

Application of Fuzzy TOPSIS to Agricultural Farm for Optimum Allocation of Different Crops

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Abstract: *In this paper we have explained some concept of fuzzy set and applied one of fuzzy model on agricultural farm for optimal allocation of different crops by considering maximization of net benefit, maximization production and maximization utilization of labour. Crisp values of the objective functions obtained from selected non-dominated solutions are converted into triangular fuzzy numbers and ranking of those fuzzy numbers are done to make a decision.*

Keywords: *Triangular fuzzy number, α -cut, Optimism index, MCDM (Multi-criteria Decision Making), Fuzzy TOPSIS (The Technique for order preference by similarity to Ideal solution)*

1. Introduction and Literature Survey

In real life situation the introduction of fuzzy logic makes the mathematical models more acceptable for decision makers. We presents general discussion of multi criteria decision method and applied the techniques to one of the important models with their fuzzy extension to the field of Agricultural sciences, where the application of the models seems to be rare. Most of the multi criteria decision method have not taken full shape or have not been tested. Mathematical model solved by various methods would provide a comparative analysis of the methods. As for the applicability of the decision analysis by these methods, at the present stage of development, it is probably more useful as a means of providing insight rather than analytical answer. It is hoped that the Decision Maker (DM) can make educated compromises and judgments based on insights generated by multi criteria decision method. The idea of fuzzy concept was first used in a scientific sense by the computer scientist Lotfi Zadeh in 1965. Fuzzy concept can generate uncertainty because they are imprecise. There are four quite distinct families of method i.e 1- the out ranking, 2-the value and utility theory based, 3-the multiple objective programming and 4-group decision and negotiation theory based method. Fuzzy concept to the extent that their meaning can never be completely and exactly specified with logical operators or objective terms and can have multiple interpretations which are in part exclusively subjective.

In this paper we have discussed the fuzzy TOPSIS (The Technique for order preference by similarity to Ideal solution) method as a fuzzy model for decision making in agricultural farm. We have taken an example of a certain agricultural farm in the state of Odisha, India for approximation of fuzzy concept on agricultural land for decision making.

The present study deals with the objective of making comparative evaluation of cropping plans so far as allocation of land is concerned. As per Hoda and Kapoor[12]and Chen[5] different area have selected for different crops in the distribution centre. The application of fuzzy multiobjective linear programming to aggregate production

planning has applied by Wang [18]. Here the methodology, so developed, are applied to an existing major irrigation project, Distributary No.1, Mahanadi-Taladanda Canal, Cuttack, Odisha, India. A total of 18 crops were considered in a pilot area under three conflicting objectives, namely, maximization of net benefit, maximization of agricultural production and maximum utilization of labour. Different constraints such as land availability, water, fertilizer, labour availability are considered. The response of the farmers and authorities are obtained through a questionnaire. Depending on their response, assessment of weight of each criterion has been obtained. Geometric mean approach is adopted to aggregate the individual opinion to formulate the group opinion. Analytical Hierarchy Process is employed to obtain the weight of the three criteria. Optimization of each individual objective is performed with linear programming algorithm. The pay off matrix is obtained to obtain the upper and lower bound of each objective. The maximization of net benefit is taken as the main objective in the constraint method formulation due to the higher importance attributed to it by the farmers and the authorities. Non-dominated solutions are generated by parametrically varying the bounds. Initially, a large number of non-dominated solutions are generated. Different alternatives are ranked and proper weightage are given. Considering the total weights of each alternative few alternatives are selected and cluster analysis is employed to reduce the non-dominated alternatives to a manageable alternatives for more convenient analysis. Then for decision making we have also followed Fuller and Carlsson [9] principle. The following table gives the selected alternative policies for further analysis in MCDM (Multi-Criteria Decision Making) context.

2. Basic Preliminaries

In many decision-making process data play an important role. But in most cases the pertinent data and the sequence of possible actions are not precisely known. Therefore it is required to use fuzzy data to decision-making process. The merit of using a fuzzy approach is to assign the relative importance of attributes using fuzzy numbers instead of crisp numbers. Triangular fuzzy number with lower, modal and upper values has an edge over other fuzzy numbers.

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A real fuzzy number M is described as any fuzzy subset of the real line R with membership function μ_m , which possesses the following properties.

- (a) μ_m is continuous mapping from R to the closed interval [0, 1]
- (b) $\mu_m(x) = 0$ for all $x \in (-\infty, a]$
- (c) μ_m is strictly increasing on [a, b]
- (d) $\mu_m(x) = 1$ for all $x \in [b, c]$
- (e) μ_m is strictly decreasing on [c, d]
- (f) $\mu_m(x) = 0$ for all $x \in [d, \infty)$

where a, b, c, d are real numbers. We may let $b = c$. In this work, we have used triangular fuzzy numbers whose membership function

$\mu_m : R \rightarrow [0, 1]$ is defined as

$$\mu_m(x) = \frac{x}{m-l} - \frac{l}{m-l}, \quad x \in [l, m] \text{ or}$$

$$\mu_m(x) = \frac{x}{m-u} - \frac{u}{m-u}, \quad x \in [m, u] \text{ or}$$

$$\mu_m(x) = 0, \text{ otherwise}$$

where $l \leq m \leq u$ and l & u stand for the lower and upper values of the support of the fuzzy number M respectively and m for the modal value. A triangular fuzzy number with lower, modal and upper values is expressed as (l, m, u)

Fuzzy operations were first introduced by Dubois and Prade [7]. Other researchers, such as Laarhoven and Pedrycz [14], Buckley [4] and Boender *et al* [3] treated a fuzzy version of the AHP by using the fuzzy operations introduced by Dubois and Prade [7].

The basic operations on fuzzy triangular numbers, which were developed and used, are defined as follows.

$$\tilde{n}_1 \oplus \tilde{n}_2 = (n_{1l} + n_{2l}, n_{1m} + n_{2m}, n_{1u} + n_{2u}) \text{ for addition}$$

$$\tilde{n}_1 \otimes \tilde{n}_2 = (n_{1l} \times n_{2l}, n_{1m} \times n_{2m}, n_{1u} \times n_{2u}) \text{ for multiplication}$$

$$(-)\tilde{n}_1 = (-n_{1l}, -n_{1m}, -n_{1u}) \text{ for negation}$$

$$\frac{1}{\tilde{n}_1} \cong \left(\frac{1}{n_{1u}}, \frac{1}{n_{1m}}, \frac{1}{n_{1l}} \right) \text{ for division}$$

$$\ln(\tilde{n}_1) \cong (\ln(n_{1l}), \ln(n_{1m}), \ln(n_{1u})) \text{ for natural logarithm}$$

$$\exp(\tilde{n}_1) \cong (\exp(n_{1l}), \exp(n_{1m}), \exp(n_{1u})) \text{ for exponentiation}$$

where

$$(\tilde{n}_1) = (n_{1l}, n_{1m}, n_{1u}) \text{ and } (\tilde{n}_2) = (n_{2l}, n_{2m}, n_{2u})$$

represent two triangular fuzzy numbers with lower, modal and upper values and \cong denotes approximation. For the special case of raising of triangular fuzzy number to the power of another triangular fuzzy number, the following approximation was used.

$$\tilde{n}_1^{\tilde{n}_2} \cong (n_{1l}^{n_{2l}}, n_{1m}^{n_{2m}}, n_{1u}^{n_{2u}})$$

3. Ranking of Triangular Fuzzy Numbers

The problem of ranking fuzzy members appears very often in the literature. As each method of ranking fuzzy numbers has its advantage over the others in certain situations it is very difficult to determine which method is the best one. In fuzzy decision making problems, fuzzy number must be ranked before an action is taken by decision maker. Some important factors in deciding which ranking method is the most appropriate for a given situation include the complexity of the algorithm, its flexibility, accuracy, ease of interpretation and the shape of the fuzzy numbers which are used. The development in ranking fuzzy numbers are made by S.H Nasser et al [16]. Baas and Kwakernaak [2] first introduced a method for comparing fuzzy numbers. M.Yaghoobi et al [15] made comparison of fuzzy numbers with ranking fuzzy number. Detyniecki. and Yager [8] introduced the ranking of fuzzy numbers using alpha weighted valuation. Tong and Boissone [17] introduced the concept of a dominance measure. Geldermann et al. [10] introduced fuzzy out ranking for environmental assessment. This method was also later adopted by Buckley [4]. According to Zhu and Lee [19] this ranking method is less complex and still effective. B Asady [1] revised the method of ranking of fuzzy numbers based on deviation degree. It allows a decision maker to implement it without difficulty. However, a given problem may require different method. Here we have discussed ranking of triangular fuzzy numbers using α -cut. In this technique, the irregular fuzzy numbers are further defuzzified into crisp values to determine the order of the alternatives.

Definition of α -cut : The α -cut of fuzzy number M is defined as

$$M^\alpha = \{ x : \mu_m(x) \geq \alpha \} \text{ where, } x \in R, \alpha \in [0, 1]$$

M^α is a non-empty bounded closed interval contained in R and it can be denoted by

$$M^\alpha = [M_L^\alpha, M_U^\alpha], \tag{3.34}$$

where M_L^α and M_U^α are the lower and upper bounds of the closed interval respectively. For example, if $M = (a, b, c)$ be the triangular fuzzy number, then the α -cut of M can be expressed as

$$M^\alpha = [M_L^\alpha, M_U^\alpha] = [(b-a)\alpha + a, (b-c)\alpha + c] \tag{3.35}$$

and its graphical representation is shown in Fig

Given two fuzzy numbers A and B, $A, B \in R^+$, the α -cuts of A and B are

$$A^\alpha = [A_L^\alpha, A_U^\alpha] \text{ and } B^\alpha = [B_L^\alpha, B_U^\alpha]$$

respectively.

Some main operations of A and B can be expressed as follows :

$$(A \oplus B)^\alpha = [A_L^\alpha + B_L^\alpha, A_U^\alpha + B_U^\alpha]$$

$$(A \ominus B)^\alpha = [A_L^\alpha - B_U^\alpha, A_U^\alpha - B_L^\alpha]$$

$$(A \otimes B)^\alpha = [A_L^\alpha \cdot B_L^\alpha, A_U^\alpha \cdot B_U^\alpha]$$

$$(A \oslash B)^\alpha = \left[\frac{A_L^\alpha}{B_U^\alpha}, \frac{A_U^\alpha}{B_L^\alpha} \right]$$

$$(A \otimes r)^\alpha = [A_L^\alpha \cdot r, A_U^\alpha \cdot r], r \in R$$

Chu [6] introduced a fuzzy number interval arithmetic based fuzzy MCDM algorithm. Using the above α – cut concept, the fuzzy performance matrices are transformed to interval performance matrices. The α – cut is known to incorporate the experts or decision maker’s confidence over his preference or the judgment. The α -cut value ranges from 0 to 1 stating that if the α -cut = 1 then the expert is highly certain about his knowledge regarding a phenomenon over which he expresses his performances and the outcome will be a single value having the membership 1 in the fuzzy performance set. Then the further steps are not needed. But

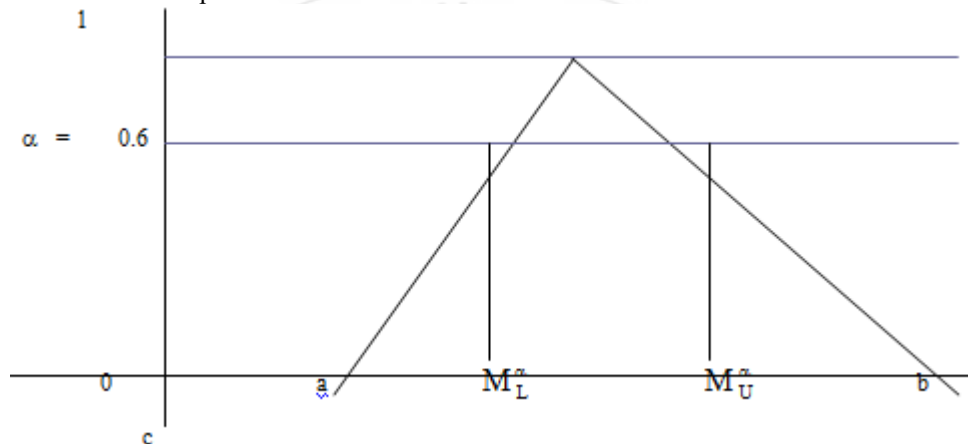


Figure 6.1: Alpha cut operation on triangular fuzzy number $M = (a, b, c)$

4. Implementation of Triangular Fuzzy Numbers

For successful inclusion of uncertainties into the solution procedure, the fuzzy numbers that are used to represent the uncertain model parameters must be implemented in an appropriate form.

Considering a definite uncertain parameter a , measured data for the parameter is assumed to be available from which a normalized distribution function can be derived. In most cases, the data approximately show a Gaussian distribution. The uncertainty in the parameter a can be modeled by a fuzzy number \bar{a} with the membership function $\mu_{\bar{a}}(x)$ of the form

$$\mu_{\bar{a}}(x) = \exp\left(\frac{-(x - m_a)^2}{2\sigma_a^2}\right)$$

where m_a and σ_a are the mean value and standard deviation of Gaussian distribution.

The original fuzzy number \bar{a} with the membership function $\mu_{\bar{a}}(x)$ can be approximated by a symmetric triangular fuzzy

number when the α -cut is less than 1, it indicates that there exists uncertainty; the expert is obviously uncertain about the decisions he made. The α -cut = 0 expresses the highest levels of uncertainty and then the possible performance will be whole support of the fuzzy performance. Any value of α other than 1 needs further evaluation to get the crisp performance.

The crisp performance matrix is obtained by applying the optimism index λ . If $[M_L^\alpha, M_U^\alpha]$ represents the interval performance corresponding to a triangular fuzzy number M using α -cut, then, the crisp performance c is obtained as

$$c = \lambda [M_U^\alpha] + (1 - \lambda) [M_L^\alpha] \text{ where } \lambda \in [0, 1]$$

In our work we have used this technique to rank different alternatives.

number \bar{a}_t with the membership function $\mu_{\bar{a}_t}(x)$ that can be obtained by postulating

$$\mu_{\bar{a}_t}(m_a) = \mu_{\bar{a}}(m_a) = 1$$

$$\text{and } \int_{-\infty}^{\infty} \mu_{\bar{a}_t}(x) dx = \int_{-\infty}^{\infty} \mu_{\bar{a}}(x) dx$$

The membership function $\mu_{\bar{a}_t}$ of the triangular fuzzy number is then defined by

$$\mu_{\bar{a}_t}(x) = \max\left\{0, 1 - \frac{|x - m_a|}{\sigma}\right\} \text{ with } \sigma =$$

$\sqrt{2\pi} \sigma_a$ which can also be expressed in the following form

$$\bar{a}_t = \langle m_a - \sigma, m_a, m_a + \sigma \rangle$$

5. Fuzzy MCDM Methods

Initially weight of each criterion is calculated as triangular fuzzy number. Basing on the data collected in form of questionnaire from the farmers and officials which is given

in table -1 then the weights of different criterion is calculated as follows by using the formula

Table 1: Pay-off Matrix

Criteria Policies	Labour in lakhs man days	Production in lakhs of quintals	Net Benefit in crores of rupees
P1	1.127460	2.8791235	3.7921340
P2	1.122080	2.9161725	3.8450680
P3	1.140924	2.9717465	3.6726790
P4	1.109613	2.9902711	3.6507440
P5	1.178825	2.7827819	4.0853980
P6	1.150925	2.8229123	4.0931130

(mean – $\sqrt{2\pi} \times$ S.D, mean, mean + $\sqrt{2\pi} \times$ S.D)
 Labour: (0.0697, 0.1430, 0.2227)
 Production: (0.1897, 0.3260, 0.4623)
 Benefit: (0.4249, 0.5310, 0.6371)

where sum of modal values of all criteria is equal to 1 and S.D is standard deviation. Then the crisp values of different objectives in decision matrix (Table-2) are converted into triangular fuzzy numbers.

Table 2: Fuzzy decision matrix

Policies	labour			Production			Benefit		
	Lower	Modal	Upper	Lower	Modal	Upper	Lower	Modal	Upper
RP1	1.018337	1.127460	1.236583	2.733950	2.879124	3.024298	3.640964	3.792134	3.943304
RP2	1.012957	1.122080	1.231203	2.770999	2.916173	3.061347	3.693898	3.845068	3.996238
RP3	1.065521	1.140924	1.216327	2.889852	2.971747	3.053642	3.511859	3.672679	3.833499
RP4	1.034210	1.109613	1.185016	2.908376	2.990271	3.072166	3.489924	3.650744	3.811564
RP5	1.083944	1.178825	1.273706	2.640862	2.782782	2.924702	3.983247	4.085398	4.187549
RP6	1.056044	1.150925	1.245806	2.680992	2.822912	2.964832	3.990962	4.093113	4.253933

5.1 Fuzzy TOPSIS

Hwang and yoon [13] developed TOPSIS (The Technique for Order Preference by Similarity of Ideal Solution) as an alternative to the ELECTRE method. The basic concept of this method is that the selected best alternative should have the shortest distance from the ideal solution and farthest distance from ideal solution in a geometrical (Euclidean) sense.

TOPSIS assume that each attribute has a tendency toward monotonically increasing or decreasing utility. Therefore it is easy to locate the ideal and negative ideal solutions. The Euclidean distance is used to evaluate the relative closeness of alternatives to the ideal solution. Thus the preference order of alternatives is derived by comparing these relative distances. TOPSIS method consist the following steps

Step-1. Construct the normalized decision matrix

This step converts the various attribute dimensions into non-dimensional attributes as the ELECTRE method. An element r_{ij} of the normalized decision matrix R is calculated as follows.

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^m x_{ij}^2}}$$

Step-2. Construct the weighted normalized decision matrix:

A set of weights $W = (w_1 w_2 \dots w_n)$ such that $\sum w_i = 1$ specified by the decision maker, is used in conjunction with the previous normalized decision matrix to determine the weighed normalized matrix V defined as $V = (v_{ij}) = (r_{ij} w_j)$

Step-3. Determine the ideal and negative ideal solution:

The ideal A^* and the negative ideal (A^-) solution are defined as follows.

$$A^* = \{(\max_i v_{ij} \mid j \in J), (\min_i v_{ij} \mid j \in J)\} \text{ for } I = 1, 2, 3 \dots m$$

$$= (V_1^* V_2^* V_3^* \dots V_8^*)$$

$$A^- = \{(\min_i v_{ij} \mid j \in J), (\max_i v_{ij} \mid j \in J)\} \text{ for } I = 1, 2, 3, \dots M$$

$$= (V_1^- V_2^- V_1^- \dots V_8^-)$$

Where $J = \{j = 1, 2, \dots, n \mid j \text{ associated with the benefit criteria}\}$

And $J^1 = \{j = 1, 2, \dots, n \mid j \text{ associated with the cost criteria}\}$

For benefit criteria, the DM desires to have a maximum value among the alternatives. For cost criteria, however, the decision maker desires to have a minimum value among them. Obviously, A^* indicates the least preferable alternative or negative – ideal solution.

Step-4. Calculate the separation measure

In this step the concept of the n – dimensional Euclidian distance is used to measure the separation distances of each alternative to the ideal solution and negative ideal solution. The corresponding formulas are

$$s_i^* = \sqrt{\sum (v_{ij} - v_j^*)^2}$$

for $I = 1, 2, 3 \dots, m$

where s_i^* is the separation of alternative I from the ideal solution and

$$s_i^- = \sqrt{\sum (v_{ij} - v_j^-)^2}$$

for $i = 1, 2, \dots, m$

where s_i^- is the separation of alternative i form the negative – ideal solution.

Step-5. Calculate the relative closeness to the ideal solution

The relative closeness of alternative A_i with respect to the ideal solution A^* is defined as follows.

$$C_i^* = \frac{S_i}{S_i + s_i} \quad 0 \leq C_i^* \leq 1$$

$i = 1, 2, \dots, m$

Evidently $C_i^* = 1$ if and only if $A_i = A^*$

And $C_i^* = 0$ if and only if $A_i = A^-$

Step-6. Rank the preference order

The best satisfied alternative can now be decided according to the preference rank order of C_i^* . It is the one, which has the shortest distance to the ideal solution. The way the alternative are processed in the previous steps reveals that if an alternative has the shortest distance to the ideal solution, then this alternative is guaranteed to have the longest distance to the negative ideal solution.

6. Solution

As per the above steps of the method the following results are obtained using the normalized pay-off matrix given in the following Table

Table 3

Policies Weight	Labour	Production	Benefit
	0.1290	0.3248	0.5462
RP1	1.1274600	2.8791235	3.7921340
RP2	1.2220800	2.9161725	3.8450680
RP3	1.1409240	2.9717465	3.6726790
RP4	1.1096130	2.9902711	3.6507440
RP5	1.1788250	2.7827819	4.0853980
RP6	1.1509250	2.8229123	4.0931130

(i) Ideal solutions

1. Ideal solution $A^* = \{1.178825, 2.9902711, 4.093113\}$
2. Negative ideal solution $A^- = \{1.109613, 2.7827819, 3.650744\}$

(ii) Separation measures

S_1^*	0.324932	S_1^-	0.172021
S_2^*	0.265022	S_2^-	0.236030
S_3^*	0.421766	S_3^-	0.194667
S_4^*	0.447751	S_4^-	0.207484
S_5^*	0.207633	S_5^-	0.440130
S_6^*	0.169668	S_6^-	0.446103

(iii) Calculation of relative closeness to the ideal solution

The relative closeness of alternative RPI with respect to the

ideal solution A^* is obtained as $C_i^* = \frac{S_i^*}{S_i^* + S_i^-}$. The value of C_i^* and the rank of corresponding policy are given in the following table.

Table 4: Rank of policies by TOPSIS method

Policy	C_i^*	Rank
RP1	0.346152	4
RP2	0.471069	3
RP3	0.315796	6
RP4	0.31661	5
RP5	0.679462	2
RP6	0.724462	1

7. Conclusion

In this paper we have applied one fuzzy decision making processes to an agricultural farm for allocation of land for 18 crops to get maximum net benefit, maximum agricultural production and maximum utilization of agricultural labour. On few chosen policies, the fuzzy MCDM method is applied and it is found that one particular policy (RP6) bags the first rank, which can be taken as the best compromising solution.

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