(k, d)–Super Root Square Mean Labeling

Akilandeswari. K

Abstract: Let G be a (p, q) graph and f : V(G) → {k, k + d, k + 2d, ..., k + d(p + q - 1)} be an injection. For each edge e = uv, let 
f^*(e) = \sqrt{\frac{(u^2 + v^2)k}{2}} or \sqrt{\frac{(u^2 + v^2)k}{2}}. Then f is called (k, d)-Super root square mean labeling if 
f(V) U \{f^*(e) : e \in E(G)\} = \{k, k + 1, k + 2, ..., p + q + k - 1\}. A graph that admits a (k, d)-Super root square mean labeling is called (k, d)-Super root square mean graph. In this paper, we investigate (k, d)-Super root square mean labeling of some path related graphs.

Keywords: Super root square mean labeling, Super root square mean graph, k-Super root square mean labeling, k-Super root square mean graph, (k, d)-Super root square mean labeling, (k, d)-Super root square mean graph, path, comb, ladder and triangular snake

1. Introduction

We begin with simple, finite, connected and undirected graph G (V, E) with p vertices and q edges. For a detailed survey of graph labeling we refer to Gallian [1]. Terms are not defined here are used in the sense of Harary [2]. S. Somasundram and R. Ponraj introduced mean labeling of graphs in [4]. R.Ponraj and D. Ramya introduced Super root square mean labeling of graphs in [3]. Root square mean labeling was introduced by S.S. Sandhya, R.Ponraj and S. Anusa [5]. The concept of super root square mean labeling was introduced and studied by K. Thirugnanasambandam et al. [6]. In this paper, I extend k-Super root square mean labeling to (k, d)– Super root square mean labeling and investigate (k, d)-Super root square mean labeling of path, comb, ladder and triangular snake. Throughout this paper k and d denote any integer greater than one equal to 1. For brevity, I use (k, d)-SRSML for (k, d)-Super root square mean labeling.

2. Main Results

Definition 2.1
Let G be a (p, q) graph and f : V(G) → {1, 2, ..., p + q} be an injection. For each edge e = uv, let 
f^*(e) = \sqrt{\frac{(u^2 + v^2)}{2}}. Then f is called Super root square mean labeling if 
f(V) U \{f^*(e) : e \in E(G)\} = \{1, 2, ..., p + q\}. A graph that admits a Super root square mean labeling is called Super root square mean graph.

Definition 2.2
Let G be a (p, q) graph and f : V(G) → {k, k + 1, k + 2, ..., p + q + k - 1} be an injection. For each edge e = uv, let 
f^*(e) = \sqrt{\frac{(u^2 + v^2)k}{2}}. Then f is called k-Super root square mean labeling if 
f(V) U \{f^*(e) : e \in E(G)\} = \{k, k + 1, k + 2, ..., p + q + k - 1\}. A graph that admits a k-Super root square mean labeling is called k-Super root square mean graph.

Definition 2.3
Let G be a (p, q) graph and f : V(G) → {k, k + d, k + 2d, ..., k + d(p + q - 1)} be an injection. For each edge e = uv, let 
f^*(e) = \sqrt{\frac{(u^2 + v^2)k}{2}}. Then f is called (k, d)-Super root square mean labeling if 
f(V) U \{f^*(e) : e \in E(G)\} = \{k, k + 1, k + 2, ..., p + q + k - 1\}. A graph that admits a (k, d)-Super root square mean labeling is called (k, d)-Super root square mean graph.

Theorem 2.4
Any path P_n is a (k, d)-Super root square mean graph.

Proof:
Let V(P_n) = \{v_i : 1 \leq i \leq n\} and 
E(P_n) = \{e_{i+1} = (v_i, v_{i+1}) : 1 \leq i \leq n - 1\} be the vertices and edges of P_n respectively.
Define f : V(P_n) → \{k, k + d, k + 2d, ..., k + d(2n - 2)\} by 
f(v_i) = k + d(2i - 2); 1 \leq i \leq n.
Now the induced edge labels are 
f^*(e_i) = k + d(2i - 1); 1 \leq i \leq n - 1.
Here p = n and q = n-1.
Clearly 
(f(V) U \{f^*(e) : e \in E(P_n)\}) = \{k, k + d, ..., k + d(2n - 2)\}.
So f is a (k, d)-Super root square mean labeling. Hence P_n is a (k, d)-Super root square mean graph.

Example 2.5
(25, 1)-Super root square mean labeling of P_6 is given in figure 2.1:

Figure 2.1: (25, 1)-SRSML of P_6
Observation 2.6
If $G$ is a $(k, d)$-Super root square mean graph, then $k$ and $k+2d$ must be the labels of the adjacent vertices of $G$ since an edge should get label $k+d$.

Definition 2.7
The graph $P_n \odot K_2$ is called a comb.

Theorem 2.8
Any comb $P_n \odot K_2$ is a $(k, d)$-Super root square mean graph.

Proof:
Let $V(P_n \odot K_2) = \{v_i, u_i; 1 \leq i \leq n\}$ and $E(P_n \odot K_2) = \{(v_i, v_{i+1}); 1 \leq i \leq n - 1\} \cup \{(v_1, u_1); 1 \leq i \leq n\}$ be the vertices and edges of $P_n \odot K_2$ respectively. Define
$$f: V(P_n \odot K_2) \to \{k, k+d, k+2d, \ldots, k+d(4n-2)\}$$
by $f(u_i) = \begin{cases} k + 4d(i-1); & \text{if } i \text{ is odd} \\ k + 2d(i-1); & \text{if } i \text{ is even} \end{cases}$ and $f(v_i) = \begin{cases} k + 2d(i-1); & \text{if } i \text{ is odd} \\ k + 4d(i-1); & \text{if } i \text{ is even} \end{cases}$. Now the induced edge labels are
$$f^*(e) = \begin{cases} k + d(i-1); & \text{if } i \text{ is odd} \\ k + d(5i-3); & \text{if } i \text{ is even} \end{cases}$$
Clearly $f$ is a $(k, d)$-Super root square mean labeling.

Example 2.9
$(50, 2)$-Super root square mean labeling of $P_6 \odot K_2$ is given in figure 2.2:

Figure 2.2: $(50, 2)$-SRSML of $P_6 \odot K_2$

Definition 2.10
The product graph $P_2 \times P_n$ is called a ladder and it is denoted by $L_n$.

Theorem 2.11
Any ladder $L_n$ is a $(k, d)$-Super root square mean graph.

Proof:
Let $V(L_n) = \{v_i, u_i; 1 \leq i \leq n\}$ and $E(L_n) = \{(v_i, v_{i+1}); 1 \leq i \leq n - 1\} \cup \{(u_1, u_2); 1 \leq i \leq n\}$ be the vertices and edges of $L_n$ respectively. Define $f: V(L_n) \to \{k, k+d, k+2d, \ldots, k+d(5n-3)\}$ by $f(u_i) = \begin{cases} k + 2d(i-1); & \text{if } i \text{ is odd} \\ k + 3d(i-1); & \text{if } i \text{ is even} \end{cases}$ and $f(v_i) = \begin{cases} k + 3d(i-1); & \text{if } i \text{ is odd} \\ k + 2d(i-1); & \text{if } i \text{ is even} \end{cases}$. Now the induced edge labels are
$$f^*(e) = \begin{cases} k + d(i-1); & \text{if } i \text{ is odd} \\ k + d(5i-4); & \text{if } i \text{ is even} \end{cases}$$
Clearly $f$ is a $(k, d)$-Super root square mean labeling.

Definition 2.13
A triangular snake $(T_n)$ is obtained from a path by identifying each edge of the path with an edge of the cycle $C_3$.

Theorem 2.14
Triangular snake, $T_n$ is a $(k, d)$-Super root square mean graph.

Proof:
Let $V(T_n) = \{v_i; 1 \leq i \leq n - 1\} \cup \{u_i; 1 \leq i \leq n\}$ and $E(T_n) = \{(v_i, v_{i+1}); 1 \leq i \leq n - 1\} \cup \{(u_1, v_1); 1 \leq i \leq n\}$ be the vertices and edges of $T_n$ respectively. Define $f: V(T_n) \to \{k, k+d, k+2d, \ldots, k+d(5n-3)\}$ by $f(u_i) = \begin{cases} k + 2d; & i = 1 \\ k + d(5i-5); & 2 \leq i \leq n \end{cases}$ and $f(v_i) = \begin{cases} k + d; & i = 1 \\ k + d(5i-2); & 2 \leq i \leq n - 1 \end{cases}$. Now the induced edge labels are
$$f^*(e) = \begin{cases} k + d(5i-3); & 2 \leq i \leq n - 1 \\ k + d(5i-4); & 1 \leq i \leq n \end{cases}$$
Clearly $f$ is a $(k, d)$-Super root square mean labeling.
Hence $T_n$ is a $(k, d)$-Super root square mean graph.

**Example 2.15**

$(500, 3)$-Super root square mean labeling of $L_6$ is given in figure 2.3:

![Figure 2.4: (500, 3)-SRSML of $T_5$](image)

3. Conclusion

- Every Super root square mean labeling is a $k$-Super root square mean labeling.
- $(k, d)$-Super root square mean labeling is a Super root square mean labeling if $k = 1$ and $d = 1$.

References