Polarization Mode Dispersion Effects on the Multimode Graded-Index Optical Fibers

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1. Introduction

Light consists of coupled electric and magnetic fields which are spatially and temporally varying periodically. Perpendicular to the direction of propagation, the vectors of electric field can be projected on an orthogonal basis to a horizontal and vertical component. In isotropic media, both components propagate at the same speed and the original polarization of the light is conserved. However, in anisotropic media (physical properties depend on the spatial direction) this is not the case and the two polarization components can travel with different speed causing time delay between the two directions of polarization and thus the dispersion [1, 2].

In case the refractive index depends on the spatial direction, the physical phenomenon is called birefringence [3]. Contrary to the classical types of dispersion, like the chromatic dispersion, it is not possible to fix a certain value for polarization mode dispersion (PMD) [4]. Due to the manifold of parameters which affect PMD and which change over time and interact in a very complicated and unpredictable way, the PMD can be only described in a statistical model. It has been found that PMD does not scale linearly with the length of a fiber, but with its square root. PMD is about 0.2 ps/km for a perfect uniform and symmetrical fiber, the propagation speed would arrive at the far end of the fiber at the same time. In real fibers, however, there are always small stresses on the fiber that make the refractive index slightly different for light of two orthogonal polarizations, and the arrival time, therefore, depends slightly on polarization. The resulting dispersion tends to be small, because light of one polarization is rather easily coupled (usually within a few meters) into the orthogonal polarization by fiber bends and irregularities. This gives rise to a fairly uniform propagation speed, with statistical fluctuations from the average that increase with fiber length [6,7].

Now, assume that there is negligible polarization dependent loss, so that we can use the principal states model [8] to characterize first-order PMD. Under this model, there exist a pair of orthogonal input PSP’s, |a> and |α> , and a pair of orthogonal output PSP’s |b> and |b−>, where all of PSP’s are expressed Jones vectors. If an arbitrary polarized field \( \vec{E}_a(t) = \vec{E}_a(t) \) |a> is input to the fiber, this input field can be projected onto the two PSP’s as \[ \vec{E}_a(t) = \sqrt{\gamma} E_1(t) a^+ + \sqrt{1-\gamma} E_2(t) a^- \] \[ (1) \]

where \( \gamma \) is PMD power splitting ratio, given by \( \gamma = |a^+|^2 \). In terms of first-order PMD, the output field of the fiber takes the form \[ \vec{E}_b(t) = \sqrt{\gamma} E_1(t - \tau_p - \Delta \tau / 2) b^+ + \sqrt{1-\gamma} E_2(t - \tau_p + \Delta \tau / 2) b^- \]

(2)

where \( \tau_p \) is the polarization independent group delay, and \( \Delta \tau \) is the DGD among the two PSP’s. The fiber transfer function for first-order PMD (often called PMD impulse response) in the time and frequency domains are given by [10]

\[ h(t) = \sqrt{\gamma} \delta(t - \Delta \tau / 2) b^+ + \sqrt{1-\gamma} \delta(t + \Delta \tau / 2) b^- \]

(3)

\[ H_z(w) = \sqrt{\gamma} e^{-iw \Delta \tau / 2} b^+ + \sqrt{1-\gamma} e^{iw \Delta \tau / 2} b^- \]

(4)

The root mean square width of this impulse response can be calculated as...
and the minimum width occurs if \( \gamma = 0, 1 \), i.e. \( \sigma_{\text{PMD}} = 0 \). In other words, if the input SOP is in direction of PSP’s, then the pulse will not face any broadening.

### 3. Pulse Spreading

We have been using the time delay of waves to characterize the dispersion of a fiber. An alternative approach is to consider quantitatively the pulse spreading that is induced by the dispersion. In this approach, we can imagine measuring the pulse width at the input and at the output and attributing the increase in pulse width to the dispersion. The pulse width of rms is defined as the standard deviation of the pulse width and is related to the first and second moments of the pulse by

\[
\sigma_{\text{rms}}^2 = \int_{-\infty}^{\infty} t^2 p(t) dt - \left( \int_{-\infty}^{\infty} t p(t) dt \right)^2
\]

Many optical pulses are symmetric or are assumed to be. The mean value of the first moment of a symmetric input pulse is zero; so, for symmetric pulses the mean-square pulse width is

\[
\sigma_{\text{rms}}^2 = \int_{-\infty}^{\infty} t^2 p(t) dt
\]

The pulse width at the output of the fiber is a combination of the initial pulse width and the pulse spreading of the fiber caused by dispersion. For the mean-square pulse width, the effects are combined according to

\[
\sigma_{\text{rms}}^2 = \sigma_{\text{mod}}^2 + \sigma_{\text{ch}}^2 + \sigma_{\text{DL}}^2
\]

Similar results are available in terms of the pulse width of rms of the dispersion terms, where the results are combined as

\[
\sigma_{\text{rms}}^2 = \sigma_{\text{mod}}^2 + \sigma_{\text{ch}}^2
\]

where \( \sigma_{\text{conv}} \) stands for the conventional polarizations.

We will analyze these expressions for the dispersion induced pulse spreading. The rms pulse spread for material dispersion is certain by

\[
\sigma_{\text{ch}} = \tau_{\text{f}}(\lambda) - \tau_{\text{f}}(\lambda_0) = \Delta \lambda D_{\text{ch}}
\]

On the other hand, the pulse spreading due to modal dispersion is defined as [11]

\[
\sigma_{\text{mod}} = L |D_{\text{mod}}|
\]

From both modal and chromatic dispersions, i.e. Eqs.(13) and (14), one can obtain the total dispersion. Instead of adding them directly, it is given by the following square sum expression

\[
\sigma_{\text{conv}} = L \sqrt{D_{\text{mod}}^2 + D_{\text{ch}}^2}
\]

Note that, the total increasing of pulse width is proportional to the fiber length. That is; the higher fiber lengths will be induced higher broadening.

The time spread of a pulse due to PMD is found to obey

\[
\sigma_{\text{PMD}} = D_{\text{PMD}} \sqrt{\gamma (1 - \gamma) \Delta t}
\]

This corresponds to the well-known “random walk problem” in statistics, which is characterized by a distribution with a width proportional to the square root of the number of steps. Typical values for communications fiber are \( (0.2 \pm 2) \text{ps km} \). The origin of \( \sqrt{\gamma} \) dependence lies in the statistical nature of the processes. Light is coupled randomly from mode to mode, or from one polarization to another, resulting in statistical fluctuations from an average. This corresponds to the well-known “random walk problem” in statistics, which is characterized by a distribution with a width proportional to the square root of the number of steps [12].

To determine the total time spread of a pulse, we must combine the various sources of fiber dispersion. The way that we combine them depends on whether they are correlated or uncorrelated. For example, material and waveguide dispersions are correlated because they both depend on wavelength. In this case, we add these dispersion directly, as

\[
D_{\text{total}} = D_{\text{mod}} + D_{\text{ch}} + D_{\text{DL}}
\]

To obtain the total chromatic dispersion. However, intermodal dispersion, chromatic dispersion and PMD do not share any common origin, and therefore uncorrelated. In this case, we must add the time spread in quadrature as follows

\[
\sigma_{\text{PMD}} = \sqrt{\sigma_{\text{mod}}^2 + \sigma_{\text{ch}}^2 + \sigma_{\text{DL}}^2}
\]

In many situations, one of these three terms dominates and the others can be neglected. For example, in step-index multimode fiber, the \( \sigma_{\text{mod}} \) term usually dominates. The contribution from PMD can be neglected, but can be significant in long fiber spans when very monochromatic light sources are used.
4. Polarization Mode Dispersion and Chromatic Dispersion

The pulses that propagate inside single mode fiber are affected by two types of dispersion which are chromatic dispersion and PMD. Note that the effecting of the two types of dispersion happen in the same time, so to give a distinct sense of the two types of dispersion we decided to obtain the effects in the frequency domain, see Fig.1.

The initial pulse, \( \widetilde{U}(0,w) = 3\{U(0,T)\} \), first faces the effect of chromatic dispersion (the transfer function \( H_1(w) \)) to obtain \( H_1(w)\widetilde{U}(0,w) \). Note that, the chromatic dispersion does not depend on SOP therefore the input SOP (the Jones vector \(|a\rangle\)) will not change. Next, the pulse divides into two orthogonal components towards PSP’s \(|a^+\rangle \) and \(|a^-\rangle \) under the effects of PMD. The component in the direction \(|a^+\rangle\) will face the effects of the function \( H_{2a}(w) \) to obtain the pulse \( H_{2a}(w)H_1(w)\widetilde{U}(0,w) \) and in the same time the SOP will change from \(|a^+\rangle\) to \(|b^+\rangle\). On the other side, the pulse in the direction \(|a^-\rangle\) faces the effects of the function \( H_{2f}(w) \) to yield \( H_{2f}(w)H_1(w)\widetilde{U}(0,w) \) and also the SOP will change from \(|a^-\rangle\) to \(|b^-\rangle\). We should not forget that the input or output PSP does remain orthogonal when the polarization dependent loss is absent. Finally, the vector sum of two components will give the final pulse \( H_2(w)H_1(w)\widetilde{U}(0,w) \).

![Figure 1: Effects of chromatic dispersion (CD) and PMD on the input pulse \( \widetilde{U}(0,w) \), where \( H_1(w) \) represents the transfer function of CD, \( H_2(w) \) represents the transfer function of PMD, and \(|a\rangle\) and \(|b\rangle\) are the input and output SOP’s.](image)

The transfer function of a lossless fiber, when PMD is absent and only the chromatic dispersion is considered, can be expressed in the frequency domain as [10]

\[
H_1(w) = \exp\left[\frac{i\beta_1 w}{2} - \frac{i\beta_2 w^3}{6}\right] \tag{17}
\]

where \( \beta_1 = -\lambda^2 D_{ch} (\lambda^2) / 2\pi c \) and \( \lambda \) is light wavelength. For a chirped normalized Gaussian pulse [9,1]

\[
U(0,T) = A \exp\left(-\frac{1+i\zeta T^2}{2T_o^2}\right) \tag{18}
\]

where \( A \) is constant. The Fourier transform of Eq.(18) takes the form

\[
U(z,T) = \frac{A_o}{\sqrt{\pi}} \exp\left(-\frac{w^2 T_o^2}{2(1+i\zeta)}\right) \int_{-\infty}^{\infty} e^{-i\omega T_o^2/2(1+i\zeta)} \exp\left[\frac{i\beta_1 w^2}{2} + \frac{i\beta_2 w^3}{6}\right] e^{-i\omega T_o^2/2(1+i\zeta)} \left|b^+\rangle\right. + \left\langle b^-\right| \right. \tag{20}
\]

where \( A_o = \frac{AT_o^2}{\sqrt{2(1+i\zeta)}} \). Introduce the variable \( x = wT_o \) as a new integration variable, where

\[
p^2 = \frac{T_o^2}{2} \left( \frac{1}{1+i\zeta} - \frac{i\beta_2 x}{T_o^2}\right) \tag{21}
\]
The parameter $x^2$ can be eliminated with another transformation $x = u / \sqrt{b - i / b}$.

$$U(z,T) = \frac{A_z}{\sqrt{\pi}} \left[ \int_{-\infty}^{\infty} \exp \left( -x^2 + \frac{ib}{3} x^3 - i \frac{(T + \Delta \tau / 2)}{p} x \right) dx \right] \left\{ \int_{-\infty}^{\infty} \exp \left( -x^2 + \frac{ib}{3} x^3 - i \frac{(T - \Delta \tau / 2)}{p} x \right) dx \right\}$$

(22)

where $b = \beta z / (2p^3)$. The parameter $x^2$ can be eliminated with another transformation $x = u / \sqrt{b - i / b}$.

Using the method proposed in Ref. [13], the resulting integral can be written in terms of Airy function $Ai(x)$ as

$$U(z,T) = \frac{2A_z}{\sqrt{\pi}} \left\{ \sqrt{\gamma} \Psi_+(z,T) |b^+ > + \sqrt{1-\gamma} \Psi_-(z,T) |b^- > \right\}$$

(23)

with the form of Eq.(7), but the pulse which results from the vector sum of the two orthogonal components will face a broadening that can be determined from Eq.(24). Eq.(26d) represents the nonlinear phase that generates through the propagation in optical fiber. The nonlinear phase as a function of time differs from one component to another by the amount $\Delta \tau$, but in the frequency domain they remain the same and adding the same value of noise to both components. Using Eq.(26d), one can find the frequency chirp of the two components as follows

$$\delta w_\pm(T) = \frac{\partial \psi_\pm(z,T)}{\partial T} = \frac{\beta z}{T_0} \left\{ \frac{\beta z}{T_0} \right\}$$

(27)

Eq.(27) means that the new frequencies generated are similar for the two components and the difference in the mathematical forms, i.e. $T \pm \Delta \tau / 2$, means that one of the components advances the other by time $\Delta \tau$. Eq.(20) to Eq.(23) explain that the pulse amplitude will decrease by increasing the propagation distance. These equations will convert to the same equations in Ref.[9] with ignoring the effects of PMD. The value of DGD, $\Delta \tau$, changes randomly from one optical fiber to another and from one segment to another (according to Rayleigh distribution) and

$$<\Delta \tau> = \sigma_{PMD} = \sqrt{L D_{PMD}}$$

On the other hand, the splitting ratio $\gamma$ can be determined from the polarization vectors, where $\sqrt{\gamma}$ represents the projection of $|a^+>$ onto $|a>$ (i.e. $\sqrt{\gamma} = |a^+|a>$). We can also write $\gamma$ as a function of Stokes vectors as

$$\gamma = |a^+|a|^2 = \frac{1 + \hat{P} \cdot \hat{\sigma}^2}{2} = \cos^2(\theta / 2)$$

(28)

where $\hat{P} = \hat{a}^+ |\hat{\sigma}|a^+ >$ is the Stokes vector of the slow PSP, $\hat{S} = \hat{a} |\hat{\sigma}|a >$ is the Stokes vector of the input SOP, $\hat{\sigma}$ is the spin vector [14], and $\theta$ is the random angle between $\hat{P}$ and $\hat{S}$. Substituting Eq.(28) into (23), yields

The parameter $T_1$ represents the pulse width after the effects of chromatic dispersion where it is the same for the two orthogonal components. The width of each of component will not increase under the effects of PMD (this coincides
$$U(z,T) = \frac{2A_0}{3b} \sqrt{\pi} \left| b^+ \right| \cos(\theta/2) + \left| b^- \right| \sin(\theta/2)$$

The Jones vectors $|b^+>$ and $|b^->$ are orthogonal, i.e. $<b^-|b^+> = 0$. That is enough to assume a random form to one of them to find the other. For example, if $|b^+> = [x \quad iy]^T$ then $|b^-> = [iy \quad x]^T$ keeping in mind that all the polarization vectors have unit values, i.e. $x^2 + y^2 = 1$, here $t$ represents the matrix transpose. The width of reconstructed pulse can be found as: according to Fig.(1) and using the above equations, the pulse width after the effects of chromatic dispersion is $T_1$. That is; the broadening ratio (BR) for the reason that chromatic dispersion is $T_1/T_o$. Next, the input pulse has a width $T_1$ which will increase by the amount $\sigma_{PMD}$ due to the PMD, so the BR due to PMD is $\sigma_{PMD}/T_1$.

5. Results and Discussion

Fig. (2) represents the function shape for different distances for a number of assortment coefficients $\beta_2, \beta_3$. It’s clarify from the figure that the efficacy of $\beta_2$ is centralize in pulse broadening. This broadening increase with the distance increasing. While $\beta_2$ efficacy represented by shifted pulse center and appearing waviness in its preceding border, so this waviness increases with the propagation distance increasing. The two effects admixture with other will be caused in the pulse broadening, waviness, and shifted its center in depending on the propagation distance and the two factors $\beta_2, \beta_3$.

Fig. (3) represents the same concept to a constant propagation distance with changing the input pulse $T_o$. With presence of the same antecedent effects and with notice that $T_o$ increasing represented in decreasing chromatic dispersion effects appearance. This is because of the time declination of the broad pulse is so little in comparison with the narrow pulses. Fig. (4) represented the pulse shape by using the same formations with a length and $T_o$ constancy and chirp changing. We notice that there is a negative or positive chirp which will trade with addition more complication to the pulses behavior that is represented by a different shifts and a different wavelike behavior.

Fig. (5) represents the resultant pulses shape with a constant $T_o$ and neglected $\beta_3$ effect for a number of lengths and for a different chirp values. From figure we notice that the pulse may be compressed or broad depending on $L$, chirp, $\beta_2$. Where it may be compression the first propagation distance and then the pulse may be broads. In general the compression is happen within a short distance only then the broadening is follow it. And the compression did not back a new. The equalizing between $\beta_2$, L, chirp values may be enable in controlling with the pulses behavior. As long as $\beta_2$ participate in the broadening and the remainder coefficient $\beta_3$ and the higher represents with the distortion, therefore we will center our recognition on $\beta_2$ and considered it the chromatic dispersion with neglecting of the highest orders effects where we want to calculate the pulses broadening.

Fig. (6) represents the resultant pulse shape by using a number of pulses of $\theta$ angle and $\Delta \tau$. The $\theta$ angle represents the basis of the weight which each power component take. For this reason we note for each $\theta$ angle the resultant pulse may be turn to left or right with dependent on $\Delta \tau$ value. This turning is go to be zero at $\theta = \pi/2$ in which the power is equilibrate on two axis. $\Delta \tau$ Magnitude represents by antecedence or retardation of the generated pulse. With dependent on the $\theta$ angle. In general form the decreasing in process in power is because of there is a stationary magnitude of dispersion $\beta_2 = 10 ps^2/km$ and surely increasing of this magnitude caused a larger broadening. Actually the PMD did not caused broadening to any of the two components but the directional summation between the two components will produce shape have some of broadening.

Fig. (7) illustrate the relation between the power peak and the $\theta$ angle in existence of different $T_o$ and $\Delta \tau$. Notice that $T_o$ increasing signify decreasing of effect this is a legitimate thing as long as $\Delta \tau$ will be small in scale to $T_o$. From other side, the reliability of $\Delta \tau = 0$ denoted that PMD is lost, and the power peak effects only in existence of $\beta_2$ and there is a pulse peak independent on $\theta$. As that appearance in blue line. With $\Delta \tau$ increasing PDM starts in appearance and the power being at least it possible at $\theta = \pi/2$. As long as the broadening in this angle is the maximum.

Fig.(8) represents the relation between the pulse width and $\theta$ in apparent number of values of $\beta_2$, $\Delta \tau$. From figure notice that the $\beta_2$ increasing mean that the pulse width increasing this known with existence of width independent on $\theta$ when $\Delta \tau = 0$ (PMD=0) with addition PMD then the pulse width achieve another increasing at the range $0 < \theta < \pi$, then the increasing will be maximum at $\theta = \pi/2$. For this reason the pulse width will
be\[ T_1 = T_0 \sqrt{1 + \left( \frac{\beta_2 z}{T_0^2} \right)^2} \]
without existence of PMD and there is another broadening adds to it in PMD presence. This last broadening being a maximum at \( \theta = \frac{\pi}{2} \) and increase with \( \Delta J \) increasing. With note that the increasing because of PMD will be maximum at \( \beta_2 = 0 \) and with increasing of \( \beta_2 \) its effect will be decrease. Because of it is will be small in measurement with the broadening caused by \( \beta_2 \).

Fig.(9) represents the power as a function of \( \theta \) with a different values of \( \beta_2, \Delta \tau, \beta_2, \Delta \tau \). Clarify from figure that is the power peak decrease with \( \beta_2 \) increasing because of its broadening as it mentioned above. At \( \beta_2 = 0 \) the power have effecting only with PMD. Thus, it will be stationary at \( \Delta \tau = 0 \) and decline with a maximum it possible at \( \theta = \frac{\pi}{2} \) which the maximum broadening have happen. At the large values of \( \beta_2 \), the PMD effect will be not discrete as long as the broadening of \( \beta_2 \) is so large in measure to the broadening caused by PMD.

Finally fig. (11) represents the pulse width as a function of \( \beta_2 \) in existence the different values of \( \theta, \Delta \tau \). It is clarify from figure that the increasing in \( \beta_2 \) will causes an increasing in the pulse width and for all \( \theta, \Delta \tau \) cases. Because it is simply independent on \( \theta, \Delta \tau \). From other side we notice that PMD addition pertinent with the \( \theta \) angle value where at \( \theta = 0 \), indeed deferent \( \Delta \tau \) values did not causes any changing. With \( \theta \) increasing we notice that PMD cases an additional changing have a maximum summit when \( \beta_2 \approx 0 \). In general the maximum \( \Delta \tau \) cases superior changing.
Figure 2: The resultant pulse shape by using a number of pulses of $\theta$ angle and $\Delta \tau$. 

$\theta=0 \, \text{ps}^2/\text{km}$

$\theta=\pi/6$

$\theta=\pi/3$

$\theta=\pi/2$
Figure 3: Peak power as a function of $\theta$ for different $\Delta \tau$ and $T_0$. 
Figure 4: Pulse width as a function of $\theta$ for different values of $\delta t$ and $\beta_2$. 
Figure 5: Peak power as a function of $\theta$ for different $\Delta \tau$ and $\beta_2$. 
Figure 6: the peak locale as a function to $\theta$ for a number values of $\beta_2, \Delta \tau$. 
Figure 7: represents the pulse width as a function of $\beta_2$ in existence the different values of $\theta, \Delta \tau$

References


