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(k, d)–Super Root Square Mean Labeling

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Abstract: ***Let G be a (p, q) graph and*** $f: V(G) \to \{k, k+d, k+2d, \dots, k+d(p+q-1)\}$ ***be an injection. For each edge e = uv, let*** $f^*(e) = \left\lceil \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rceil$ ***or*** $\left\lfloor \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rfloor$, ***then f is called(k, d)-Super root square mean labeling if*** $f(V) \cup \{f^*(e): e \in E(G)\} = \{k, k+d, k+2d, \dots, k+d(p+q-1)\}$. ***A graph that admits a (k, d)-Super root square mean labeling is called (k, d)-Super root square mean graph. In this paper, we investigate(k, d)-Super root square mean labeling of some path related graphs.***

Keywords: Super root square mean labeling, Super root square mean graph, k-Super root square mean labeling, k-Super root square mean graph, (k, d)-Super root square mean labeling, (k, d)-Super root square mean graph, path, comb, ladder and triangular snake

1. Introduction

We begin with simple, finite, connected and undirected graph G (V, E) with p vertices and q edges. For a detailed survey of graph labeling we refer to Gallian [1]. Terms are not defined here are used in the sense of Harary [2]. S. Somasundram and R. Ponraj introduced mean labeling of graphs in [4]. R.Ponraj and D. Ramya introduced Super mean labeling of graphs in [3]. Root square mean labeling was introduced by S.S. Sandhya, R.Ponraj and S. Anusa [5]. The concept of super root square mean labeling was introduced and studied by K. Thirugnanasambandam et al. [6]. In this paper, I extend k-Super root square mean labeling to (k, d) – Super root square mean labeling and investigate (k, d)-Super root square mean labeling of path, comb, ladder and triangular snake. Throughout this paper k and d denote any integer greater than on equal to 1. For brevity, I use (k, d)-SRSML for (k, d)-Super root square mean labeling.

2. Main Results

Definition 2.1
Let G be a (p, q) graph and $f: V(G) \to \{1, 2, \dots, p+q\}$ be an injection. For each edge e = uv, let $f^*(e) = \left\lceil \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rceil$ or $\left\lfloor \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rfloor$, then f is called **Super root square mean labeling** if $f(V) \cup \{f^*(e): e \in E(G)\} = \{1, 2, \dots, p+q\}$. A graph that admits a Super root square mean labeling is called **Super root square mean graph.**

Definition 2.2
Let G be a (p, q) graph and $f: V(G) \to \{k, k+1, k+2, \dots, p+q+k-1\}$ be an injection. For each edge e = uv, let $f^*(e) = \left\lceil \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rceil$ or $\left\lfloor \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rfloor$, then f is called **k-Super root square mean labeling** if $f(V) \cup \{f^*(e): e \in E(G)\} = \{k, k+1, k+2, \dots, p+q+k-1\}$
A graph that admits a k-Super root square mean labeling is called **k-Super root square mean graph.**

Definition 2.3
Let G be a (p, q) graph and $f: V(G) \to \{k, k+d, k+2d, \dots, k+d(p+q-1)\}$ be an injection. For each edge e = uv, let $f^*(e) = \left\lceil \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rceil$ or $\left\lfloor \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rfloor$, then f is called **(k,d)-Super root square mean labeling** if $f(V) \cup \{f^*(e): e \in E(G)\} = \{k, k+d, k+2d, \dots, k+d(p+q-1)\}$
A graph that admits a (k, d)-Super root square mean labeling is called **(k, d)-Super root square mean graph.**

Theorem 2.4
Any path P_n is a (k, d)-Super root square mean graph.

Proof:
Let $V(P_n) = \{v_i; 1 \le i \le n\}$ and $E(P_n) = \{e_i = (v_i, v_{i+1}): 1 \le i \le n-1\}$ be the vertices and edges of P_n respectively.
Define $f: V(P_n) \to \{k, k+d, k+2d, \dots, k+d(2n-2)\}$ by $f(v_i) = k + d(2i-2); 1 \le i \le n$.
Now the induced edge labels are
$f^*(e_i) = k + d(2i-1); 1 \le i \le n-1$.
Here p = n and q = n-1.
Clearly
$f(V) \cup \{f^*(e): e \in E(P_n)\} = \{k, k+d, \dots, k+d(2n-2)\}$.
So f is a (k, d)-Super root square mean labeling. Hence P_n is a (k, d)-Super root square mean graph.

Example 2.5
(25, 1)-Super root square mean labeling of P_6 is given in figure 2.1:

Figure 2.1: (25, 1)-SRSML of P_6

Observation 2.6
If G is a (k, d)-Super root square mean graph, then k and k+2d must be the labels of the adjacent vertices of G since an edge should get label k+d.

Definition 2.7
The graph $P_n \odot K_1$ is called a comb.

Theorem 2.8
Any comb $P_n \odot K_1$ is a (k, d)-Super root square mean graph.
Proof:
Let $V(P_n \odot K_1) = \{v_i, u_i; 1 \leq i \leq n\}$ and
$E(P_n \odot K_1) = \{e_i = (v_i, v_{i+1}); 1 \leq i \leq n-1\} \cup$
$\{e_i' = (v_i, u_i); 1 \leq i \leq n\}$
be the vertices and edges of $P_n \odot K_1$ respectively.
Define
$f: V(P_n \odot K_1) \to \{k, k+d, k+2d \ldots, k+d(4n-2)\}$ by

$$f(u_i) = \begin{cases} k+4d(i-1); 1 \leq i \leq n, if\ i\ is\ odd \\ k+2d(2i-1); 1 \leq i \leq n, if\ i\ is\ even \end{cases}$$

$$f(v_i) = \begin{cases} k+4d(i-1); 1 \leq i \leq n, if\ i\ is\ even \\ k+2d(2i-1); 1 \leq i \leq n, if\ i\ is\ odd \end{cases}$$

Now the induced edge labels are
$f^*(e_i) = k+d(4i-1); 1 \leq i \leq n-1$
$f^*(e_i') = k+d(4i-3); 1 \leq i \leq n$
Here p = 2n and q = 2n-1.
Clearly
$f(V) \cup \{f^*(e): e \in E(P_n \odot K_1)\} = \{k, k+d, k+2d \ldots, k+d(4n-2)\}$
So f is a (k, d)-Super root square mean labeling.
Hence $P_n \odot K_1$ is a (k, d)-Super root square mean graph.

Example 2.9
(50, 2)-Super root square mean labeling of $P_6 \odot K_1$ is given in figure 2.2:

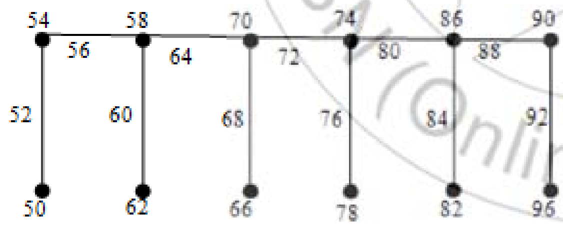

Figure 2.2: (50, 2)-SRSML of $P_6 \odot K_1$

Definition 2.10
The product graph $P_2 \times P_n$ is called a ladder and it is denoted by L_n.

Theorem 2.11
Any ladder L_n is a (k, d)-Super root square mean graph.
Proof:
Let $V(L_n) = \{v_i, u_i; 1 \leq i \leq n\}$ and
$E(L_n) = \{e_i = (u_i, u_{i+1}); 1 \leq i \leq n-1\} \cup$
$\{e_i' = (v_i, v_{i+1}); 1 \leq i \leq n-1\} \cup$
$\{e_i'' = (u_i, v_i); 1 \leq i \leq n\}$ be the vertices and edges of L_n respectively.
Define $f: V(L_n) \to \{k, k+d, k+2d, \ldots, k+d(5n-3)\}$ by

$$f(u_i) = \begin{cases} k+5d(i-1); 1 \leq i \leq n, if\ i\ is\ odd \\ k+d(5i-3); 1 \leq i \leq n, if\ i\ is\ even \end{cases}$$

$$f(v_i) = \begin{cases} k+5d(i-1); 1 \leq i \leq n, if\ i\ is\ even \\ k+d(5i-3); 1 \leq i \leq n, if\ i\ is\ odd \end{cases}$$

Now the induced edge labels are

$$f^*(e_i) = \begin{cases} k+d(5i-1); 1 \leq i \leq n-1, if\ i\ is\ odd \\ k+d(5i-2); 1 \leq i \leq n-1, if\ i\ is\ even \end{cases}$$

$$f^*(e_i') = \begin{cases} k+d(5i-1); 1 \leq i \leq n-1, if\ i\ is\ even \\ k+d(5i-2); 1 \leq i \leq n-1, if\ i\ is\ odd \end{cases}$$

$f^*(e_i'') = k+d(5i-4); 1 \leq i \leq n$
Here p = 2n and q = 3n-2.
Clearly
$f(V) \cup \{f^*(e): e \in E(L_n)\} = \{k, k+d, k+2d, \ldots, k+d(5n-3)\}$
So f is a (k, d)-Super root square mean labeling.
Hence L_n is a (k, d)-Super root square mean graph.

Example 2.12
(50, 2)-Super root square mean labeling of L_6 is given in figure 2.3:

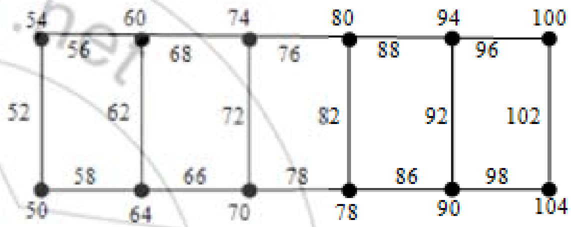

Figure 2.3: (50, 2)-SRSML of L_6

Definition 2.13
A triangular snake (T_n) is obtained from a path by identifying each edge of the path with an edge of the cycle C_3.

Theorem 2.14
Triangular snake, T_n is a (k, d)-Super root square mean graph.

Proof:
Let $V(T_n) = \{v_i; 1 \leq i \leq n-1\} \cup \{u_i; 1 \leq i \leq n\}$ and
$E(T_n) = \{e_i = (u_i, u_{i+1}); 1 \leq i \leq n-1\} \cup$
$\{e_i' = (v_i, u_i); 1 \leq i \leq n-1\} \cup$
$\{e_i'' = (u_{i+1}, v_i); 1 \leq i \leq n-1\}$ be the vertices and edges of T_n respectively.
Define $f: V(T_n) \to \{k, k+1, k+2, \ldots, 5n+k-5\}$ by

$$f(u_i) = \begin{cases} k+2d\ \ ; i = 1 \\ k+d(5i-5); 2 \leq i \leq n \end{cases}$$

$$f(v_i) = \begin{cases} k\ \ ; i = 1 \\ k+d(5i-2); 2 \leq i \leq n-1 \end{cases}$$

Now the induced edge labels are

$$f^*(e_i) = \begin{cases} k+4d\ \ ; i = 1 \\ k+d(5i-3); 2 \leq i \leq n-1 \end{cases}$$

$f^*(e_i') = k+d(5i-4); 1 \leq i \leq n-1$

$$f^*(e_i'') = \begin{cases} k+3d\ \ ; i = 1 \\ k+d(5i-1); 2 \leq i \leq n-1 \end{cases}$$

Here p = 2n-1 and q = 3n-3.
Clearly
$f(V) \cup \{f^*(e): e \in E(T_n)\} = \{k, k+d, k+2d, \ldots, k+d(5n-5)\}$
So f is a (k, d)-Super root square mean labeling.

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Hence T_n is a (k, d)-Super root square mean graph.

Example 2.15
(500, 3)-Super root square mean labeling of L_6 is given in figure 2.3:

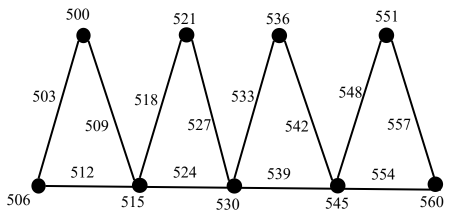

Figure 2.4: (500, 3)-SRSML of T_5

3. Conclusion

- Every Super root square mean labeling is a k- Super root square mean labeling.
- (k, d)- Super root square mean labeling is a Super root square mean labeling if k = 1and d = 1.

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