r) \cap (F,E))) = \prod_B P_{r(fu)}((H_r,\tau_r,E_r) \cap (F,E))$ which is a finite product of soft j-open sets and hence P_{fu} is soft j-open. Thus, the F.W.S. topological product $(H,\tau,E)=\prod_B(H_r,\tau_r,E_r)$ is a F.W.S. j-open, where $j \in \{\alpha,S,P,b,\beta\}$.

Remark 3.8. If (H,τ,E) is F.W.S. open (resp. F.W.S. j-open) then the second projection $\pi_2: (H,\tau,E) \times_B (K,\sigma,L) \to (K,\sigma,L)$ is soft open (resp. Soft j-open) for all F.W.S. topological space (K,σ,L). Because for every nonempty Soft open (resp. Soft open, Soft j- open and Soft j-open) set $(F,E) \times_B (F,L) \subset (H,\tau,E) \times_B (K,\sigma,L)$, we have

$\pi_2((F,E)\times_B (F,L))$=$(F,L)$ is soft open (resp. Soft j-open, Soft open and Soft j-open), where $j \in \{\alpha, S, P, b, \beta\}$ We will use this in the proof of the following results.

Proposition 3.9. Let $\phi : (H,\tau,E) \to (K,\sigma,L)$ be a fibrewise soft function, where (H,τ,E) and (K,σ,L) are F.W.S. topological spaces over (B,Ω,G). Let $id_H \times \phi: (H,\tau,E)\times_B (H,\tau,E) \to (H,\tau,E)\times_B (K,\sigma,L)$. If $id_H \times \phi$ is soft open and that (H,τ,E) is F.W.S. open, (K,σ,L) is F.W.S. j-open. Then ϕ it self is j- open, where $j \in \{\alpha, S, P, b, \beta\}$.

Proof. Consider the following commutative figure.

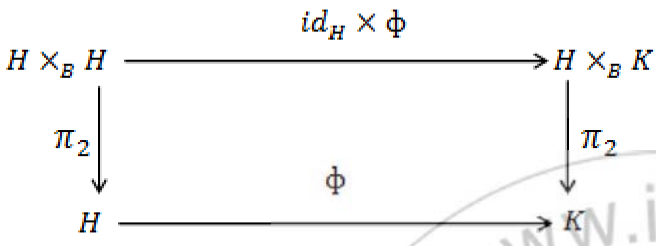

Figure 1: Diagram of Proposition 3.9

The projection on the left is surjective and soft j-open, since (K,σ,L) is F.W.S. j-open, while the projection on the right is soft j- open, since (H,τ,E) is F.W.S. j- open. Therefore, $\pi_2 \circ (id_H \times \phi) = \phi \circ \pi_2$ is soft j-open, and so ϕ is soft j-open, by Proposition (2.3) as asserted, where $j \in \{\alpha, S, P, b, \beta\}$.

Proposition 3.10. Let $\phi : (H,\tau,E) \to (K,\sigma,L)$ be a soft j-continuous fibrewise surjection, where (H,τ,E) and (K,σ,L) are F.W.S. topological spaces over (B,Ω,G).

Proof. Suppose that $\phi : (H,\tau,E) \to (K,\sigma,L)$ is soft j-continuous fibrewise surjection and (H,τ,E) is F.W.S. j-closed (resp. F.W.S. j-open) i.e., the projection $P_{H(fu)} : (H,\tau,E) \to (B,\Omega,G)$ is soft j-closed (resp. Soft j-open). To show that (K,σ,L) is F.W.S. j-closed (resp. F.W.S. j-open) i.e., the projection $P_{K(qd)} : (K,\sigma,L) \to (B,\Omega,G)$ is soft j-closed (resp. Soft j-open). Let(G,E) be a soft closed (resp. soft open) subset of K_b, where $b \in B$. Since ϕ is soft continuous fibrewise, then $\phi^{-1}(G,E)$ is soft closed (resp. Soft open) subset of H_b. Since $P_{H(fu)}$is soft j-closed (resp. Soft j-open), then $P_{H(fu)}(\phi(G,E))$ is soft j-closed (resp. Soft j-open) in (B,Ω,L). But $P_{H(fu)}(\phi(G,E))$=$(P_{H(fu)} \circ \phi^{-1})$ (G,E)=$P_{K(fu)}(G,E)$ is soft j-closed (resp. Soft j-open) in (B,Ω,G). Thus $P_{K(fu)}$ is Soft j-closed (resp. Soft j-open) and (K,σ,L) is F.W.S. j-closed (resp. F.W.S. j-open), where $j \in \{\alpha, S, P, \beta, b\}$.

Proposition 3.11. Let (H,τ,E) be a F.W.S. topological space over (B,Ω,G). Suppose that (H,τ,E) is F.W.S. j-closed (resp. F.W.S j-open) over (B,Ω,G). Then (H_{B^*},E_{B^*}) is F.W.S. j-closed (resp. F.W.S. j-open) over (B^*,Ω^*,G^*) for each subspace (B^*,Ω^*,G^*) of (B,Ω,G), where $j \in \{\alpha, S, P, b, \beta\}$.

Proof. Suppose that (H,τ,E) is a F.W.S. j-closed (resp. F.W.S. j-open) i.e., the projection $P_{fu} : (H,\tau,E) \to (B,\Omega,G)$ is soft j-closed (resp. Soft j-open). To show that $(H_{B^*},\tau_{B^*},E_{B^*})$ is F.W.S. j-closed (resp. F.W.S. j-open) over (B^*,Ω^*,G^*) i.e., the projection $P_{B^*(fu)} : (H_{B^*},\tau_{B^*},E_{B^*}) \to (B^*,\Omega^*,G^*)$ is soft j-closed (resp. Soft j-open). Now, let (N,E) be a Soft closed (resp. Soft open) subset of (H,τ,E), then $(G,E) \cap (H_{B^*},\tau_{B^*},E_{B^*})$ is soft closed (resp. Soft open) in subspace $(H_{B^*},\tau_{B^*},E_{B^*})$and $P_{B^*(fu)}$ $((G,E) \cap$ $(H_{B^*},\tau_{B^*},E_{B^*})$ =$P_{fu}((G,E) \cap (H_{B^*},\tau_{B^*},E_{B^*})$=$P_{fu}(G,E) \cap (B^*,\Omega^*,G^*)$ which is soft j-closed (resp. Soft j-open) set in(B^*,Ω^*,G^*). Thus $P_{B^*(fu)}$ is soft j-closed (resp. Soft j-open) and $(H_{B^*},\tau_{B^*},E_{B^*})$ is F.W.S. j-closed (resp. F.W.S. j-open) over(B^*,Ω^*,G^*), where $j \in \{\alpha, S, P, b, \beta\}$.

4. Fibrewise Soft Near Compact and Locally Soft Near Compact Spaces.

In this section, we study fibrewise soft near compact and fibrewise locally soft near compact spaces as a generalizations of well-known concepts soft near compact and locally soft near compact topological spaces.

Definition 4.1. The function $\phi : (H,\tau,E) \to (K,\sigma,L)$ is called soft near proper (briefly S. j-proper) function if it is S. j-continuous, closed and for each $(k) \in K, \phi^{-1}(k)$ is compact set, where $j \in \{\alpha, S, P, b, \beta\}$.

For example, let (R,τ,E) where τ is the topology with basis whose members are of the form (a,b) and $(a,b) - N$, N = $\{1\backslash n; n \in Z^+\}$ and E =N Define $\phi : (R,\tau,E) \to (R,\sigma,L)$ by $\phi\,(F,E)$=(F,E), then ϕ is soft j-proper function, where $j \in \{\alpha, S, P, b, \beta\}$.

If $\phi : (H,\tau,E) \to (K,\sigma,L)$ is fibrewise and S. j-proper function, then ϕ is said to be fibrewise S. j-proper function, where $j \in \{\alpha, S, P, b, \beta\}$.

Definition 4.2. The F.W.S. topological space (H,τ,E) over (B,Ω,G) is called fibrewise soft j-compact (briefly, F.W.S.j-compact) if the projection P_{fu} is soft j-proper, where $j \in \{\alpha, S, P, b, \beta\}$.

Proposition 4.3. The F.W.S. topological space (H,τ,E) over (B,Ω,G) is F.W.S. j-compact iff (H,τ,E) is fibrewise soft closed and every fibre soft of (H,τ,E) is soft j-compact, where $j \in \{\alpha, S, P, b, \beta\}$.

Proof. $(\Longrightarrow)$ Let (H,τ,E) be a F.W.S. j-compact space, then the projection $P_{fu} : (H,\tau,E) \to (B,\Omega,G)$ is soft j-proper function i.e., P_{fu} is soft closed and for each $b \in B$, $H_b$is soft j-compact. Hence (H,τ,E) is fibrewise soft closed and every fibre soft of (H,τ,E) is soft j-compact, where $j \in \{\alpha, S, P, b, \beta\}$.

($\Leftarrow$) Let (H,τ,E) be F.W.S. closed and every fibre soft of (H,τ,E) is soft j-compact, then the projection $P_{fu} : (H,\tau,E) \to (B,\Omega,G)$ is closed and it is clear that P_{fu} is S. j-continuous, also for each $b \in B$, H_b is soft j-compact. Hence (H,τ,E) is F.W.S. j-compact, where $j \in \{\alpha, S, P, b, \beta\}$.

Proposition 4.4. Let (H,τ,E) F.W.S. topological space over (B,Ω,G). Then is F.W.S. j-compact iff for each fibre soft H_b of (H,τ,E) and each covering Γ_E of H_b by soft open sets of H there exists a soft nbd (N,G) of b such that a finite subfamily of Γ_E covers $H_{(N,G)}$, where $j \in \{\alpha, S, P, b, \beta\}$.

Proof . ($\Rightarrow$) Let (H,τ,E) be F.W.S. j-compact space, then the projection $P_{fu} : (H,\tau,E) \to (B,\Omega,G)$ is soft j-proper function, so that H_b is soft j-compact for each $b \in B$. Let Γ_E be a covering of H_b by soft open sets of H for each $b \in B$ and let $H_{(N,G)}=\cup H_b$ for each $\in (N,G)$. Since H_b is soft j-compact for each $b \in (N,G) \in B$ and the union of soft j-compact sets is soft j-compact, we have $H_{(N,G)}$ is soft j-compact. Thus, there exists a soft nbd (N,G) of b such that a finite subfamily of Γ_E covers $H_{(N,G)}$, where $j \in \{\alpha, S, P, b, \beta\}$.

($\Leftarrow$) Let (H,τ,E) be F.W.S. topological space over (B,Ω,G), then the projection $P_{fu} : (H,\tau,E) \to (B,\Omega,G)$ exist. To show that P_{fu} is soft j-proper. Now, it is clear that P_{fu} is soft j- continuous and for each $b \in B$, H_b is soft j-compact by take $H_b=H_{(N,G)}$. By Proposition (3.4), we have P_{fu} is closed. Thus, P_{fu} is soft j-proper and (H,τ,E) is F.W.S. j-compact, where $j \in \{\alpha, S, P, b, \beta\}$.

Proposition 4.5. Let $\phi : (H,\tau,E) \to (K,\sigma,L)$ be a j-proper, j-closed fibrewise soft function, where $(H,\tau,E)$and (K,σ,L) are F.W.S. topological spaces over (B,Ω,G) If (K,σ,L) is F.W.S. j-compact then so is (H,τ,E), where $j \in \{\alpha, S, P, b, \beta\}$.

Proof : Suppose that $\phi : (H,\tau,E) \to (K,\sigma,L)$ is j-proper, j-closed fibrewise soft function and (K,σ,L) is F.W.S. j-compact space i.e., the projection $P_{K(qd)} : (K,\sigma,L) \to (B,\Omega,G)$ is j-proper. To show that (H,τ,E) is F.W.S. j-compact space i.e., the projection $P_{H(fu)}:(H,\tau,E) \to (B,\Omega,G)$is soft j-proper. Now, clear that $P_{H(fu)}$ is soft j-continuous. let (F,L) be a soft closed subset of H_b, where $b \in B$. Since ϕ is closed, then $\phi(F,L)$ is soft closed subset of K_b. Since $P_{K(qd)}$ is soft closed, then $P_{K(fu)}(\phi(F,L))$ is soft closed in (B,Ω,G). But $P_{K(qd)}(\phi(F,L))=(P_{K(qd)} \circ \phi)(F,L)=P_{H(fu)}(F,L)$ is soft closed in (B,Ω,G) so that $P_{H(fu)}$ is soft closed. Let $b \in B$, since $P_{K(qd)}$ is soft j-proper, then K_b is soft j-compact. Now let $\{(F,E)_i : i \in I\}$ be a family of soft j-open sets of (H,τ,E) such that $K_b \subset \cup_{i\in I}(F,E)_i$. If $k \in K_b$, then there exist a finite soft subset $N(k)$ of I such that $\phi^{-1}(k) \subset \cup_{i\in N(k)}(F,E)_i$. Since ϕ is soft j-closed function, so by Proposition (3.4) there exist a soft j-open set $(M,L)_k$ of (K,σ,L) such that $k \in (M,L)_k$ and $\phi^{-1}((M,L)_y) \subset \cup_{i\in N(y)}(F,E)_i$. Since K_b is soft j-compact, there exist a finite subset C of K_b such that $K_b \subset \cup_{k\in c}(M,L)_k$. Hence $\phi^{-1}(K_b) \subset \cup_{k\in c}\phi^{-1}((M,L)_k \subset \cup_{k\in c}\cup_{i\in N(k)}(F,E)_i$. Thus if $N=\cup_{k\in c}N(k)$, then N is a finite subset of I and $\phi^{-1}(K_b)\cup_{i\in N}((F,E)_i)$. Thus, $\phi^{-1}(K_b)$ $=\phi^{-1}\left(P^{-1}_{K(qd)}(b)\right)=(P_{K(qd)} \circ \phi)^{-1}(b)=P^{-1}_{H(fu)}(b)=H_b$ and $H_b \subset \cup_{i\in M}(F,E)_i$ so that H_b is soft j-compact. Thus, $P_{H(fu)}$ is soft j-proper and (H,τ,E) is F.W.S. j-compact, where $j \in \{\alpha, S, P, b, \beta\}$.

The class of fibrewise soft j-compact spaces is multiplicative, where $j \in \{\alpha, S, P, b, \beta\}$, in the following sense:

Proposition 4.6. Let (H_r,τ_r,E_r) be a family of F.W.S. j-compact spaces over (B,Ω,G). Then the F.W.S. topological product $(H,\tau,E)=\prod_B(H_r,\tau_r,E_r)$ is F.W.S. j-compact, where $j \in \{\alpha, S, P, b, \beta\}$.

Proof. Without loss of generality, for finite products a simple argument can be used. Thus, let (H,τ,E) and(K,σ,L) be F.W.S. topological spaces over (B,Ω,G). If (H,τ,E) is F.W.S. j-compact then the projection $P_{(fu)} \times id_K:(H,\tau,E) \times_B (K,\sigma,L) \to (B,\Omega,G) \times_B (K,\sigma,L)\equiv(K,\sigma,L)$ is soft j-proper. If (K,σ,L) is also F.W.S.j-compact then so is $(H,\tau,E) \times_B (K,\sigma,L)$, by Proposition (3.5).

A similar result holds for finite coproducts.

Proposition 4.7. Let (H,τ,E) be F.W.S. topological space over (B,Ω,G). Suppose that (H_i,E_i), is F.W.S. j-compact for each member (H_i,E_i) of a finite covering of (H,τ,E) Then (H,τ,E) is F.W.S. j-compact, where $j \in \{\alpha, S, P, b, \beta\}$.

Proof. Let (H,τ,E) be F.W.S. topological space over (B,Ω,G) then the projection $P_{fu} : (H,\tau,E) \to (B,\Omega,G)$ exist. To show that P_{fu} is soft j-proper. Now, it is clear that P_{fu} is soft j-continuous. Since (H_i,E_i) is F.W.S. j-compact, then the projection $P_{i(fu)}:(H_i,E_i) \to (B,G)$ is soft closed and for each $b \in B$, $(H_{i_b},E_{i_b})$is soft j-compact for each member (H_i,E_i) of a finite covering of (H,τ,E). Let (F,L) be a soft closed subset of (H,τ,E), then $P_{fu}(F,L) = \cup P_{i(fu)}((H_i,E_i) \cap (F,L)$ which is a finite union of soft closed sets and hence P_{fu} is soft closed. Let $b \in B$, then $H_b = \cup(H_i)_b$ which is a finite union of soft j-compact sets and hence H_b is soft j-compact. Thus, P_{fu} is soft j-proper and (H, τ, E) is F.W.S. j-compact, where $j \in \{\alpha, S, P, b, \beta\}$.

Definition 4.8. A F.W.S. topological space (H, τ, E) over (B, Ω, G) is called fibrewise soft j-irresolute (briefly, F.W.S. j-irresolute) if the projection P_{fu} is soft j-irresolute, where $j \in \{\alpha, S, P, b, \beta\}$.

Proposition 4.9. Let $\phi : (H, \tau, E) \to (K, \sigma, L)$ be a soft j-continuous, soft j-irresolute fibrewise surjection, where (H, τ, E) and (K, σ, L) are F.W.S. topological spaces over (B, Ω, G). If (H, τ, E) is F.W.S. j-compact then so is (K, σ, L), where $j \in \{\alpha, S, P, b, \beta\}$.

Proof. Suppose that $\phi : (H, \tau, E) \to (K, \sigma, L)$ is a soft j-continuous, soft j-irresolute fibrewise surjection and (H, τ, E) is F.W.S. j-compact i.e., the projection $P_{H(fu)} : (H, \tau, E) \to (B, \Omega, G)$ is soft j-proper. To show that (K, σ, L) is F.W.S. j-compact i.e., the projection $P_{K(qd)} : (K, \sigma, L) \to (B, \Omega, G)$ is soft j-proper. Now, it is clear that $P_{K(qd)}$ is soft j-continuous. Let (F, L) be a closed subset of K_b, where $b \in B$. Since ϕ is soft j-continuous F.W.S. topological space over (B, Ω, G), then $\phi^{-1}(F, L)$ is soft closed subset of H_b. Since $P_{H(fu)}$ is soft closed, then $P_{H(fu)}(\phi^{-1}(F, L))$is soft closed in (B, Ω, G) But $P_{H(fu)}(\phi^{-1}(F, L)) = (P_{H(fu)} \circ \phi^{-1})(F, L) = P_{K(qd)}(F, L)$ is soft closed in (B, Ω, G), hence $P_{K(qd)}$ is soft closed. For any point $b \in B$, we have $K_b = \phi(H_b)$ is soft j-compact because H_b is soft j-compact and the image of a soft j-compact subset under soft j-irresolute function is soft j-compact. Thus, $P_{K(qd)}$ is soft j-proper and (K, σ, L) is F.W.S. j-compact, where $j \in \{\alpha, S, P, b, \beta\}$.

Proposition 4.10. Let (H, τ, E) be F.W.S. j-compact space over (B, Ω, G). Then $(H_{B^*}, \tau_{B^*}, E_{B^*})$ is F.W.S. j-compact space over (B^*, Ω^*, G^*) for each soft subspace (B^*, Ω^*, G^*) of (B, Ω, G), where $j \in \{\alpha, S, P, b, \beta\}$.

Proof. Suppose that(H, τ, E) is F.W.S. j-compact i.e., the projection $P_{fu} : (H, \tau, E) \to (B, \Omega, G)$ is soft j-proper. To show that $(H_{B^*}, \tau_{B^*}, E_{B^*})$ is F.W.S. j-compact space over (B^*, Ω^*, G^*) i.e., the projection $P_{B^*(fu)} : (H_{B^*}, \tau_{B^*}, E_{B^*}) \to (B^*, \Omega^*, G^*)$ is soft j-proper. Now, it is clear that $P_{B^*(fu)}$ is soft j-continuous. Let (F, L) be a soft closed subset of (H, τ, E), then $(F, L) \cap (H_{B^*}, \tau_{B^*}, E_{B^*})$ is soft closed in soft subspace $(H_{B^*}, \tau_{B^*}, E_{B^*})$ and $P_{B^*(fu)}((F, L) \ni (H_{B^*}, \tau_{B^*}, E_{B^*}) = P_{fu}((F, L) \cap (H_{B^*}, \tau_{B^*}, E_{B^*}) = P_{fu}(F, L) \cap (B^*, \Omega^*, G^*)$ which is soft closed set in (B^*, Ω^*, G^*), hence $P_{B^*(fu)}$ is soft closed. Let $b \in B^*$, then $(H_{B^*})_b = H_b \cap H_{B^*}$ which is soft j-compact set in(H_{B^*}, E_{B^*}). Thus, $P_{B^*(fu)}$ is soft j-proper and $(H_{B^*}, \tau_{B^*}, E_{B^*})$ is F.W.S. j-compact over (B^*, Ω^*, G^*), where $j \in \{\alpha, S, P, b, \beta\}$.

Proposition 4.11. Let (H, τ, E) be F.W.S. topological space over (B, Ω, G). Suppose that $(H_{Bi}, E_{Bi})$is F.W.S. j-compact over (B_i, G_i) for each member (B_i, G_i) of a soft j-open covering of (B, Ω, G) Then $(H, \tau, E)$is F.W.S. j-compact over (B, Ω, G), where$j \in \{\alpha, S, P, b, \beta\}$.

Proof. Suppose that $(H, \tau, E)$is F.W.S. topological space over (B, Ω, G), then the projection $P_{fu} : (H, \tau, E) \to (B, \Omega, G)$ exist. To show that P_{fu} is soft j-proper. Now, it is clear that P_{fu} is soft j-continuous. Since (H_{Bi}, E_{Bi}) is F.W.S. j-compact over (B_i, G_i), then the projection $P_{i(fu)} : (H_{Bi}, E_{Bi}) \to (B_i, G_i)$ is soft j-proper for each member (B_i, G_i) of a soft j-open covering of (B, Ω, G). Let (F, L) be a soft closed sub-set of (H, E), then we have $P_{fu}(F, L) = \cup P_{Bi(fu)}((H_{Bi}, E_{Bi}) \cap (F, L))$which is a union of soft closed sets and hence P_{fu} is soft closed. Let $b \in B$ then $H_b = \cup(H_{Bi})_b$ for every $b = \{b_i\} \in B_i$. Since $(H_{Bi_b}, E_{Bi_b})$is soft j-compact in (H_{Bi}, E_{Bi}) and the union of soft j-compact sets is soft j-compact over (B, Ω, G), where$j \in \{\alpha, S, P, b, \beta\}$.

In fact the last result is also holds for locally finite soft j-closed coverings, instead of soft j-open coverings.

Proposition 4.12. Let $\phi : (H, \tau, E) \to (K, \sigma, L)$ be a fibrewise soft function, where (H, τ, E) and (K, σ, L) are F.W.S. topological spaces over (B, Ω, G). If (H, τ, E) is F.W.S. j-compact and $id_H \times \phi : (H, \tau, E) \times_B (H, \tau, E) \to (H, \tau, E) \times_B (K, \sigma, L)$ is soft j-proper and soft j-closed then ϕ is soft j-proper, where $j \in \{\alpha, S, P, b, \beta\}$.

Proof. Consider the commutative figure shown below

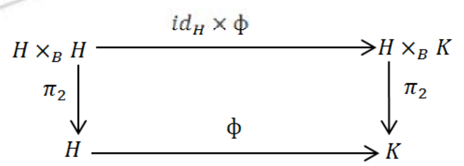

Figure 1: Diagram of Proposition 4.12.

If (H, τ, E) is F.W.S. j-compact then π_2 is soft j-proper. If $id_H \times \phi$is also soft j-proper and j-closed then $\pi_2 \circ (id_H \times \phi) = \phi \circ \pi_2$ is soft j-proper, and so ϕ itself is soft j-proper, where $j \in \{\alpha, S, P, b, \beta\}$. The second new concept in this section is given by the following:

Definition 4.13. The F.W.S. topological space (H, τ, E) over (B, Ω, G) is called fibrewise locally soft j- compact (briefly, F.W. L. S. j-compact) if for each point h of H_b, where

$b \in B$, there exists a soft nbd (N,G) of b and an soft open set $(F,E) \subset H_{(N,G)}$ of h such that the closure of (F,E) in $H_{(N,G)}$ (i.e., $H_{(N,G)} \cap Cl(F,E)$) is F.W.S. j-compact over (N,G), where $j \in \{\alpha, S, P, b, \beta\}$.

Remark 4.14. F.W.S j-compact spaces are necessarily F.W.L.S. j-compact by taken $W = B$ and $H_W = H$. But the conversely is not true for example, let (H, τ_{dis}, E) where H and E is infinite set and τ_{dis} is discrete soft topology, then (H, τ_{dis}, E) F.W.L.S. j-compact over (R, Ω, G), since for each $h \in H_b$, where $b \in R$, there exists a soft nbd (N,G) of b and an soft open $(F,E) \subset H_{(N,G)}$ of (H, τ_{dis}, E)such that $Cl(F,E) = (F,E)$ in $H_{(N,G)}$ is F.W.S. j-compact over (N,G). But (H, τ, E) is not F.W.S. j-compact space over (R, Ω, G), where $j \in \{\alpha, S, P, b, \beta\}$.

Proposition 4.15. Let $\phi : (H, \tau, E) \to (H^*, \tau^*, E^*)$ be a closed fibrewise soft embedding, where (H, τ, E) and (H^*, τ^*, E^*) are F.W.S. topological spaces over (B, Ω, G). If (H^*, τ^*, E^*) is F.W.L.S. j-compact then so is (H, τ, E), where $j \in \{\alpha, S, P, b, \beta\}$.

Proof. Let $h \in H_b$, where $b \in B$. Since (H^*, τ^*, E^*) is F.W.L.S. j-compact there exists a soft nbd (N,G) of b and an open $(F,E) \subset H^*_{(N,G)}$ of $\phi(h)$ such that the closure $H^*_{(N,G)}$ of (F,E) in $(H^*_{(N,G)}, \tau^*_{(N,G)}, E^*_{(N,G)})$ is F.W.S.j-compact over (N,G). Then $\phi^{-1}(F,E) \subset H_{(N,G)}$ is an soft open set of h such that the closure $H_{(N,G)} \cap Cl(\phi^{-1}(F,E)) = \phi^{-1}(H^*_{(N,G)} \cap Cl(F,E))$ of $\phi^{-1}(F,E)$in $H_{(N,G)}$ is F.W.S.j-compact over (F,E). Thus, (H, τ, E) is F.W.L.S. j-compact, where $j \in \{\alpha, S, P, b, \beta\}$.

The class of F.W.L.S. j-compact spaces is finitely multiplicative, where $j \in \{\alpha, S, P, b, \beta\}$, in the following sense.

Proposition 4.16. Let (H_r, τ_r, E_r) be a finite family of F.W.L.S. j-compact spaces over (B, Ω, G) Then the F.W.S. topological product $(H, \tau, E) = \prod_B (H_r, \tau_r, E_r)$is F.W.L.S. j-compact, where $j \in \{\alpha, S, P, b, \beta\}$.

Proof. The proof is similar to that of Proposition (4.6).

References

[1] A. A. Abo Khadra, S. S. Mahmoud and Y. Y. Yousif, fibrewise near topological spaces, Journal of Computing, USA, Vol4, Issue 5, , pp. 223-230 May (2012).

[2] M. M. Akdag, A. Ozkan, Soft α-open sets and soft α-continuous functions. Abstr. Anal. Appl. Art ID 891341, 1-7 (2014)

[3] M. Akdag and A. Ozkan, Soft β − open sets and soft β − continuous functions, e Scientific World Journal,vol. 2014, Article ID 843456 , 6 pages, (2014).

[4] M. Akdag and A. Ozkan, Soft b-open sets and soft b-continuous functions, Math Sci 8:124 DOI 10.1007/s40096-014-0124-7, (2014)

[5] M. I. Ali, F. Feng, X. Liu, W. K. Min and M. Shabir, On some new operations in soft set theory, Computer and Mathematics with Applications,57, 1547-1553, (2009).

[6] I. Arockiarani, A. Arokialancy, Generalized soft gb-closed sets and soft gsb-closed sets in soft topological spaces, Int. J. Math. 4(2), 1–7, Arch. (2013)

[7] Aygunoglu and H. Aygun, Some notes on soft topological spaces, Neural Comput and Applic., Neural Comput. and Applic, 21(1), 113-119, (2012)

[8] N. Cagman and S. Enginoglu, Soft set theory and uniint decision making, European Journal of Operational Research, 207, 848-855, (2010)

[9] N. Çağman, S. Karataş Enginoğlu, Soft topology Computers and Mathematics with Applicatons,. 62:351-358 (2011) .

[10] B. Chen, Soft semi-open sets and related properties in soft topological spaces, Appl. Math. Inf. Sci. 7, No. 1, 287-294 (2013).

[11] S. Hussain and B. Ahmedi, Some properties of soft topological spaces, Neural Comput and Appl., 62, 4058-4067, (2011).Inf. Sci. 7(1), 287–294 (2013)

[12] I. M. James, Fibrewise Topology, Cambridge University Paress (1989).

[13] D. V. Kovkov, V. M. Kolbanov and D. A. Molodtsov, Soft sets theory-based optimization, Journal of Computer and Systems Sciences International, 46, 872-880, (2007).

[14] M. J. Das, PK, On soft topological space via semiopen and semiclosed soft sets, Math. J. 54, 221-236(2014)

[15] P. K. Maji, R. Biswas and A. R. Roy, Soft set theory, Computer and Math. with Appl., 555-562, (2003)

[16] M., D, Soft set theory-First results, Comput. Math. Appl. 37(4–5), 19–31 (1999)

[17] D. Molodtsov, V.Y. Leonov, D.V. Kovkov, Soft sets technique and its application, Nechetkie Sistemy i Myagkie Vychisleniya 1 (1) 8–39(2006).

[18] D. Pei and D. Miao, From soft sets to information systems, in, X. Hu, Q. Liu, A. Skowron, T. Y. Lin, R. R. Yager, B. Zhang (Eds.), Proceedings of Granular Computing, in: IEEE, 2, 617-621, (2005).

[19] S. S. Mahmoud and Y. Y. Yousif, fibrewise near separation axioms, International Mathematical Forum, Hikari Ltd, Bulgaria, Vol.7, No.35, , pp.1725-1736(2012)

[20] S. M., M. Chiney, S. K. Samanta, Urysohn's lemma and Tietze's extension theorem in soft topology , Volume 2, No. 1 pp. 1—130 July (2011) .

[21] M. Shabir and M. Naz, On soft topological spaces, Comput. Math. Appl.,61, 1786-1799, (2011)

[22] Y. Y. Yousif, Some result on fibrewise Lindelöf and locally Lindelöf Topological space, Ibn Al-haitham Journal science, 191-198, Vol. 22, No. 3, pp. 191-198, (2009)

[23] Y. Y. Yousif, Some result on fibrewise Topological space, Ibn Al-haitham Journal for pure and Applied science. University of Baghdad – Collage of Education - Ibn Al-haitham Vol. 21, No. 2, pp. 118-132, (2008).

[24] Y. Yumak and A. K. Kaymakc, Soft β-open sets and their aplications, submitted, http://arxiv.org/abs/1312.6964.
[25] I. Zorlutuna, M. Akdag, W. K. Min and S. Atmaca, Remarks on soft topological spaces, Annals of Fuzzy Mathematics and Informatics, 3(2), 171-185, (2012).