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On the ωb-Separation Axioms

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Abstract: ***The main aim of our paper is introduced new type of axiom separation by using the open sets of type $\omega b-open$ and studied some it is main concepts, and we proved and explained , some theorems related it, and we introduced definition of spaces $\omega b-R_0$ −space, $\omega b-R_1-space.$, also Explained between them .***

Keywords: axiom separation, $\omega-open, b-open, \omega b-open, \omega b-R_0$ −space, $\omega b-$ R_1 −space

1. Introduction and Preliminaries

The concept of ω-open sets in topological spaces was introduced in1982 by Hdeib [10], In1996 Andrjivic [12] gave a new type of generalized open set In topological space called b-open sets, In 2008,Noiri,Al-Omari and Noorani [11] introduced the conceptof ωb −open and The complement of an$\omega b-open$ set is said to be ωb − closed [11] the intersection of all $\omega b-closed$ sets of X containing A is called the ωb-closure of A and is denoted by $\overline{A}^{\omega b}$ The union of all $\omega b-open$ sets. of X contained in A is called the ωb-interior of A and is denoted by $A^{\circ\omega b}$,In this work we gave a different conceptof the separation axiom using by $\omega b-open$set and we introduced proposition, remarks,theorems of this concept,also we study relation betweenωb −separation axiom and $\omega b-R_i-spaces$.In order to proved our result we need the following Definitions and result.

Definition (1.1): [10] A subset A is said to be ω-open set if for each x∈A ,there exists an open set $U_x$such that $x \in U_x$and $U_x - A$ is countable the complement of ω-open set is called ω-closed set

Definition (1.2): [12] Let X be topological space and A is called b-open set in X, if and only if $A \subseteq \overline{A^\circ} \cup \overset{\circ}{\overline{A}}$ the complement of b-open set is called b-closed and it is easy to see that A is b- closed set iff$\overline{A^\circ} \cap \overset{\circ}{\overline{A}} \subseteq A$

Definition (1.3):[11] A subset A of a space X is said to be ωb-open if for every $x \in A$,there exists a b-open subset $U_x - X$containing x such that $U_x - A$ is countable ,the complement of an ωb-open subset is said to be ωb-closed.

Definition (1.4):[13] Let $f{:}\,X \to Y$ be a function of a space X,into a space Y then f is called an open function if f(A) is an open set in Y for every open set A in X .

Definition (1.5): [13] Let f:X→ Y be a function of a space X, into a space Y ,then f is called an closed function if f(A) is an closed set in Y for every closed set A in X .

Definition(1.6):[2] A space X is called T_1-space if for each $x \neq y$ in X There exists open sets U and V such that $x \in U, y \notin U$ and $y \in V, x \notin V$.

Definition (1.7):[3] A space X is called bT_1-space if for each $x \neq y$ in X There exists b-open sets such thatU and V such that $x \in U$, $y \notin U$ and $y \in V, x \notin V$.

Definition (1.8): [4] A space (X,T) is called a door space if every subset it is either open or closed

Example (1.9): The space (X, T) for $X = \{a, b\}$ and $\tau = \{X, \emptyset, \{a\}\}$,is a door space.

Definition (1.10): [14] A topological space (X,T) is said to be R_0 if every open set contains the closure of each of its single tons.

Definition (1.11):.[5] A space X is called T_2-space (Hausdorff space) if for each $x \neq y$ in Xthere exists disjoint an open sets U, V such that $x \in U, y \in V$

Definition (1.12):[3] A space X i called bT_2-space (b-Hausdorff space) if for each $x \neq y \; in \; X$ there existsdisjoint an b-open sets U, Vsuch that $x \in U, y \in V$.

Proposition (1.13):. [7] It is Clear that every Hausdorff space is b-Hausdorff space

Definition(1.14): [14] A topological space (X, τ) is said to be R_1space if for x and y in X,with $\overline{(\{X\})} \neq \overline{(\{y\})}$, there exists disjoint open set U and V such that$\overline{\{X\}} \subset U$ and $\overline{\{y\}} \subset V$

Definition (1.15): [6] A space X is said to be regular space if for each $x \in X$ and A closed subset such that $x \notin A$ there exist disjoint open sets U, V such that $x \in U$ and $A \subseteq V$

Definition (1.16) [3] A space Xis said to be b-regular space if for each $x \in X$ and A closed subset such that $x \notin A$ there exist disjoint b-open sets U, Vsuch that $x \in U$ and $A \subseteq V$.

Remark (1.17): [7] It is clear that each regular space is b-regular space However, a b-regular space is notregular in general and a as is the following

Example (1.18)

Let $X = \{1,2,3,4\}$, $\tau = \{X, \emptyset, \{4\}, \{2.3\}\{2,3,4\}\}$ then

$BO(X) =$

$X, \emptyset, \{2\}, \{3\}, \{2,4\}, \{3,4\}, \{1,2,4\}, \{1,3,4\}, \{4\}. \{2,3\}, \{2,3,4\}, \{1,4\}, \{1,2,3\}$

is b-regular space ,but Xis not regular since$\{1,2,3\}$ is closed set$4 \notin \{1,2,3\}$ and thus do not exists disjoint open sets which separate them in X,

Definition (1.19): [1] Let X be a space and $A \subseteq X$, A is called regular open set in X if $A = \overset{\circ}{\overline{A}}$ The complement of regular open set is called regular closed and it is easy to see that A is regular closed if $A = \overline{A^{\circ}}$.

Definition(1.20): [8]A Topological space X is called almost regular space if for each x in X and regular closed set C such that $x \notin C$, there exist disjoint open sets U, Vsuch that $x \in U$, $C \subseteq V$

Definition (1.21): [9]A Topological space X is called normal space if where every $C_1$and $C_2$are disjoint closed subset in X there exists disjoint open sets V_1, V_2, with$C_1 \subseteq V_1$, and $C_2 \subseteq V_2$

Definition (1.22): [3] A topological space X is called b-normal space if for every disjoint closed set C_1, C_2there exist disjoint b-open sets V_1, V_2such that $C_1 \subseteq V_1, C_2 \subseteq V_2$

Remark (1.23): [7] It is clear that every normal space is b-normal, but the converse is not true in general.

Example (1.24): Let X ={1,2,3,4,5} , $\tau = \{X, \emptyset, \{1\}, \{3,4\}, \{1,3,4\}, \{1,2,4,5\}, \{4\}, \{1,4\}\}$
BO(X) =
{X , ∅, {1}, {3,4}, {1,3,4}, {1,2,4,5}, {4}, {1,4}, {1,2,4}, {1,4,5}, {1,2,3,4},

{1,3,4,5}, {2,4,5}, {2,3,4,5}, {1,6}, {2,4}, {4,5}, {3,4,5}, {2,3,4}, {1,2,5}, {1,2}}
It is clear that X is b-normal space but is not normal. In Fact, the disjoint closed sets {3}, {2,5} cannot be separated by open sets in X .

2. ωb-Separation Axiom

Definition (2.1) A functionf: X ⟶ Y is said to be ωb-open for every open subset A of X if f(A) is an ωb-open set in Y.

Definition (2.2)A function $f: X \longrightarrow Y$ is said to beωb-closed for every closed subset A of X if $f(A)$ is an ωb-closed set in Y.

Definition (2.3) Let $f: X \to Y$ be a function of a space X into a space Ythen f is called an ωb- continuous function if $f^{-1}(A)$ is an ωb-open set in X for every open set A in Y.

Definition(2. 4): Let f: X → Y be a function of a topological space (X, τ) into a topological space (Y, τ′), then fis called an ωb-irresolute function if is and $f^{-1}(A)$ is an ωb −open set in X for every ωb-open set A in Y.

Definition (2.5): Let Y be subspace of space X,A subset B of space X is said to be un ωb − open setin Y. if for every $x \in B$ there exists ab − open subset U_x in X contain x such that $U_x - B$ *is* countable.

Definition (2.6): Let X be a space. and A ⊆ X The intersection of all ωb-closed sets of X containing A iscalled the ωb-closure of A defined by

$$\overline{A}^{\omega b} = \cap \{B: B\ \omega b\text{-closed in } X \text{ and } A \subseteq B\}$$

Definition (2.7) Let X be a space and $x \in X, A \subseteq X$. The point x is called ωb-limit point of A if every ωb-open set containing x contains a point of A distinct from x We call the set of all ωb-limit point of A the ωb-derived set of A and denoted by $\acute{A}^{\omega b}$ Therefore $x \in \acute{A}^{\omega b}$ iff for every ωb-open set V in X, such that $x \in V$ such that $(V \cap A) - \{x\} \neq \emptyset$.

Proposition (2.8): Let X be a space and $A \subseteq B \subseteq X$ then

$1 - \overline{A}^{\omega b} = A \cup \acute{A}^{\omega b}$

$2 -$ $A\omega b$-closed set iff $\acute{A}^{\omega b} \subseteq A$

Proof:

1 − Let $x \in \acute{A}^{\omega b}$, $x \notin \overline{A}^{\omega b}$ there existsωb − open set U such that $U \cap A = \emptyset, (U \cap A) - \{x\} = \emptyset$ therefore $x \notin \acute{A}^{\omega b}$ is contradiction thus $x \in \overline{A}^{\omega b}$ hence $\acute{A}^{\omega b} \subseteq \overline{A}^{\omega b}$ where $\acute{A}^{\omega b} \cup \overline{A}^{\omega b} \subseteq \overline{A}^{\omega b}$ Conversely: Let $x \in \overline{A}^{\omega b}$ Then either $x \in A$ or $x \notin A$, if $x \in A$ then $x \in A \cup \acute{A}^{\omega b}$,complete $x \notin A$, since $x \in \overline{A}^{\omega b}$ then for all Uωb − open set contains x such that , $U \cap A \neq \emptyset$ since $x \notin A$ then $(U \cap A) - \{x\} \neq \emptyset$,Then $x \in \acute{A}^{\omega b}$, then .$x \in A \cup \acute{A}^{\omega b}$hence$\overline{A}^{\omega b} \subseteq A \cup \acute{A}^{\omega b}$,then $\acute{A}^{\omega b} = A \cup \acute{A}^{\omega b}$ 2 − Let A be an ωb −closed set .To prove$\acute{A}^{\omega b} \subseteq A$,. Let $x \notin A$ then $x \in A^c$, since A ωb −closed set then A^cωb −open set $A \cap A^c = \emptyset$,then$(A \cap A^c) - \{x\} = \emptyset$, then $x \notin \acute{A}^{\omega b}$ thus $\acute{A}^{\omega b} \subseteq$ A Conversely : Let$\acute{A}^{\omega b} \subseteq A$,To prove Aωb-closed set, Since $\acute{A}^{\omega b} = A \cup \acute{A}^{\omega b}$then $\acute{A}^{\omega b} = A$ thus A ωb-closed set

Definition (2.9): A space X is called ωbT_1-space if for each $x \neq y$ in X There exists ωb-open sets U and Vsuch that $x \in U, y \notin U$ and $y \in V, x \notin V$.

Proposition (2.10): Every T_1-space is bT_1-space

Proof:
Let (X, τ) be bT_1-space and x ,$y \in X \ni x \neq y$,Then there exists two *open* sets U,V such that
$x \in U, y \notin U$and $y \in V, x \notin V$ since every *open* set is b −open set,thus. U , V are two b-open set such that $x \in U, y \notin U$and $y \in V, x \notin V$ therefore (X,τ) be bT_1-space.

Remark (2.11): but the .converse is not true in general, in fact from **Example(2.12)**
Let X = {1,2,3} , τ = {∅, X, {1}, {2}, {1,2}}., BO(X) = {∅, X, {1}, {2}, {1,2}, {1,3}, {2,3}},then 1,3 ∈ X ∃U ∈ τ ∋ 1 ∈ Uand 3 ∉ Ubut ∄ V ∈ τ ∋ 3 ∈ V and 1∉ V thus (X ,τ) is notT_1-spacesuch that ∀x ,y ∈ X∃ U,V are two b-open set such that x ∈ U ,y ∉ U, y ∈ V, x ∉ V therefore (X, τ) is bT_1-space

Proposition (2.13): Every T_1-space is ωbT_1-space

Proof:
Similar to prove of **Proposition (2.10)** But the converse is not true in general ,in fact from **Example(2.12)**it is easy to check that is ωbT_1-space but not T_1-space

Proposition (2.14): Every bT_1-space is ωbT_1-space

Proof:
Similar to prove of **Proposition (2.10).** But the converse is not true in general ,in fact from **Example (2.15)**it is easy to check that is ωbT_1-space but not bT_1-space.

Let $X = N, \tau = \{G: 1 \in G\} \cup \{\emptyset\}, BO(X) = \{\emptyset, N, 1 \in G, 1,3 \in N$, is not exists two b-open sets $\ni 1 \in U$ and $3 \notin U$but $3 \in V$ and $1 \notin V$ then is notbT_1 −space since $\omega BO(X) = \{A \subseteq N\}$therefore is$\omega bT_1$ −space.

Theorem (2.16): If M subspace of X (where M is open subset of X) ,Then M is ωbT_1-space if X is ωbT_1-space

Proof:
Let x ,y $\in M \ni x \neq y$ since X is ωbT_1-space then ∃ two ωb-open sets U,V such that$x \in$ U but $y \notin U$and $y \in V$ but $x \notin V$ Let $A = U \cap M, B = V \cap M$,thus A,B are ωb-openset in M and $x \in A$ but $y \notin A$ and $y \in B$ but $x \notin B$therefore M is ωbT_1-space.

Theorem (2.17): Let $f: X \longrightarrow Y$ be *a* ωb-irresolute injective map, If Y is ωbT_1-space, then X is ωbT_1-space.

Proof:
Let $x, y \in X \ni x \neq y$ then $f(x), f(y) \in Y$ and $f(x) \neq f(y)$SinceY,is ωbT_1-space. then there . exists two $\omega b-open$ sets U,V in Y such that $f(x) \in U$ but$f(y) \notin U$,and $f(y) \in V$ but $f(x) \notin V$ thus $x \in f^{-1}(u)$but$y \notin f^{-1}(u)$and $y \in f^{-1}(v)$, but $x \notin f^{-1}(v)$since f ωb-irresolute, hence$f^{-1}(u)$, $f^{-1}(v)$ areωb −open thereforeX is ωbT_1 − space.

Proposition(2.18): Let X be a Topological space, Then X is ωbT_1 − space iff{x} is ωb −closed set for each $x \in X$

Proof:
Let X be ωbT_1 −space and let $x \in X$ and let $y \notin \{x\}$.Since X is ωb-T_1-space then there exists an ωb − open set Vsuch that then $y \in V, x \notin V$ then $V \cap \{x\} = \emptyset$,It is $(V - y) \cap \{x\} = \emptyset$ and hence, $y \notin \{x\}'^{\omega b}$ thus$\{x\}'^{\omega b} \subseteq \{x\}$ and hence$\overline{\{x\}}^{\omega b} = \{x\} \cup \{x\}'^{\omega b}$=$\{x\}$ so that, $\{x\}$isωb −closed set for each$x \in X$ by Last Proposition(2.8) , Conversely: assume that $\{x\}$ is ωb-closed set for each $x \in X$, Let $x \neq y$ in X then $X - \{x\} = V$ is ωb-open set such that $y \in V, x \notin V$
Let $X - \{y\} = U$,hence Uis ωb-open set which is contains x, Therefore X ωbT_1-space.

Theorem(2.19): Let $f: X \to Y$ be an bijective ωb-open function,If X is T_1-space then Y is ωbT_1 − space

Proof:
Let $y_1, y_2 \in Y \ni y_1 \neq y_2$,since$f$onto function then $x_1, x_2 \in X \ni f(x_1) = y_1, f(x_2) = y_2, x_1 \neq x_2$ Since X is T_1- space∃ U, V open sets in X $\ni x_1 \in U$ *but* $x_2 \notin U$ and$x_2 \in V$but$x_1 \notin V$ hence f is ωb-open $\ni f(U), f(V)$two are ωb-open set in Ythen $f(x_1) = y_1 \in f(U)$ but$f(x_2) = y_2 \notin f(U)$.and $f(x_2) = y_2 \in f(V)$ but$f(x_1) = y_1 \notin f(V)$ since every open sets isωb −open thus $f(U), f(V)$are twoωb-open therefore $Y$is ωbT_1 − space.

Theorem(2.20): Let $f: X \to Y$ be an one-to-one ωb-continuous function.If $Y$is T_1-space then X is ωbT_1 − space

Proof:
Let.$x_1, x_2 \in X \ni x_1 \neq x_2$, since $f: X \to Y$ is one-to-one function and $x_1 \neq x_2$ then$f(x_1) \neq f(x_2)$and$f(x_1), f(x_2) \in Y$,since Y is T_1-space∃ U, Vopen sets in Y $\ni f(x_1) \in U$ but $f(x_2) \notin U$ and $f(x_2) \in V$ but $f(x_1) \notin V$ since f is , ωb −continuous function then$f^{-1}(U), f^{-1}(V)$ areωb −open set in X , since $f(x_1) \in U$ thus $x_1 \in f^{-1}(U)$and since$f(x_2) \notin U$ then$x_2 \notin f^{-1}(U)$and since $f(x_2) \in V$ then$x_2 \in f^{-1}(V)$since$f(x_1) \notin V$,thus$x_1 \in f^{-1}(V)$ therefore$X$ is ωbT_1 −space.

Definition (2.21):A Topological space (X,T) is said to be $\omega b - R_0$ if every ωb − open set contains the ωb −closure of each of its singletons.

Theorem (2.22): The topological door space is $\omega b - R_0$ if and only if it is ωbT_1 − space

Proof:
Let x,y are distinct points in X Since (X,T) is door space then $\{x\}$is open or closedif$\{x\}$is openhence ωb −open in X,Let $V = \{X\}$then $x \in V$and$y \notin$ V since(X,T) is $\omega b - R_0$ space

thus $\overline{(\{X\})}^{\omega b} \subset V$ hence $X \notin X \setminus V$, $y \in y \setminus V$ Therefore $X \setminus V$ ωb − open subset of X if $\{x\}$is closedhence it is ωb −closed $y \in X \setminus \{X\}$and $x \notin X \setminus \{X\}$ *is* ωb −open set in X. Since (X,T) is $\omega b - R_0$ space, then $\overline{(\{y\})}^{\omega b} \subset X \setminus \{X\}$Let $V = X \setminus \overline{(\{y\})}^{\omega b}$ thus $x \in V$but $y \notin V$and $V\omega$b −open set in X, therefore (X,T) is ωbT_1 − spaceConversely:
let (X,T) be ωbT_1 − space and Let V be an ωb −open set of X and $x \in V$ for each $y \in X \setminus V$ there is an ωb −open set V_y such that, $x \notin V_y$but$y \in V_y$then$\overline{(X)}^{\omega b} \cap V_y = \phi$ for each $y \in X \setminus V$ thus$\overline{(X)}^{\omega b} \cap (\cup_{y \in x \setminus v} V_y) = \phi$ hence$y \in V_y$, $X \setminus V \subset (\cup_{y \in x \setminus v} V_y)$, $\overline{(\{X\})}^{\omega b} \subset V$,Therefore (X,T) is $\omega b - R_0$.

Remark (2.23) :but the converse Proposition **(1.13)** is not true in general ,as the following **Example (2.24)** shows:
Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$, $BO(X) = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$ then b,c $\in X \ni U \in \tau \ni b \in U$, but $\nexists V \in \tau \ni C \in V, U \cap V = \phi$ then (X, τ) is not T_2-spacesuch that∀x , $y \in X \ni$ U,V are two b-open set hence $x \in U, y \in V, U \cap V = \phi$ therefore (X,τ) is bT_2-space

Definition (2.25): A space X is called ωbT_2-space (ωb-Hausdorff space) if for each $x \neq y$ in X there exists disjoint an ωb-open sets U, V such that $x \in U, y \in V$

Remark (2.26): It is Clear that every Hausdorff space is ωbT_2-space but the converse is not true in general as the following **Example(2.24)** it is easy to check that is ωbT_2-space but not T_2-space

Remark (2.27): Every bT_2-space is ωbT_2-space, but the converse is not true in general asthe following:

Example (2.28): Let $X = N, \tau = \{A \subseteq X : A^c finite\} \cup \emptyset$ $\overline{\{1\}^{\circ}} \cup \overline{\{1\}}^{\circ} = \phi \Longrightarrow \{1\} \not\leq \overline{\{1\}^{\circ}} \cup \overline{\{1\}}^{\circ} \therefore \{1\}$ isnot b-open Let A = {1} and 1∈ U = $N - \{2\}$ then U is b-open set contain 1, since N-B is countable. then A is ωb −open Since 1,2 ∈ N is not exists to b −open sets U, V such that 1 ∈ U, 2 ∈ V, and $U \cap V = \emptyset$, then is not bT_2 −space since $\omega BO(X) = \{A : A \subseteq N\}$ thus It is ωbT_2-space

Theorem (2.29): Let $f: X \longrightarrow Y$ be a function

1- If f is a bijection and f ωb-open, X is T_2-space then Y is ωbT_2-space

2- If f is injective and ωb-continuous ,Y is T_2-space then X is ωbT_2-space.

Proof:

Let $f: X \longrightarrow Y$ be a function

1- suppose f is ωb-open and X is T_2-space Let $y_1 \neq y_2 \in Y$ since f is bijective

Then *there exist* x_1, x_2 *in X suchthat* $f(x_1) = y_1$ *and* $f(x_2) = y_2$ *and* $x_1 \neq x_2$

Since X is T_2-space then *there exists* disjointopen sets U and V in X , *such that* ($x_1 \in U$ and $x_2 \in V$)Since f ωb-open $f(U)$ and $f(V)$ are ωb-open sets in Y hence $f(x_1) \in f(U)$ and $y_2 = f(x_2) \in f(V)$ Again since f is bijective $f(U)$ and $f(V)$ are disjoint in Y, thus Y is ωbT_2-space

2- suppose f: X ⟶ Y is ωb-continuous and T_2-space, Let $x_1, x_2 \in X$ with $x_1 \neq x_2$, $f(x_1) = y_1$ and $f(x_2) = y_2$ since f is one-to-one Since f is , $y_1 \neq y_2$ since Y is T_2 − space then there exists open sets U and V containg y_1 and y_2 respectively Since f is ωb-continuousbijective $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint ωb − open. containg x_1 and x_2 respectively thus X is ωbT_2 − space. respectively

thus X is ωbT_2 − space.

Theorem (2.30): Every ωbT_2 − space is ωbT_1 − space Let (X,τ) be a ωbT_2 − space ,let x and y be two disjoint distinct in X since X. ωbT_2 − spacethere exists disjoint ωb-open set U and V such that x ∈ U and y ∈ V since U and V are disjoint x ∈ U but y ∉ U and y ∈ V but x ∉ V hence X is ωbT_1 − space

Theorem (2.31): Let M be open subspace of X ,Then M is ωbT_2 − space if X is ωbT_2 − space

Proof:

Let x ,$y \in M, x. \neq y$ *then* x, y ∈ X SO ∃ B_1, B_2 such that $B_1 \cap B_2 = \phi \ni x \in B_1$ $y \in B_2$ where B_1, B_2 *are* ωb −open set set in X Let $E_1 = B_1 \cap M$ $E_2 = B_2 \cap M$

are ωb − open set *subset* in *M* and $x \in E_1, y \in E_2$ *then* $E_1 \cap E_2 = (B_1 \cap M) \cap (B_2 \cap M) = (B_1 \cap B_2) \cap M = \phi \cap M = \phi$ hence *M* is ωbT_2 − space

Theorem (2.3): Let $f: X \longrightarrow Y$ be bijective ωb − irresolute function and X is ωbT_2 − space, then (X, τ_2) is ωbT_2 − space

Proof:

Suppose f: (X, τ) ⟶ (Y, τ) is bijective and f is ωb − irresolute and (Y, τ_2) is ωbT_2 − space

Let $x_1, x_2 \in X$ with $x_1 \neq x_2$ since f is bijective then $y_1 = f(x_1) \neq f(x_2) = y_2$ for some $y_1, y_2 \in Y$ since (Y, τ_2) is ωbT_2 − space there exists disjoint ωb − open set U and V such that $y_1 = f(x_1) \in U$ and $y_2 = f(x_2) \in V$ again f is bijective $x_1 = f^{-1}(y_1) \in f^{-1}(U)$ and $x_2 = f^{-1}(y_2) \in f^{-1}(V)$ since f is ωb − irresolute $f^{-1}(U)$ and $f^{-1}(V)$ ωb − irresolute $f^{-1}(U)$ and $f^{-1}(V)$ are ωb − open set in (X, τ) also f is bijective U ∩ V = ∅ implies that $f^{-1}(U) \cap f^{-1}(V) = f^{-1}(U \cap V) = f^{-1}(\phi) = \phi$ is follows (X, τ_2) is ωbT_2 − space.

Definition (2.33):

Topological space (X, τ) is said to be $\omega b - R_1$ space if for x and y in X with $\overline{(\{X\})}^{\omega b} \neq \overline{(\{y\})}^{\omega b}$ there.exists.disjoint ωb −open.set U.and.V such that $\overline{\{X\}}^{\omega b} \subset U$ *and* $\overline{\{y\}}^{\omega b} \subset V$

Theorem (2.3): The Topological door space is $\omega b - R_1$ if and only if it is ωbT_2 − space

Proof:

Let x and y be two distinct points in X, Since X is door space for each x in the set $\{X\}$ is open or closed If $\{X\}$ is open Since Since $\{X\} \cap \{y\} = \phi$ *then* $\{X\} \cap \overline{\{y\}}^{\omega b} = \phi$ Thus $\overline{\{X\}}^{\omega b} \neq \overline{\{y\}}^{\omega b}$ If $\{X\}$ is closed So it is ωb − closed and $\overline{\{X\}}^{\omega b} \cap \{y\} = \{X\} \cap \{y\} = \phi$ Therefore $\overline{\{X\}}^{\omega b} \neq \overline{\{y\}}^{\omega b}$ We have (X, τ) is $\omega b - R_1$ space so that ,there are disjoint ωb −open set U and V such that $x \in \overline{\{X\}}^{\omega b} \subset U$ and $y \in \overline{\{y\}}^{\omega b} \subset V$ so X is ωbT_2 − space

Conversely:

Let x and y be any points in X with $\overline{\{X\}}^{\omega b} \neq \overline{\{y\}}^{\omega b}$ by Theorem (2.30) so by so by Proposition (2.18) hence $\overline{\{X\}}^{\omega b} = \{X\}$ and $\overline{\{y\}}^{\omega b} = \{y\}$ this implies x≠ y since X is ωbT_2 − space. there are two disjoint ωb − open sets U and V such that $\overline{\{X\}}^{\omega b} = \{X\} \subset U$ and $\overline{\{y\}}^{\omega b} = \{y\} \subset V$ This proves X is $\omega b - R_1$ space .

Corollary- (2.35): Let (X ,τ) be a topological door space Then if X is $\omega b - R_1$ space then it is $\omega b - R_0$ space

Proof:

Let X be an $\omega b - R_1$ door space.Then by Theorem (2.34) then X is $\omega bT_2 -$ space thus by Theorem (2.30)so thatby Theorem (2.22) therefore X is $\omega b - R_0$ space

Definition (2.36):

A space X is said to be ωb-regular space if for each $x \in X$ and A closed set such that $x \notin A$ there exist disjoint ωb-open sets U, V such that $x \in U$and $A \subseteq V$

Remark (2.37): It is clear that each regular space is ωb-regula but the converse is not true ingeneral,in fact from **Example (1.18)** is easy to check that ωb-regular space is not regular

Proposition (2.38): A Topological space X is ωb-regular space iff for every $x \in X$ and each open set U in Xsuch that $x \in U$ there exists an ωb-open set L such that $x \in L \subseteq \overline{L} \subseteq U$.

Proof:

Let X be ωb-regular space and $x \in X$,U an open set in X such that $x \in U$ Then U^can closed set in X and $x \notin U^c$,Then there exists disjoint ωb-open set L, V thus$x \in L$, $U^c \subseteq V$ therefore $x \in L \subseteq L^{-\omega b} \subseteq V^C \subseteq U$Conversely: let $x \in X$ and M be a closed set in X such that $x \notin M$.Then M^c is an open set in Xand $x \in M^c$,Then there exists an ωb-open set L such that $x \in L \subseteq L^{-\omega b} \subseteq M^c$ Thus $x \in L$, $M \subseteq (L^{-\omega b})^c$ and L , $(L^{-\omega b})^c$are disjoint ωb-open set Therefore X is ωb-regular

Definition (2.39):

A topological space X is called almost ωb-regular space if for each x in X and regular closed set C such that $x \notin C$ there exist disjoint $\omega b -$ open sets U, V such that $x \in U, C \subseteq V$

Proposition (2.40):

A space is almost X is ωb-regular space iff for every x in X and each regular open set U in X then$x \in U$ there exists an $\omega b -$ open set L such that $x \in L \subseteq L^{-\omega b} \subseteq U$

Proof:

Let X be almost ωb-regular space and $x \in X$, U regular open set in X then $x \in U$, hence U^c regular closed set in X and $x \notin U^c$ thus there exist disjoint $\omega b -$ open set V, L such that $x \in V$, $U^c \subseteq L$,Therefore $x \in V \subseteq V^{-\omega b} \subseteq L^{c-\omega b} = L^c \subseteq U$.

Conversely:
Let $x \in X$ and C be a regular closed set in Xthen$x \notin C$ hence C^c regular open set in X and $x \in C^c$,Thus there exists an ωb-open set V such that $x \in V, C \subseteq (V^{-\omega b})^c$ and V, $(V^{-\omega b})^c$are disjoint $\omega b -$ open set. Therefore X is almost ωb-regular space.

Definition (2.41):

A Topological space $X$is called ωb-normal space if for every disjoint closed sets C_1, C_2 there exist disjoint ωb-open sets V_1, V_2such that $C_1 \subseteq V_1, C_2 \subseteq V_2$.

R.emark (2.42): It is clear that every normal space is ωb-normal, but the converse is not true, in fact from :

Example (1.24) it is easy to check ωb-normal space but not normal

Proposition(2.43): Topological space X is ωb-normal space iff for every closed set $D \subseteq X$ andeach open set U in Xsuch that $D \subseteq U$ there exists an ωb-open set V such that $D \subseteq V \subseteq V^{-\omega b} \subseteq U$.

Proof:

Let X be ωb-normal space and Let D be closed set and U open set in $X \ni D \subseteq U$ Then D, $U^c$are disjoint closed sets in X Since X is ωb-normal space then there exists disjoint ωb-open sets V, L such that $D \subseteq V, U^c \subseteq L$, Thus , $D \subseteq V \subseteq V^{-\omega b} \subseteq L^{c-\omega b} = L^c \subseteq U$

Conversely:
Let D_1, D_2 be disjoint closed sets in X then D_2^c is open set in X and $D_1 \subseteq D_2^c$ thus there exists an ω b-open Set V suchthat.$D_1 \subseteq V \subseteq V^{-\omega b} \subseteq D_2^c$ hence $D_1 \subseteq V, D_2 \subseteq (V^{-\omega b})^c$and V, $(V^{-\omega b})^c$aredisjoint $\omega b -$ open sets Therefore X is $\omega b -$ normal space

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