International Journal of Science and Research (IJSR)
ISSN (Online): 2319-7064
Index Copernicus Value (2015): 78.96 | Impact Factor (2015): 6.391

k-Odd Sequential Harmonious Labeling of Some Special Graphs

B. Gayathri[1], D. Muthuramakrishnan[2]

[1]Periyar E.V.R. College, Tiruchirappalli, India

[2]National College, Tiruchirappalli, India

Abstract: ***Graham and Sloane [7] introduced the harmonious graphs and Singh & Varkey [8] introduced the odd sequential graphs. Gayathri & Hemalatha [2] introduced even sequential harmonious labeling of graphs. We studied even sequential harmonious labeling of trees in [3]. In [4], we have extended this notion to k-even sequential harmonious labeling graphs. It is further studied in [5]. k-even sequential harmonious labeling of some cycle related graphs are studied in [6]. Also, we have introduced k-odd sequential harmonious labeling of graphs in [5]. In this paper, we investigate k-odd sequential harmonious labeling of some graphs.***

Keywords: k- odd sequential Harmonious Graphs

1. Introduction

All graphs in this paper are finite, simple and undirected. The symbols V(*G*) and E(*G*) denote the vertex set and the edge set of a graph *G*. The cardinality of the vertex set is called the order of *G*. The cardinality of the edge set is called the size of *G*. A graph with *p* vertices and *q* edges is called a (*p*, *q*) graph.

A **graph labeling** is an assignment of integers to the vertices or edges or both subject to certain conditions. If the domain of the mapping is the set of vertices or edges then the labeling is called **vertex or edge labeling**.

Graph labeling was first introduced in late 1960's. In the recent years, dozens of graph labeling techniques have been studied in over 1200 papers.

Labeled graphs serve as useful models for a broad range of applications such as coding theory, X-ray crystallography, radar, astronomy, circuit design, communication network addressing etc. [1]

In [4], we have defined *k*-even sequential harmonious labeling. It is further studied in [5,6] .
Also, we have introduced k-odd sequential harmonious labeling of graphs in [5].

We say that a labeling is a *k*-odd sequential harmonious labeling if there exists an injection f from the vertex set *V* to $\{k-1, k+1, ..., k+2q-1\}$ such that the induced mapping f^* from the edge set E to $\{2k-1, 2k+1, ..., 2k+2q-3\}$ defined by

$$f^*(uv) = \begin{cases} f(u)+f(v)+1, & \text{if } f(u)+f(v) \text{ is even} \\ f(u)+f(v), & \text{if } f(u)+f(v) \text{ is odd} \end{cases}$$

are distinct. A graph *G* is said to be an *k*-odd sequential harmonious graph if it admits an *k*-odd sequential harmonious labeling.

In this paper, we investigate k-odd sequential harmonious labeling of some graphs. Throughout this paper, *k* denote any positive integer ≥ 1. For brevity, we use *k*-ESHL for *k*-even sequential harmonious labeling.

2. Main Results

Definition 8.2
We say that a labeling is a ***k*–odd sequential harmonious labeling** if there exists an injection *f* from the vertex set *V* to $\{k-1, k, k+1, ..., k+2q-1\}$ such that the induced mapping f^+ from the edge set *E* to $\{2k-1, 2k+1, ..., 2k+2q-3\}$ defined by

$$f^+(uv) = \begin{cases} f(u)+f(v)+1 & \text{if } f(u)+f(v) \text{ is even} \\ f(u)+f(v) & \text{if } f(u)+f(v) \text{ is odd} \end{cases}$$

are distinct
A graph *G* is said to be a ***k*-odd sequential harmonious graph** if it admits a *k*-odd sequential harmonious labeling.

Theorem 8.3
Triangular snake T_n (n ≥ 2) is a *k*-odd sequential harmonious graph for any *k*.

Proof
Let $\{v_1, v_2, ..., v_{n+1}, u_1, u_2, ..., u_n\}$ be the vertices and $\{e_1, e_2, ..., e_n, e_1', e_2', ..., e_{2n}'\}$ be the edges of T_n which are denoted as in Fig. 8.1.

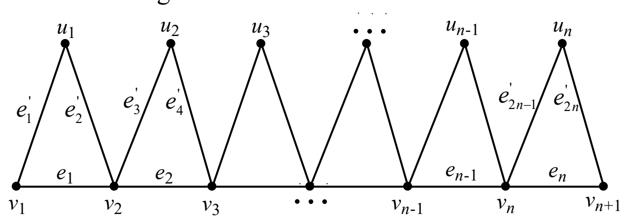

Figure 8.1: T_n with ordinary labeling

First we label the vertices as follows:
Define $f: V \rightarrow \{k-1, k, k+1, ..., k+2q-1\}$ by

For $1 \le i \le n+1$,

$$f(v_i) = k + i - 2$$

For $1 \le i \le n$,

$$f(u_i) = k + 2n + 3i - 3.$$

Then the induced edge labels are:

For $1 \le i \le n$,

$$f^+(e_i) = 2k + 2i - 3$$

For $1 \le i \le 2n$,

$$f^+(e_i') = 2k + 2n + 2i - 3$$

Therefore, $f^+(E) = \{2k - 1, 2k + 1, ..., 2k + 2q - 3\}$. So, f is a k-odd sequential harmonious labeling and hence, the graph triangular snake T_n $(n \ge 2)$ is a k–odd sequential harmonious graph for any k.

Illustration 8.4

(a) 2–*OSHL* of T_6 is shown in Fig. 8.2(a).

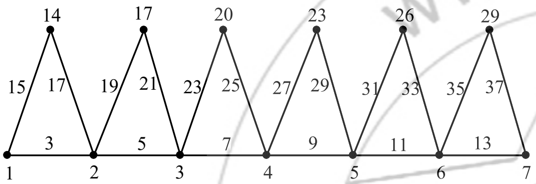

Figure 8.2(a): 2–*OSHL* of T_6

(b) 3-*OSHL* of T_7 is shown in Fig. 8.2(b).

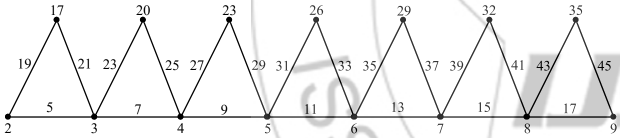

Figure 8.2(b): 3–*OSHL* of T_7

Theorem 8.5

The Festoon graph $P_m \odot nK_1$ $(m \ge 3, n \ge 2)$ is a k-odd sequential harmonious graph for any k.

Proof

Let $\{u_i : 1 \le i \le m : u_{ij}, 1 \le i \le m, 1 \le j \le n\}$ be the vertices and $\{e_i, 1 \le i \le m-1;\ e_{ij}, 1 \le i \le m, 1 \le j \le n\}$ be the edges of $P_m \odot nK_1$ which are denoted as in Fig. 8.3.

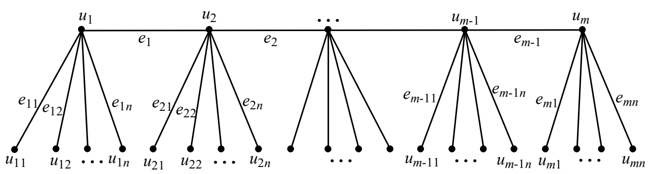

Figure 8.3: $P_m \odot nK_1$ with ordinary labeling

First we label the vertices as follows:

Define $f: V \to \{k - 1, k, k + 1, ..., k + 2q - 1\}$ by

For $1 \le i \le m$, $1 \le j \le n$,

$$f(u_{ij}) = \begin{cases} k + 2j + i(n+1) - n - 3 & i \text{ odd} \\ k + 2j + i(n+1) - 2n - 3 & i \text{ even} \end{cases}$$

For $1 \le i \le m$,

$$f(u_i) = \begin{cases} k + (n+1)i - n - 2 & i \text{ odd} \\ k + i(n+1) - 2 & i \text{ even} \end{cases}$$

Then the induced edge labels are:

For $1 \le i \le m - 1$,

$$f^+(e_i) = 2k + 2i\,(n + 1) - 3$$

For $1 \le i \le m$, $1 \le j \le n$,

$$f^+(e_{ij}) = 2k + 2j + 2i\,(n + 1) - 2n - 5$$

Therefore, $f^+(E) = \{2k - 1, 2k + 1, ..., 2k + 2q - 3\}$. So, f is a k-odd sequential harmonious labeling and hence, the Festoon graph $P_m \odot nK_1\ (m \ge 3, n \ge 2)$ is a k–odd sequential harmonious graph for any k.

Illustration 8.6

(a) 3–*OSHL* of $P_4 \odot 5K_1$ is shown in Fig. 8.4(a).

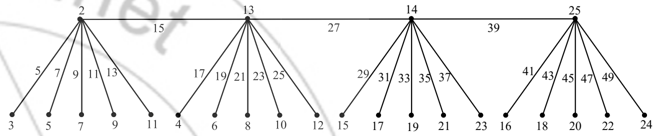

Figure 8.4(a): 3–*OSHL* of $P_4 \odot 5K_1$

(b) 4-*OSHL* of $P_3 \odot 4K_1$ is shown in Fig. 8.4(b).

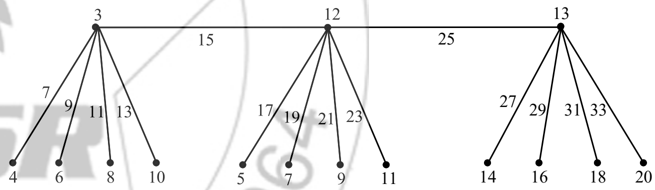

Figure 8.4(b): 4–*OSHL* of $P_3 \odot 4K_1$

Theorem 8.7

The $spl(K_{1,n})$ $(n \ge 3)$ is a k-odd sequential harmonious graph for any k.

Proof

Let $\{v, u, u_i, 1 \le i \le n\ ;\ v_i, 1 \le i \le n\}$ be the vertices and $\{e_i, 1 \le i \le 3n\}$ be the edges of $spl(K_{1,n})$ $(n \ge 3)$ which are denoted as in Fig. 8.5.

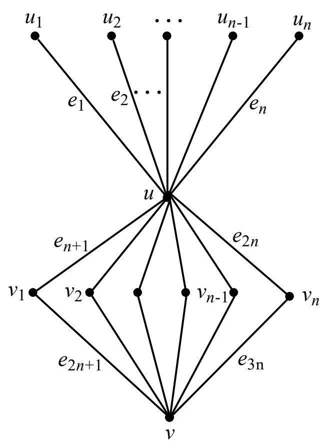

Figure 8.5: $spl(K_{1,n})$ with ordinary labeling

First we label the vertices as follows:

Define $f: V \rightarrow \{k-1, k, k+1, ..., k+2q-1\}$ by

$f(u) \quad = k-1$

$f(v) \quad = 2n+k-1$

For $1 \le i \le n$,

$f(u_i) \quad = k+2i-2$

For $1 \le i \le n$,

$f(v_i) \quad = k+2n+2i-2.$

Then the induced edge labels are:

For $1 \le i \le 3n$,

$f^+(e_i) = 2k+2i-3$

Therefore, $f^+(E) = \{2k-1, 2k+1, ..., 2k+2q-3\}$. So, f is a k-odd sequential harmonious labeling and hence, the $spl(K_{1,n})$ $(n \ge 3)$ is a k-odd sequential harmonious graph for any k.

Illustration 8.8

(a) 4-*OSHL* of $spl(K_{1,6})$ is shown in Fig. 8.6(a).

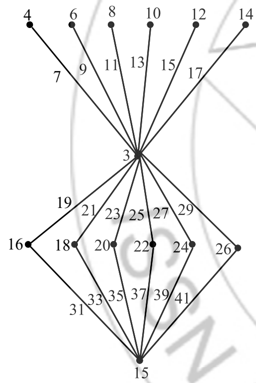

Figure 8.6(a): 4-*OSHL* of $spl(K_{1,6})$

(b) 3-*OSHL* of $spl(K_{1,5})$ is shown in Fig. 8.6(b).

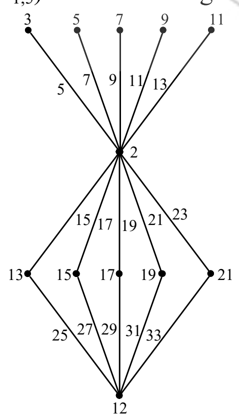

Figure 8.6(b): 3-*OSHL* of $spl(K_{1,5})$

Theorem 8.9

The graph $JE_{m,n}\,(m,n \ge 2)$ is a k-odd sequential harmonious graph for any k.

Proof

Let $\{v,\ v_i,\ 1 \le i \le m+n\ ;\ v_i^{'},\ 1 \le i \le n\}$ be the vertices and $\{e_i,\ 1 \le i \le m+n\ ;\ e_i^{'},\ 1 \le i \le n\}$ be the edges of $JE_{m,n}$ which are denoted as in Fig. 8.7.

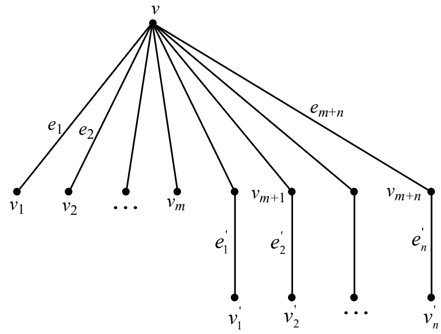

Figure 8.7: $JE_{m,n}$ with ordinary labeling

First we label the vertices as follows:

Define $f: V \rightarrow \{k-1, k, k+1, ..., k+2q-1\}$ by

$f(v) \quad = k-1$

For $1 \le i \le m+n$,

$f(v_i) \quad = k+2i-2$

For $1 \le i \le n$,

$f(v_i^{'}) \quad = k-4i+4n+1$

Then the induced edge labels are:

For $1 \le i \le m+n$,

$f^+(e_i) \quad = 2k+2i-3$

For $1 \le i \le n$

$f^+(e_i^{'}) = 2k+2(m+n)+2(n-i)-1$

Therefore, $f^+(E) = \{2k-1, 2k+1, ..., 2k+2q-3\}$. So, f is a k-odd sequential harmonious labeling and hence, the graph $JE_{m,n}\,(m,n \ge 2)$ is a k-odd sequential harmonious graph for any k.

Illustration 8.10

(a) 3−*OSHL* of $JE_{5,3}$ is shown in Fig. 8.8(a).

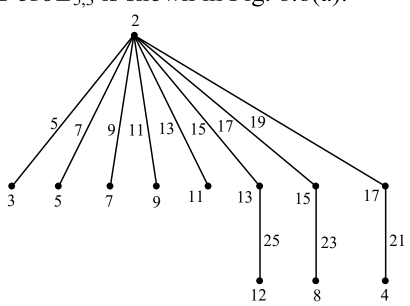

Figure 8.8(a): 3-*OSHL* of $JE_{5,3}$

(b) 2-*OSHL* of $JE_{4,5}$ is shown in Fig. 8.8(b)

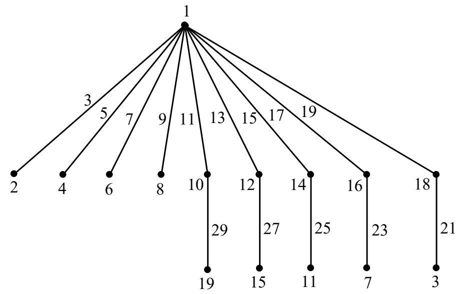

Figure 8.8(b): 2-*OSHL* of $JE_{4,5}$

Definition 8.11

The **middle graph** $\boldsymbol{M(G)}$ of a graph G is the graph whose vertex set is $V(G) \cup E(G)$ and in which two vertices are adjacent if and only if either they are adjacent edges of G or one is a vertex of G and the other is an edge incident with it.

Theorem 8.12

$M(P_n)$ $(n \geq 3)$ is a k–odd sequential harmonious graph for any k.

Proof

Let $\{v, v_i, 1 \leq i \leq n-1;\ u_i, 1 \leq i \leq n\}$ be the vertices and $\{e_i, 1 \leq i \leq n-2;\ e_i^{'}, 1 \leq i \leq 2n-2\}$ be the edges of $M(P_n)$ which are denoted as in Fig. 8.9.

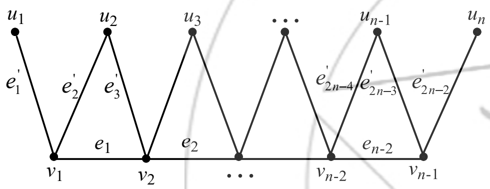

Figure 8.9: $M(P_n)$ with ordinary labeling

First we label the vertices as follows:

Define $f: V \rightarrow \{k-1, k, k+1, \ldots, k+2q-1\}$ by

$$f(v_1) = k-1$$

For $2 \leq i \leq n-1$,

$$f(v_i) = k+3i-3$$

$$f(u_1) = k$$

$$f(u_2) = k+2$$

For $3 \leq i \leq n$,

$$f(u_i) = k+3i-5$$

Then the induced edge labels are:

For $1 \leq i \leq n-2$,

$$f^{+}(e_i) = 2k+6i-3$$

For $1 \leq i \leq 2n-2$,

$$f^{+}(e_i^{'}) = \begin{cases} 2k+3i-4 & i \text{ odd} \\ 2k+3i-5 & i \text{ even} \end{cases}$$

Therefore, $f^{+}(E) = \{2k-1, 2k+1, \ldots, 2k+2q-3\}$. So, f is a k-odd sequential harmonious labeling and hence, the graph $M(P_n)$ $(n \geq 3)$ is a k–odd sequential harmonious graph for any k.

Illustration 8.13

(a) 4–*OSHL* of $M(P_6)$ is shown in Fig. 8.10(a).

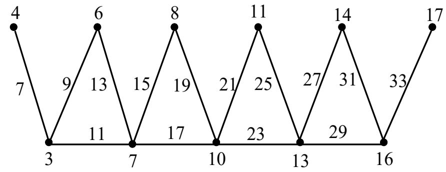

Figure 8.10(a): 4-*OSHL* of $M(P_6)$

(b) 3-*OSHL* of $M(P_7)$ is shown in Fig. 8.10(b).

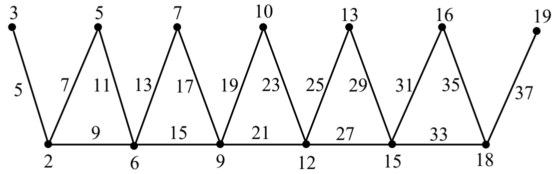

Figure 8.10(b): 3-*OSHL* of $M(P_7)$

Theorem 8.14

The graph P_n @ $K_{1,m}$ $(n \geq 3,\ m \geq 3)$ is a k-odd sequential harmonious graph for any k.

Proof

Let $\{u_1, u_2, \ldots, u_m, v_1, v_2, \ldots, v_n\}$ be the vertices and $\{e_1, e_2, \ldots, e_{n-1}, e_1^{'}, e_2^{'}, \ldots, e_m^{'}\}$ be the edges of P_n @ $K_{1,m}$ which are denoted as in Fig. 8.11.

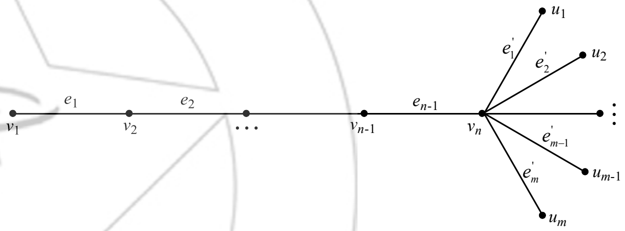

Figure 8.11: P_n @ $K_{1,m}$ with ordinary labeling

First we label the vertices as follows:

Define $f: V \rightarrow \{k-1, k, k+1, \ldots, k+2q-1\}$ by

For $1 \leq i \leq n$,

$$f(v_i) = k+i-2$$

For $1 \leq i \leq m$,

$$f(u_i) = k+2i+n-3$$

Then the induced edge labels are:

For $1 \leq i \leq n-1$,

$$f^{+}(e_i) = 2k+2i-3$$

For $1 \leq i \leq m$,

$$f^{+}(e_i^{'}) = 2k+2n+2i-5$$

Therefore, $f^{+}(E) = \{2k-1, 2k+1, \ldots, 2k+2q-3\}$. So, f is a k-odd sequential harmonious labeling and hence, the graph $P_n @ K_{1,m}\ (n, m \geq 3)$ is a k-odd sequential harmonious graphs for any k.

Illustration 8.15

(a) 4-*OSHL* of $P_5 @ K_{1,7}$ is shown in Fig. 8.12(a).

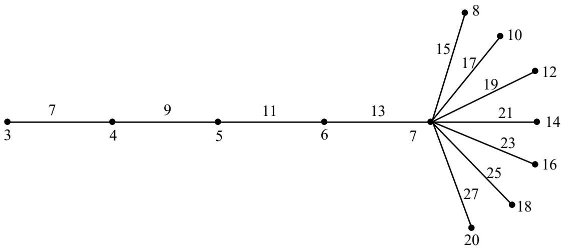

Figure 8.12(a): 4-*OSHL* of P_5 @ $K_{1,7}$

(b) 3-*OSHL* of P_4 @ $K_{1,8}$ is shown in Fig. 8.12(b).

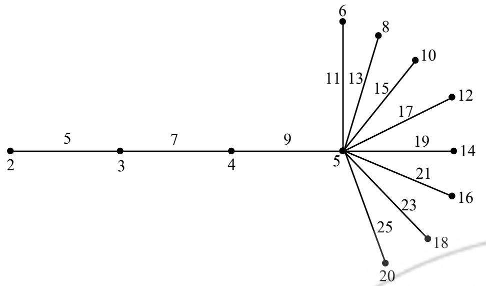

Figure 8.12(b): 3-*OSHL* of P_4 @ $K_{1,8}$

Theorem 8.16
The twig $TW(n)$ ($n \geq 4$) is a k-odd sequential harmonious graph for any k.

Proof
Let $\{v_i,\ 1 \leq i \leq n,\ u_i,\ w_i,\ 1 \leq i \leq n-2\}$ be the vertices and $\{a_i,\ 1 \leq i \leq n-1,\ b_i,\ c_i,\ 1 \leq i \leq n-2\}$ be the edges which are denoted as in Fig. 8.13.

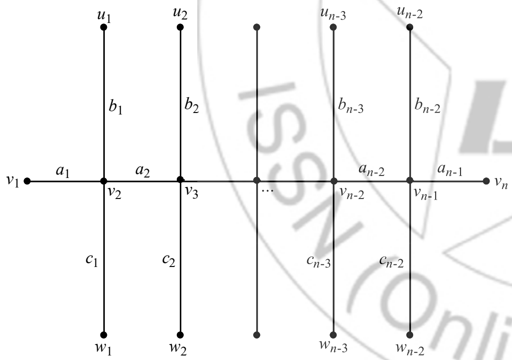

Figure 8.13: Ordinary labeling of $TW(n)$

First we label the vertices as follows:
$f: V \rightarrow \{k-1, k, k+1, ..., k+2q-1\}$ by
For $1 \leq i \leq n$,

$$f(v_i) = \begin{cases} k+3i-4 & i \text{ odd} \\ k+3i-6 & i \text{ even} \end{cases}$$

For $1 \leq i \leq n-2$,

$$f(u_i) = \begin{cases} k+3i-2 & i \text{ odd} \\ k+3i-4 & i \text{ even} \end{cases}$$

$$f(w_i) = \begin{cases} k+3i & i \text{ odd} \\ k+3i-2 & i \text{ even} \end{cases}$$

Then the induced edge labels are:
For $1 \leq i \leq n-1$,

$$f^+(a_i) = 2k+6i-7$$

For $1 \leq i \leq n-2$,

$$f^+(b_i) = 2k+6i-5$$

$$f^+(c_i) = 2k+6i-3$$

Therefore, $f^+(E) = \{2k-1, 2k+1, ..., 2k+2q-3\}$. So, f is a k-odd sequential harmonious labeling and hence, the twig $TW(n)$ ($n \geq 4$) is a k-odd sequential harmonious graphs for any k.

Illustration 8.17
(a) 1-*ESHL* of $TW(4)$ is shown in Fig. 8.14(a).

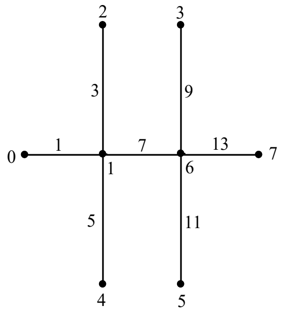

Figure 8.14(a): 1-*ESHL* of $TW(4)$

(b) 4-*ESHL* of $TW(7)$ is shown in Fig. 8.14(b).

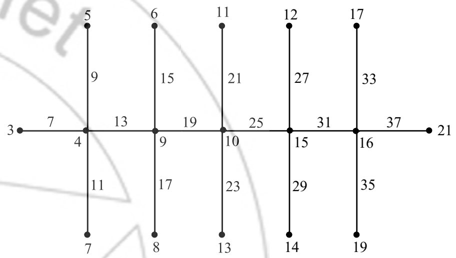

Figure 8.14(b): 4-*ESHL* of $TW(7)$

Theorem 8.18
$K_{1,n}$ ($n \geq 3$) is a k-odd sequential harmonious graph for any k.

Proof
Let $\{u,\ u_i,\ 1 \leq i \leq n\}$ be the vertices and $\{a_i,\ 1 \leq i \leq n\}$ be the edge which are denoted as in Fig. 8.15.

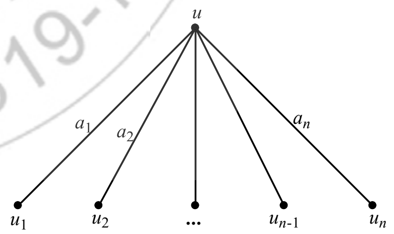

Figure 8.15: Ordinary labeling of $K_{1,n}$

First we label the vertices as follows:
Define $f: V \rightarrow \{k-1, k, ..., k+2q-1\}$ by

$$f(u) = k-1$$

For $1 \leq i \leq n$,

$$f(u_i) = k+2i-3$$

Then the induced edge labels are:
For $1 \leq i \leq n$,

$$f^+(a_i) = 2k+2i-3$$

Therefore, $f^+(E) = \{2k-1, 2k+1, ..., 2k+2q-3\}$. So, f is a k-odd sequential harmonious labeling and hence, $K_{1,n}$ ($n \geq 3$) is a k-odd sequential harmonious graphs for any k.

Illustration 8.19

(a) 1-*ESHL* of $K_{1,5}$ is shown in Fig. 8.16(a).

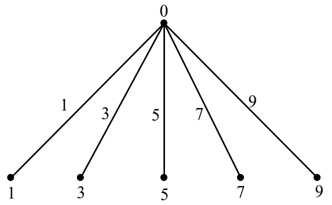

Figure 8.16(a): 1-*ESHL* of $K_{1,15}$

(b) 4-*ESHL* of $K_{1,8}$ is shown in Fig. 8.16(b).

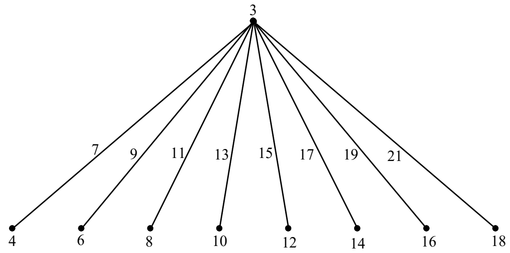

Figure 8.16(b): 4-*ESHL* of $K_{1,6}$

Theorem 8.20

The graph $TG_1(n)$ ($n \geq 3$) is a k-odd sequential harmonious graph for any k.

Proof

Let $\{u, u_i, 1 \leq i \leq n, u_1^{'}, u_2^{'}\}$ be the vertices and $\{a_i, 1 \leq i \leq n, a_1^{'}, a_2^{'}\}$ be the edges which are denoted as in Fig. 8.17.

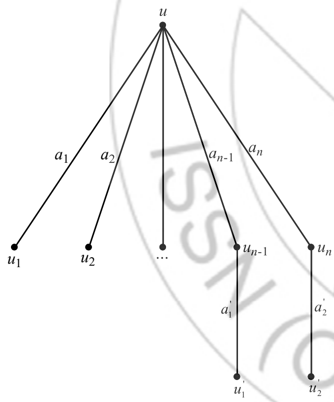

Figure 8.17: Ordinary labeling of $TG_1(n)$

First we label the vertices as follows:

Define $f: V \rightarrow \{k-1, k, k+1, ..., k+2q-1\}$ by

$$f(u) = k-1$$

For $1 \leq i \leq n$,

$$f(u_i) = k+2i-2$$

$$f(u_1^{'}) = k+5$$

$$f(u_2^{'}) = k+1$$

Then the induced edge labels are:

For $1 \leq i \leq n$,

$$f^+(a_i) = 2k+2i-3$$

$$f^+(a_1^{'}) = 2k+2n+1$$

$$f^+(a_2^{'}) = 2k+2n-1$$

Therefore, $f^+(E) = \{2k-1, 2k+1, ..., 2k+2q-3\}$. So, f is a k-odd sequential harmonious labeling and hence, the graph $TG_1(n)$ ($n \geq 3$) is a k-odd sequential harmonious graphs for any k.

Illustration 8.21

(a) 1-*ESHL* of $TG_1(5)$ is shown in Fig. 8.18(a).

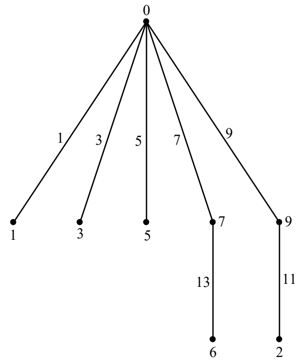

Figure 8.18(a): 1-*ESHL* of $TG_1(5)$

(b) 4-*ESHL* of $TG_1(8)$ is shown in Fig. 8.18(b).

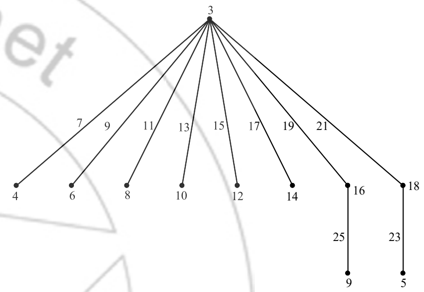

Figure 8.18(b): 4-*ESHL* of TG_1 (8)

Theorem 8.22

The graph $TG_2(n)$ ($n \geq 3$) is a k-odd sequential harmonious graph for any k.

Proof

Let $\{u, u_i, 1 \leq i \leq n, u_1^{'}, u_2^{'}, u_3^{'}\}$ be the vertices and $\{a_i, 1 \leq i \leq n, a_1^{'}, a_2^{'}, a_3^{'}\}$ be the edges which are denoted as in Fig. 8.19.

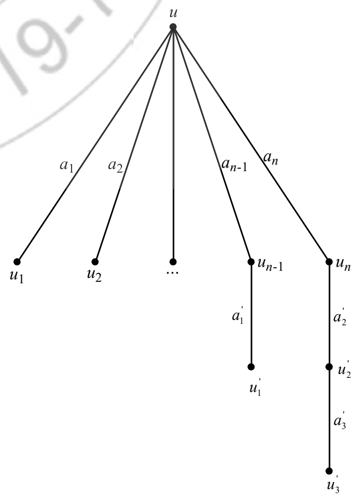

Fig. 8.19: Ordinary labeling of $TG_2(n)$

First we label the vertices as follows:

Define $f: V \rightarrow \{k-1, k, k+1, ..., k+2q-1\}$ by

$$f(u) = k-1$$

For $1 \leq i \leq n$,

$$f(u_i) = k+2i-2$$

$f(u_1') = k + 5$

$f(u_2') = k + 1$

$f(u_3') = k + 2n + 2$

Then the induced edge labels are:
For $1 \le i \le n$,

$f^+(a_i) = 2k + 2i - 3$

$f^+(a_1') = 2k + 2n + 1$

$f^+(a_2') = 2k + 2n - 1$

$f^+(a_3') = 2k + 2n + 3$

Therefore, $f^+(E) = \{2k - 1, 2k + 1, ..., 2k + 2q - 3\}$. So, f is a k-odd sequential harmonious labeling and hence, the graph $TG_2(n)$ $(n \ge 3)$ is a k-odd sequential harmonious graphs for any k.

Illustration 8.23

(a) 1-*ESHL* of $TG_2(4)$ is shown in Fig. 8.20(a).

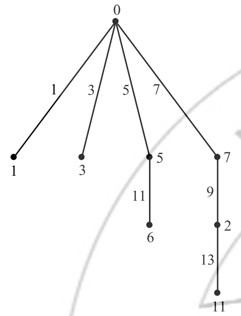

Figure 8.20(a): 1-*ESHL* of $TG_2(4)$

(b) 4-*ESHL* of $TG_2(5)$ is shown in Fig. 8.20(b).

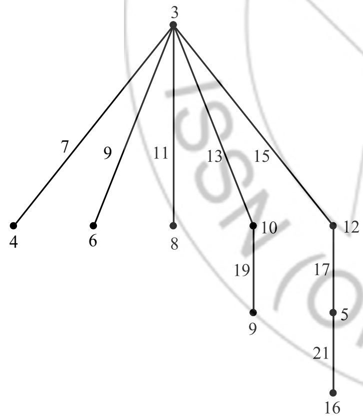

Figure 8.20(b): 4-*ESHL* of $TG_2(5)$

Theorem 8.24

The graph $TG_3(n)$ $(n \ge 3)$ is a k-odd sequential harmonious graph for any k.

Proof

Let $\{u, u_i, 1 \le i \le n, u_1', u_2', u_3', u_4'\}$ be the vertices and $\{a_i, 1 \le i \le n, a_1', a_2', a_3', a_4'\}$ be the edges which are denoted as in Fig. 8.21.

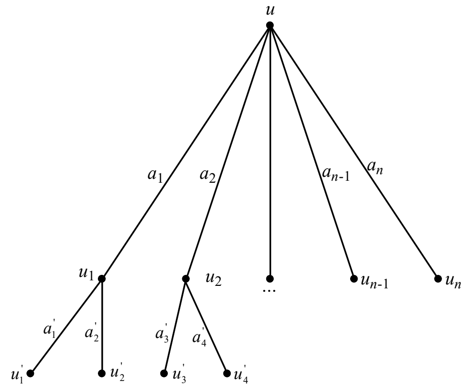

Figure 8.21: Ordinary labeling of $TG_3(n)$

First we label the vertices as follows:
Define $f : V \to \{k - 1, k, k + 1, ..., k + 2q - 1\}$ by

$f(u) = k - 1$

For $1 \le i \le n$,

$f(u_i) = k + 2n - 2i$

$f(u_1') = k + 1$

$f(u_2') = k + 3$

$f(u_3') = k + 7$

$f(u_4') = k + 9$

Then the induced edge labels are:
For $1 \le i \le n$,

$f^+(a_i) = 2k + 2n - 2i - 1$

$f^+(a_1') = 2k + 2n - 1$

$f^+(a_2') = 2k + 2n + 1$

$f^+(a_3') = 2k + 2n + 3$

$f^+(a_4') = 2k + 2n + 5$

Therefore, $f^+(E) = \{2k - 1, 2k + 1, ..., 2k + 2q - 3\}$. So, f is a k-odd sequential harmonious labeling and hence, the graph $TG_3(n)$ $(n \ge 3)$ is a k-odd sequential harmonious graphs for any k.

Illustration 8.25

(a) 1-*ESHL* of $TG_3(5)$ is shown in Fig. 8.22(a).

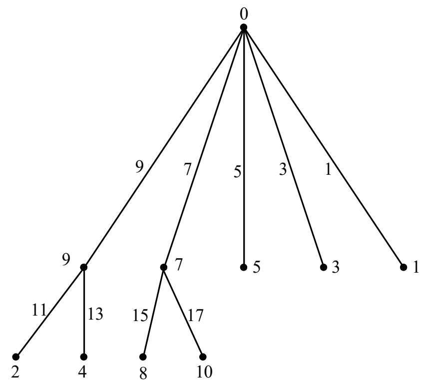

Figure 8.22(a): 1-*ESHL* of $TG_3(5)$

(b) 1-*ESHL* of $TG_3(8)$ is shown in Fig. 8.22(b).

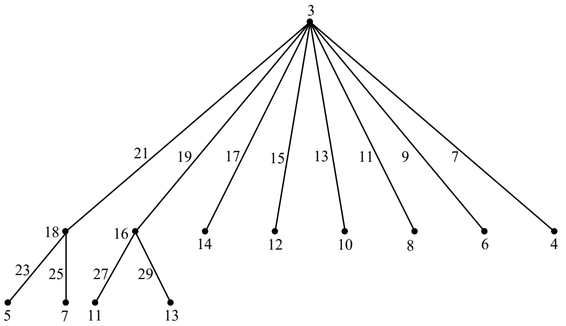

Figure 8.22(b): 4-*ESHL* of $TG_3(8)$

Theorem 8.26

The graph $TG_4(n)$ ($n \geq 3$) is a k-odd sequential harmonious graph for any k.

Proof

Let $\{u, u_i, 1 \leq i \leq n, u_1^{'}, u_2^{'}, u_3^{'}\}$ be the vertices and $\{a_i, 1 \leq i \leq n, a_1^{'}, a_2^{'}, a_3^{'}\}$ be the edges which are denoted as in Fig. 8.23.

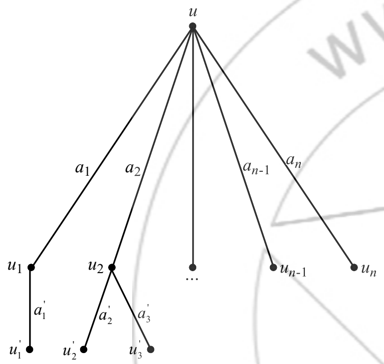

Figure 8.23: Ordinary labeling of $TG_4(n)$

First we label the vertices as follows:

Define $f: V \to \{k-1, k, k+1, ..., k+2q-1\}$ by

$$f(u) = k-1$$

For $1 \leq i \leq n$,

$$f(u_i) = k+2n-2i$$

$$f(u_1^{'}) = k+1$$

$$f(u_2^{'}) = k+5$$

$$f(u_3^{'}) = k+7$$

Then the induced edge labels are:

For $1 \leq i \leq n$,

$$f^{+}(a_i) = 2k+2n-2i-1$$

$$f^{+}(a_1^{'}) = 2k+2n-1$$

$$f^{+}(a_2^{'}) = 2k+2n+1$$

$$f^{+}(a_3^{'}) = 2k+2n+3$$

Therefore, $f^{+}(E) = \{2k-1, 2k+1, ..., 2k+2q-3\}$. So, f is a k-odd sequential harmonious labeling and hence, the graph $TG_4(n)$ ($n \geq 3$) is a k-odd sequential harmonious graphs for any k.

Illustration 8.27

(a) 1-*ESHL* of $TG_4(5)$ is shown in Fig. 8.24(a).

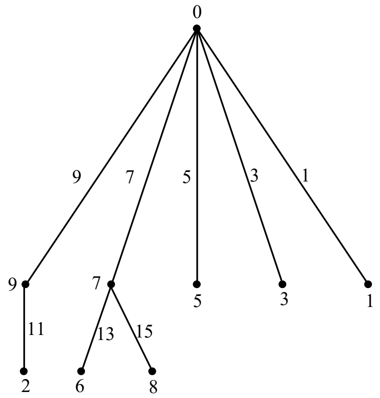

Figure 8.24(a): 1-*ESHL* of $TG_4(5)$

(b) 4-*ESHL* of $TG_4(6)$ is shown in Fig. 8.24(b).

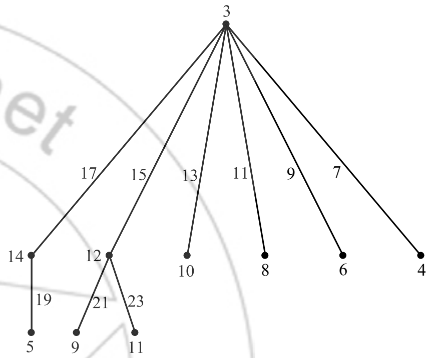

Figure 8.24(b): 4-*ESHL* of $TG_4(6)$

Theorem 8.28

The graph $TG_5(n)$ ($n \geq 4$) is a k-odd sequential harmonious graph for any k.

Proof

Let $\{u, u_i, 1 \leq i \leq n, u_1^{'}, u_2^{'}, u_3^{'}\}$ be the vertices and $\{a_i, 1 \leq i \leq n, a_1^{'}, a_2^{'}, a_3^{'}\}$ be the edges which are denoted as in Fig. 8.25.

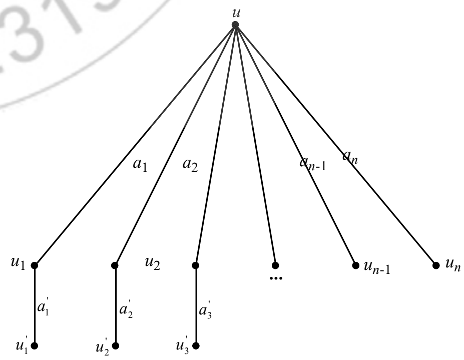

Figure 8.25: Ordinary labeling of $TG_5(n)$

First we label the vertices as follows:

Define $f: V \to \{k-1, k, k+1, ..., k+2q-1\}$ by

$$f(u) = k-1$$

For $1 \leq i \leq n$,

$$f(u_i) = k+2n-2i$$

$$f(u_1^{'}) = k+1$$

$f(u_2^{'}) = k + 5$

$f(u_3^{'}) = k + 9$

Then the induced edge labels are:

For $1 \le i \le n$,

$f^+(a_i) = 2k + 2n - 2i - 1$

$f^+(a_1^{'}) = 2k + 2n - 1$

$f^+(a_2^{'}) = 2k + 2n + 1$

$f^+(a_3^{'}) = 2k + 2n + 3$

Therefore, $f^+(E) = \{2k - 1, 2k + 1, ..., 2k + 2q - 3\}$. So, f is a k-odd sequential harmonious labeling and hence, the graph $TG_5(n)$ $(n \ge 4)$ is a k-odd sequential harmonious graphs for any k.

Illustration 8.29

(a) 1-*ESHL* of $TG_5(4)$ is shown in Fig. 8.26(a).

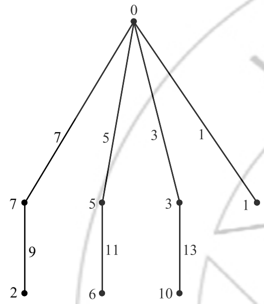

Figure 8.26(a): 1-*ESHL* of $TG_5(4)$

(b) 4-*ESHL* of $TG_5(7)$ is shown in Fig. 8.26(b).

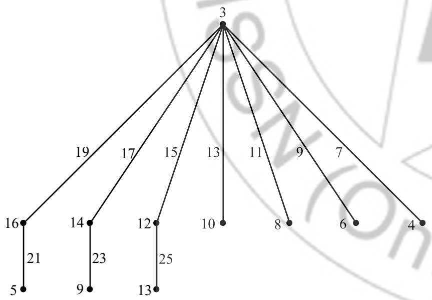

Figure 8.26(b): 4-*ESHL* of $TG_5(7)$

Theorem 8.30

One point union of a cycle with one chord and $K_{1,m}$ in a k-*OSHG* for any k when n is even and for any m.

Proof

Let $G(n)$ the cycle C_n with one chord as denoted in Fig. 8.27.

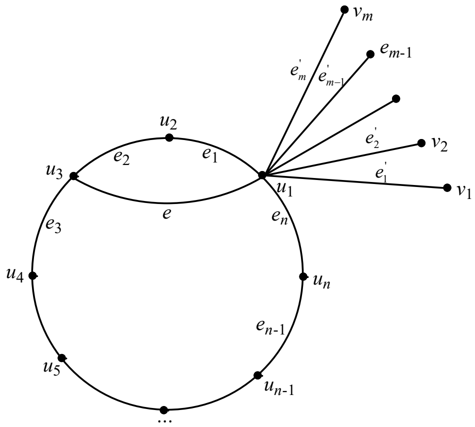

Figure 8.27: Ordinary labeling of $G(n)$

We first label the vertices as follows:

Define $f: V(G) \to \{k - 1, k, k + 1, ..., k + 2q - 1\}$ by

$f(u_1) = k - 1$

$f(u_2) = k$

For $3 \le i \le \frac{n}{2}$,

$f(u_i) = k + i - 1$

For $\frac{n+2}{2} \le i \le n$,

$f(u_i) = k + i$

For $1 \le i \le m$,

$f(v_i) = k + 2n - 1 + 2i$

Then the induced edge labels are:

$f^+(e) = 2k + 1$

$f^+(e_1) = 2k - 1$

For $2 \le i \le \frac{n-2}{2}$,

$f^+(e_i) = 2k + 2i - 1$

For $\frac{n}{2} \le i \le n - 1$,

$f^+(e_i) = 2k + 2i + 1$

$$f^+(e_n) = \begin{cases} 2k + n - 1 & \text{if } n \text{ is even} \\ 2k + n - 2 & \text{if } n \text{ is odd} \end{cases}$$

For $1 \le i \le m$,

$f^+(e_i^{'}) = 2k + 2n + 2i - 1$

Therefore $f^+(E) = \{2k - 1, 2k + 1, ..., 2k + 2q - 3\}$. So, f is a k-odd sequential harmonious labeling and hence $G(n)$ is a k-*OSHG* for any k when n is even.

Illustration 8.31

(a) 2-*OSHL* of $G(6)$ is shown in Fig. 8.28(a).

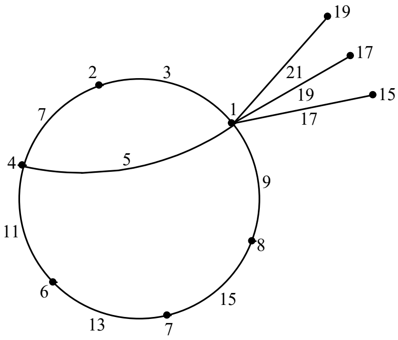

Figure 8.28(a): 2-*OSHL* of $G(6)$

(b) 3-*OSHL* of $G(7)$ is shown in Fig. 8.28(b).

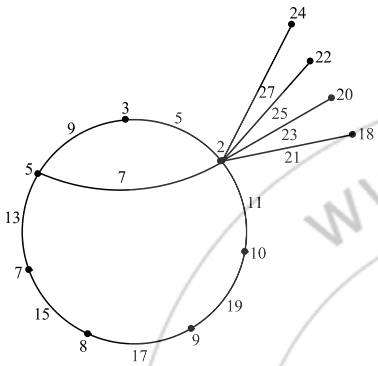

Figure 8.28(b): 3-*OSHL* of $G(7)$

Theorem 8.32

The graph G_{mn} obtained from the path P_n, by replanting each edge by m copies of $K_{1,2}$ is a k-*OSHG* for any k.

Proof

Let G_{mn} be a graph whose vertices and edges be denoted as in Fig. 8.29.

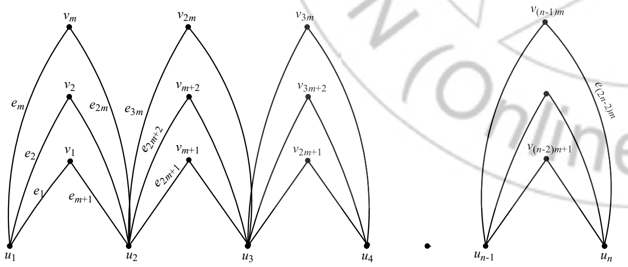

Figure 8.29: Ordinary labeling of G_{mn}

We first label the vertices as follows:

For $1 \le i \le n$,

$$f(u_i) = k + 2m(i-1) - 1$$

For $1 \le i \le (n-1)\,m$,

$$f(v_i) = k + 2i - 2$$

Then the induced edge labels are:

For $1 \le i \le (2n-2)\,m$,

$$f^+(e_i) = 2k + 2i - 3$$

Therefore, $f^+(E) = \{2k-1,\ 2k+1,\ \ldots,\ 2k+2q-3\}$. So, f is a k-odd sequential harmonious labeling and hence, G_{mn} is a k-*OSHG* for any k, m and n.

Illustration 8.33

(a) 1-*OSHL* of G_{34} is shown in Fig. 8.30(a).

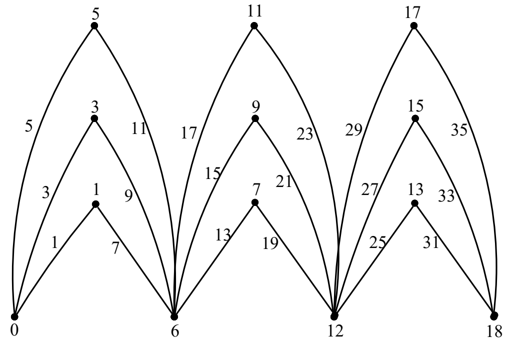

Figure 8.30(a): 1-*OSHL* of G_{34}

(b) 1-*OSHL* of G_{45} is shown in Fig. 8.30(b).

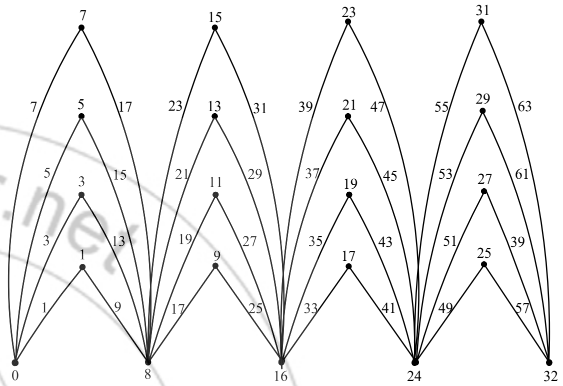

Figure 8.30(b): 1-*OSHL* of G_{45}

References

[1] J.A. Gallian, A dynamic survey of graph labeling – The Electronic Journal of Combinatorics 15(2008), #DS6.

[2] B. Gayathri and V. Hemalatha, Even sequential harmonious labeling of some graphs - Presented in national Conference hold at Govt. College for Women, Pudukkottai – 28 & 29th March 2008.

[3] B. Gayathri and D. Muthuramakrishnan, k-even sequential harmonious labeling of some tree related graphs, International Journal of Engineering Science, Advanced Computing and Bio-technology, Vol. 3, No. 2, April – June 2012, pp. 85-92.

[4] B. Gayathri and D. Muthuramakrishnan, Some results on k-even sequential harmonious labeling of graphs, Elixir Applied Mathematics, 47 (2012) 9054-9057.

[5] D. Muthuramakrishnan, Ph.D. Thesis, March 2013, k-even sequential harmonious labeling of graphs, Bharathidasan University, Tiruchirappalli-24.

[6] B. Gayathri and D. Muthuramakrishnan, k-even sequential harmonious labeling of some cycle related graphs, International Journal of Science and Research, 47 (2012) 9054-9057.

[7] R.L. Graham and N.J.A. Sloane, On addition bases and harmonious graphs, SIAM, J. Alg. Discrete Math., 1 (1980) 382-404.

[8] G.S. Singh and T.K.M. Varkey, On odd sequential and bisequential graphs, preprint.