Estimation $R_{(s,k)}$ of Generalized Rayleigh Distribution

Abbas .N .S[1], Eman .A .A[2]

Department of Mathematics, Baghdad University, College of Education for Pure Sciences (Ibn Al – Haitham)

Abstract: ***In this paper, a reliability of the multi-component system in stress-strength model $R_{(s,k)}$ will be considered ,when the stress and strength are independent identically distribution (iid), follows Generalized Rayleigh Distribution (GRD) with the unknown shape parameter α and known scale parameter λ (λ =1).Different estimation methods of $R_{(s,k)}$ for (GRD) were introduced like Maximum likelihood ,Least square and Shrinkage estimators . The comparisons among the proposed estimators were made depend on simulation based on mean squared error (MSE) criteria.***

Keywords: Generalized Rayleigh Distribution(GRD),Reliability of multi-component Stress – Strength models $R_{(s,k)}$, Maximum likelihood estimator (MLE),least square estimator (LS) ,Shrinkage estimator and mean squared error (MSE).

1. Introduction

A multi component system of k components having strength following independently and identically distribution random variable and each component experiencing a random stress was introduced by Bhattacharyya and Johnson (1974);[4].The reliability of multicomponent system model or s out of k (s-k) model denoted by $R_{(s,k)}$ when at least s ($1\leq s\leq k$) of components survive .Noted thatif s=1 and s=k corresponded, respectively to parallel and series systems.

In 2010, Rao & Kantam studied a system of k multi-component such as x and y (iid) follows the Log–Logistic distribution;[10].In the same year ,Srinivasa Rao studied estimation for the reliability of multi-component stress-strength model based on Generalized Exponential distribution;[12].

In 2012 Srinivasa Rao estimated the multi-component system of reliability for log-logistic distribution with different shape parameters;[13], also Hassan & Basheikh studied reliability estimation of stress- strength model with non-identical component strengths by using the Exponentiated Pareto distribution;[5]. In (2016),Srinivasa Rao et al estimated thereliability of multi-component system in stress-strength model when stress and strength follow Exponentiated Weibull distribution;[11]. In (2017), Hassan studied the estimation the reliability system of multi-componentin (S-S) model when each of stress and strength follows Lindley distribution; [6].In the same year, Salman and Sail estimated the reliability of multi-component system in stress-strength model using shrinkage estimation method;[15].

The model mentioned used in many applications of physics and engineering such as strength failure and the system collapse;[5].

The Generalized Rayleigh Distribution has firstlystudied byKundu and Raqab (2005);[8]. In (2014) Srinivasa Rao estimated the multi-component system of reliability for Generalized Rayleigh distribution (GRD) using maximum likelihood estimator and the reliability estimators are compared asymptotically [9].In (2015) Salman and Ameen, estimated the shape parameter of Generalized Rayleigh distribution (GRD) used Bayesian-shrinkage estimator [15],the same researches in (2016) the estimated the shape parameter of Generalized Rayleigh distribution (GRD) by using double stage minimax- shrinkage estimator[16].

The aim of this paper is to estimate the reliability of multi-component system in stress-strength model $R_{(s,k)}$ based on Generalized Rayleighdistribution with known scale parameter $(\lambda = 1)$ and the parameter α will be unknown via different estimation methods like Maximums likelihood estimator (MLE),Least square method (LS),as well as some of shrinkage methods and make comparisons among the proposed estimator methods using simulation depends on mean squared error criteria.

The probability density function (p. d. f.) of a random variable X follows GeneralizedRayleigh Distribution GRD (α,1) has the from below:

$$f(x;\alpha,\lambda) = 2\alpha\lambda^2 x e^{-(\lambda x)^2}\left(1-e^{-(\lambda x)^2}\right)^{\alpha-1};\ \text{for } x>0,\ \alpha,\beta>0 \quad (1)$$

The cumulative distribution function (c.d. f.) of x is:

$$F(x,\alpha,\lambda) = \left(1-e^{-(\lambda x)^2}\right)^{\alpha} \quad x>0 \quad (2)$$

Where,αrefer to shape parameter, λrefer to scale parameter.

Note that, when $(\lambda = 1)$, the (p .d .f) and the (c.d.f) are respectively became as below :-

$$f(x;\alpha,\lambda) = 2\alpha x e^{-x^2}\left(1-e^{-x^2}\right)^{\alpha-1};\ \text{for } x>0\ \alpha>0 \quad (3)$$

$$F(x,\alpha,\lambda) = \left(1-e^{-x^2}\right)^{\alpha} \quad ;\ x>0 \quad (4)$$

The first one who derived the reliability of a multi-component system in stress-strength model $R_{(s,k)}$were Bhattacharyya and Johnson (1974), as the following form,[4].

$R_{(s,k)}$ =P(at least s of the $X_1, X_2,\ldots, X_k$exceed Y)

Where$x_1,x_2\ldots,x_k$ are identically independent distributed (iid) with common distribution function F(x) and subjected to the common stress random variable Ywith distribution Function G(y).

Consequently the reliability of multi-component system in stress-strength model $R_{(s,k)}$ will be as below:

$R_{(s,k)} = \sum_{i=s}^{k}\binom{k}{i}\int_{o}^{\infty}(1 - F_x(y))^{i}(F_x(y))^{k-i}\,dG(y)$

And when x~ GRD (α,1) and y~ GRD(β,1) then:

$R_{(s,k)}$

$= \sum_{i=s}^{k}\binom{k}{i}\int_{o}^{\infty}(1 -$

(1−e−y2)α)i((1−e−y2)α)k−i2βye−y2

(1−e−y2)β+1dy

$R_{(s,k)} = \sum_{i=s}^{k}\binom{k}{i}\int_{o}^{\infty} 2\beta y e^{-y^2}(1 - (1 - e^{-y^2})^{\alpha})^{i}(1 -$

e−y2)αk−iα+β−1dy

And by some simplification, we get

$R_{(s,k)} = \frac{\beta}{\alpha}\sum_{i=s}^{k}\frac{k!}{(k-i)!}\left[\prod_{j=0}^{i}\left(k + \frac{\beta}{\alpha} - j\right)\right]^{-1}$; k, i, j are integers (5)

2. Estimation Methods of $R_{(s,k)}$

2.1 Maximum Likelihood Estimator (MLE)

Let $x_1, x_2, \ldots, x_n$ be a random sample of size n follows GRD $(\alpha, 1)$, then $x_{(1)} < x_{(2)} < \ldots < x_{(n)}$ the order random sample of x and $y_1, y_2, \ldots, y_m$ be a random sample of size m follows GRD $(\beta, 1)$, and $y_{(1)} < y_{(2)} < \ldots < y_{(m)}$ the order random sample of y. Then the likelihood functions:

$L = L(\alpha, \beta; x, y) = \prod_{i=1}^{n} f(x_i)\prod_{j=1}^{m} g(y_j)$

$= \prod_{i=1}^{n} 2\alpha x_i e^{-x_i^2}(1 - e^{-x_i^2})^{\alpha-1}\prod_{j=1}^{m} 2\beta y_j e^{-y_j^2}(1 -$

e−yj2)β−1

$= 2^n\alpha^n\prod_{i=1}^{n} x_i\, e^{-\sum_{i=1}^{n} x_i^2}\prod_{i=1}^{n}(1 -$

e−xi2)α−12mβmi=1nyje−j=1myj2j=1m(1−e−yj2)β−1

$\ln(l) = (n+m)ln2 + nln\alpha + \sum_{i=1}^{n} lnx_i - \sum_{i=1}^{n} x_i^2 + (\alpha - 1)\sum_{i=1}^{n} ln(1 - e^{-x_i^2}) + mln\beta + \sum_{i=1}^{n} lny_j - \sum_{j=1}^{m} y_j^2 + (\beta - 1)\sum_{j=1}^{m} ln(1 - e^{-y_j^2})$

$$\frac{d\mathrm{Ln}(l)}{d\alpha} = \frac{n}{\alpha} + \sum_{i=1}^{n} ln\left(1 - e^{-x_i^2}\right) = 0$$

$$\frac{d\mathrm{Ln}(l)}{d\beta} = \frac{m}{\beta} + \sum_{j=1}^{m} ln\left(1 - e^{-y_j^2}\right) = 0$$

Thus, the maximum likelihood estimator of the parameters α , β will be as follows:

$$\hat{\alpha}_{mle} = \frac{-n}{\sum_{i=1}^{n} ln\left(1 - e^{-x_i^2}\right)} \quad (6)$$

$$\hat{\beta}_{mle} = \frac{-m}{\sum_{j=1}^{m} ln\left(1 - e^{-y_j^2}\right)} \quad (7)$$

By substituting $\hat{\alpha}_{mle}$,$\hat{\beta}_{mle}$ in equation (5) we get the reliability estimation for $R_{(s,k)}$ model via Maximum Likelihood method:

$$\hat{R}_{(s,k)_{mle}} = \frac{\hat{\beta}_{mle}}{\hat{\alpha}_{mle}}\sum_{i=s}^{k}\frac{k!}{(k-i)!}\left[\prod_{j=0}^{i}\left(k + \frac{\hat{\beta}_{mle}}{\hat{\alpha}_{mle}} - j\right)\right]^{-1} \quad (8)$$

2.2 Least Square estimator of $R_{(s,k)}$

Let x~ GRD(α,1) and y~ GRD(β,1) ,i=1,2,……,n and j=1,2…,m then the least square estimator which minimize thesquared errors between the value of CDF $F(x_i)$ and its expected value $E(F(x_i))$.

$$S = \sum_{i=1}^{n}[F(x_i) - E(F(x_i))]^{2}$$

$F(x_i) = \left(1 - e^{-x_i^2}\right)^{\alpha}$

And, $E(F(x_i)) = P_i$

Such that; $P_i = \frac{i}{n+1}$; i=1,2,…,n

$F(x_i) = E(F(x_i))$

$\left(1 - e^{-x_i^2}\right)^{\alpha} = P_i$

$$\alpha Ln\left(1 - e^{-x_i^2}\right) - LnP_i = 0$$

$S = \sum_{i=1}^{n}[\alpha\mathrm{Ln}(1 - e^{-x_i^2}) - \mathrm{Ln}P_i]^{2}$ (9)

And, the partial derivatives of the equation (9) with respect to α:

$$\frac{\partial S}{\partial\alpha} = 2\sum_{i=1}^{n}[\alpha\mathrm{Ln}(1 - e^{-x_i^2}) - \mathrm{Ln}P_i]\,\mathrm{Ln}(1 - e^{-x_i^2})$$

And equal the result to zero,

$$2\sum_{i=1}^{n}[\alpha\mathrm{Ln}(1 - e^{-x_i^2}) - \mathrm{Ln}P_i]\,\mathrm{Ln}(1 - e^{-x_i^2}) = 0$$

Then, theleast square estimator of α will be

$$\hat{\alpha}_{LS} = \frac{\sum_{i=1}^{n}\mathrm{Ln}P_i\mathrm{Ln}\left(1 - e^{-x_i^2}\right)}{\sum_{i=1}^{n}[\mathrm{Ln}\left(1 - e^{-x_i^2}\right)]^2}\ ;\ P_i = \frac{i}{n+1},\ i=1,2,\ldots,n \quad (10)$$

And by the same procedure, one can obtainthe least square estimator of β as below

$$\hat{\beta}_{LS} = \frac{\sum_{j=1}^{m}\mathrm{Ln}P_j\mathrm{Ln}\left(1 - e^{-y_j^2}\right)}{\sum_{j=1}^{m}[\mathrm{Ln}\left(1 - e^{-y_j^2}\right)]^2}\ ;\ P_j = \frac{j}{m+1},\ j=1,2,\ldots,m \quad (11)$$

By substituting $\hat{\alpha}_{LS}$,$\hat{\beta}_{LS}$ in equation (5), we get the reliability estimation for $R_{(s,k)}$ model usingLeast squared method:

$$\hat{R}_{(s,k)_{LS}} = \frac{\hat{\beta}_{LS}}{\hat{\alpha}_{LS}}\sum_{i=s}^{k}\frac{k!}{(k-i)!}\left[\prod_{j=0}^{i}\left(k + \frac{\hat{\beta}_{LS}}{\hat{\alpha}_{LS}} - j\right)\right]^{-1} \quad (12)$$

2.3 Shrinkage Estimation Method (Sh)

Thompson in 1968, assumed the problem of shrink a usual estimator $\hat{\alpha}$ of the parameter αbased on prior information α_0 using shrinkage weight factor$\emptyset(\hat{\alpha})$, such that $0 \le \emptyset(\hat{\alpha}) \le 1$.Thompson estimating α and he believe that α_0 is closed to the true value of α. Thus, the Thompson-type of shrinkage estimator of α say $\hat{\alpha}_{sh}$ will be: [1][2][3][14][15][17]

$$\hat{\alpha}_{sh} = \emptyset(\hat{\alpha})\hat{\alpha} + (1 - \emptyset(\hat{\alpha}))\alpha_0 \quad (13)$$

This paper, employs the unbiased estimator $\hat{\alpha}_{ub}$ as a usual estimator of α and $\alpha_0 \approx \alpha$ as a prior estimation of α in equation (13).

Note that,

$$\hat{\alpha}_{ub} = \frac{n-1}{n}\hat{\alpha}_{mle} = \frac{n-1}{-\sum_{i=1}^{n} ln\left(1 - e^{-x_i^2}\right)}$$

Hence,

E $(\hat{\alpha}_{ub}) = \alpha$ and $Var(\hat{\alpha}_{ub}) = \frac{\alpha^2}{n-2}$

And,

$$\hat{\beta}_{ub} = \frac{m-1}{m}\hat{\beta}_{mle} = \frac{m-1}{-\sum_{j=1}^{m} ln\left(1 - e^{-y_j^2}\right)}$$

Implies,

E $(\hat{\beta}_{ub}) = \beta$ and $Var(\hat{\beta}_{ub}) = \frac{\beta^2}{m-2}$.

As we mentioned, $\emptyset(\hat{\alpha})$ denote the shrinkage weight factor such that $0 \le \emptyset(\hat{\alpha}) \le 1$, which may be a function of $\hat{\alpha}_{ub}$,a function of sample size (n,m) or may be constant or can be found by minimizing the mean square error of $\hat{\alpha}_{sh}$.

2-3-1 The shrinkage weight function (sh1):

In this subsection, the shrinkage weight factors as a function of sizes n and m respectively will be considered and taking the form below:

i.e. $\emptyset_1(\hat{\alpha}) = |\sin n/n|$, and $\emptyset_2(\hat{\beta}) = |sin m/m|$

Where n, and m is defined in (2-1), therefore the shrinkage estimator using shrinkage weight function of α and β which is defined in equation (13), will be respectively as bellow:

$\hat{\alpha}_{sh1} = \emptyset_1(\hat{\alpha})\hat{\alpha}_{ub} + (1-\emptyset_2(\hat{\alpha}))\alpha_0$ (14)

$\hat{\beta}_{sh1} = \emptyset_1(\hat{\beta})\hat{\beta}_{ub} + (1-\emptyset_2(\hat{\beta}))\beta_0$(15)

Also, as Thompson (1968) mentioned,α_0 and β_0 are closed to the real value of α *and* βrespectively.

Then, the shrinkage estimation of the reliability of a multi-component system in (s-s) model which is defined in equation (5) using shrinkage weight functiondepends on (14),(15) will be:

$$\hat{R}_{(s,k)_{sh1}} = \frac{\hat{\beta}_{sh1}}{\hat{\alpha}_{sh1}} \sum_{i=s}^{k} \frac{k!}{(k-i)!} \left[\prod_{j=0}^{i} (k + \frac{\hat{\beta}_{sh1}}{\hat{\alpha}_{sh1}} - j) \right]^{-1} \quad (16)$$

2.3.2 Constant shrinkage weight factor (sh2)

We suggest in this subsection constant shrinkage weight factor $\varphi_i(\hat{\alpha}_i) = 0.1$. Therefore, the shrinkage estimator using specific constant weight factor will be as follows

$$\hat{\alpha}_{sh2} = \emptyset_1(\hat{\alpha})\hat{\alpha}_{ub} + (1-\emptyset_2(\hat{\alpha}))\alpha_0 \quad (17)$$

$$\hat{\beta}_{sh2} = \emptyset_1(\hat{\beta})\hat{\beta}_{ub} + (1-\emptyset_2(\hat{\beta}))\beta_0 \quad (18)$$

Substitute equation (17) and (18) in equation (5) to obtain the shrinkage estimation of $R_{(s,k)}$ using the above constant shrinkage weight factor as bellow:

$$\hat{R}_{(s,k)_{sh2}} = \frac{\hat{\beta}_{sh2}}{\hat{\alpha}_{sh2}} \sum_{i=s}^{k} \frac{k!}{(k-i)!} \left[\prod_{j=0}^{i} (k + \frac{\hat{\beta}_{sh2}}{\hat{\alpha}_{sh2}} - j) \right]^{-1} \quad (19)$$

2.3.3 Modified Thompson type shrinkage weight function (th)

In this subsection, we modify the shrinkage weight factor which was consideredby Thompson in 1968 as bellow:

$$\gamma(\hat{\alpha}) = \frac{(\hat{\alpha}_{ub}-\alpha_0)^2}{(\hat{\alpha}_{ub}-\alpha_0)^2 + Var(\hat{\alpha}_{ub})} *0.001 \quad (20)$$

$$\gamma(\hat{\beta}) = \frac{(\hat{\beta}_{ub}-\beta_0)^2}{(\hat{\beta}_{ub}-\beta_0)^2 + Var(\hat{\beta}_{ub})} * 0.001 \quad (21)$$

Therefore, the shrinkage estimator ofα and β using modified shrinkage weight factor are respectively as bellow:

$$\hat{\alpha}_{th} = \gamma(\hat{\alpha})\hat{\alpha}_{ub} + (1-\gamma(\hat{\alpha}))\alpha_0 (22)$$

$$\hat{\beta}_{th} = \gamma(\hat{\beta})\hat{\beta}_{ub} + \left(1-\gamma(\hat{\beta})\right)\beta_0 (23)$$

Then the shrinkage estimation of $R_{(s,k)}$ in equation (5) based on modified Thompson type shrinkage weight factorand equations (22) and(23) will be :

$$\hat{R}_{(s,k)_{th}} = \frac{\hat{\beta}_{th}}{\hat{\alpha}_{th}} \sum_{i=s}^{k} \frac{k!}{(k-i)!} \left[\prod_{j=0}^{i} (k + \frac{\hat{\beta}_{th}}{\hat{\alpha}_{th}} - j) \right]^{-1} \quad (24)$$

3. Simulation Study

In this section, numerical results were studied to compare the performance of the different estimators of reliability, using different sample size = (10, 40, 60 and 100), based on 1000 replicationvia MSE criteria. For this purpose, Mote Carlo simulation was employed by generate the random sample from continuous uniform distribution defined on the interval (0,1) as $u_1, u_2, \ldots, u_n$; $v_1, v_2, \ldots, v_m$. ; [7].

Transform uniform random samples to follows GRD (α,1) using (c. d. f.) as below:

$$F(x) = (1 - e^{x_i^2})^{\alpha}$$

$$U_i = (1 - e^{x_i^2})^{\alpha}$$

$x_i = \sqrt{-\ln(1 - U_i^{\frac{1}{\alpha}})}$, i=1, 2, 3…., n

And, by the similartechnique, obtain

$y_j = \sqrt{-\ln(1 - V_j^{\frac{1}{\beta}})}$, j=1, 2, 3…., m

Compute the $R_{(s,k)}$ in equation (5),compute the maximum likelihood estimator of $R_{(s,k)}$ using equation (8) , Least square(LS) estimator of$R_{(s,k)}$ using equation (12)and shrinkage estimators using equations(16) ,(19)and(24).

Based on (L=1000) Replication, we calculate the MSE for all proposed estimation methods of $R_{(s,k)}$ as follows:

$$MSE = \frac{1}{L} \sum_{i=1}^{L} (\hat{R}_{(s,k)_i} - R_{(s,k)})^2$$

Note that , we consider (s,k)=(1,3),(2,3) and (2,4) and all the result were put it in the tables below:

Table 1: $R_{(s,k)}$and $\hat{R}_{(s,k)}$when (s,k)=(1,3), α=2, β=6 ,α_0=2.001and β_0=6.001

(n,m)	$R_{(s,k)}$	$\hat{R}_{(s,k)_{mle}}$	$\hat{R}_{(s,k)_{sh1}}$	$\hat{R}_{(s,k)_{sh2}}$	$\hat{R}_{(s,k)_{th}}$	$\hat{R}_{(s,k)_{ls}}$
(10,10)	0.50000	0.49799	0.50004	0.49999	0.50008	0.44819
(10,40)	0.50000	0.50647	0.50348	0.51963	0.50027	0.20087
(10,60)	050000	0.51404	0.50121	0.52225	0.50029	0.10127
(10,100)	0.50000	0.50935	0.50113	0.52362	0.50031	0.08091
(40,10)	0.50000	0.48944	0.45899	0.42999	0.49939	0.79300
(40,40)	0.50000	0.50360	0.50015	0.50044	0.50008	0.43949
(40,60)	0.50000	0.50349	0.50054	0.50889	0.50016	0.40280
(40,100)	0.50000	0.50246	0.50086	0.51573	0.50023	0.27636
(60,10)	0.50000	0.48827	0.43469	0.39300	0.49888	0.70411
(60,40)	0.50000	0.49820	0.49769	0.48753	0.49998	0.64017
(60,60)	0.50000	0.50095	0.50009	0.50018	0.50008	0.52018
(60,100)	0.50000	0.50125	0.50059	0.51046	0.50018	0.48928
(100,10)	0.50000	0.48668	0.39516	0.33765	0.49791	0.89773
(100,40)	0.50000	0.49707	0.49302	0.46439	0.49972	0.77479
(100,60)	0.50000	0.49729	0.49921	0.48343	0.49993	0.63902
(100,100)	0.50000	0.50001	0.50008	0.50008	0.50008	0.55478

Table 2: MSE for $\hat{R}_{(s,k)}$ when (s,k)=(1,3), α=2, β=6,α_0=2.001, β_0=6.001, and $R_{(s,k)}$= 0.50000.

(n,m)	$\hat{R}_{(s,k)_{mle}}$	$\hat{R}_{(s,k)_{sh1}}$	$\hat{R}_{(s,k)_{sh2}}$	$\hat{R}_{(s,k)_{th}}$	$\hat{R}_{(s,k)_{ls}}$	Best
(10,10)	0.0122415	0.0000461	0.0001523	0.00000001	0.0094491	Th
(10,40)	0.0080630	0.0000358	0.0004643	0.00000008	0.0911235	Th
(10,60)	0.0071005	0.0000233	0.0005671	0.00000009	0.1591028	Th
(10,100)	0.0069813	0.0000245	0.0006341	0.00000009	0.1760394	Th
(40,10)	0.0077948	0.0019567	0.0055928	0.00000048	0.0886426	Th
(40,40)	0.0031309	0.0000012	0.0000327	0.000000009	0.0197627	Th
(40,60)	0.0026898	0.0000009	0.0001016	0.00000003	0.0109528	Th
(40,100)	0.0021241	0.0000013	0.0002641	0.00000006	0.0529908	Th
(60,10)	0.0075204	0.0047910	0.0125682	0.00000153	0.0669452	Th
(60,40)	0.0024163	0.0000065	0.0001965	0.000000004	0.0208359	Th
(60,60)	0.0020972	0.00000006	0.0000216	0.000000008	0.0024943	Th
(60,100)	0.0017719	0.0000004	0.0001237	0.00000003	0.0081858	Th
(100,10)	0.0071269	0.0119328	0.0279335	0.00000514	0.1590156	Th
(100,40)	0.0021879	0.0000522	0.0013509	0.00000009	0.0769482	Th
(100,60)	0.0016981	0.0000007	0.0003079	0.000000009	0.0229289	Th
(100,100)	0.0012287	0.00000004	0.0000126	0.000000008	0.0391408	Th

Table 3: $R_{(s,k)}$ and $\hat{R}_{(s,k)}$ when (s,k)=(2,3), α=2, β=6 ,α_0=2.001and β_0=6.001.

(n,m)	$R_{(s,k)}$	$\hat{R}_{(s,k)_{mle}}$	$\hat{R}_{(s,k)_{sh1}}$	$\hat{R}_{(s,k)_{sh2}}$	$\hat{R}_{(s,k)_{th}}$	$\hat{R}_{(s,k)_{ls}}$
(10,10)	0.20000	0.21613	0.20018	0.20035	0.20008	0.11582
(10,40)	0.20000	0.21784	0.20335	0.21827	0.20024	0.01073
(10,60)	0.20000	0.21681	0.20100	0.22004	0.20026	0.00866
(10,100)	0.20000	0.21862	0.20113	0.22172	0.20027	0.09390
(40,10)	0.20000	0.19491	0.16535	0.14348	0.19945	0.44348
(40,40)	0.20000	0.20273	0.20005	0.19999	0.20007	0.24214
(40,60)	0.20000	0.20630	0.20046	0.20793	0.20014	0.11436
(40,100)	0.20000	0.20474	0.20076	0.21416	0.20021	0.17313
(60,10)	0.20000	0.20018	0.14864	0.12053	0.19904	0.76934
(60,40)	0.20000	0.20220	0.19798	0.18930	0.19998	0.26961
(60,60)	0.20000	0.20218	0.20007	0.20006	0.20007	0.15703
(60,100)	0.20000	0.20350	0.20053	0.20935	0.20016	0.12126
(100,10)	0.20000	0.19657	0.12111	0.08738	0.19817	0.87368
(100,40)	0.20000	0.20113	0.19392	0.17032	0.19976	0.47129
(100,60)	0.20000	0.20151	0.19932	0.18601	0.19994	0.47233
(100,100)	0.20000	0.19985	0.20006	0.19992	0.20007	0.07794

Table 4: MSE for $\hat{R}_{(s,k)}$ when (s,k)=(2,3), α=2,β=6,α_0=2.001,β_0=6.001 and$R_{(s,k)}$=0.20000

(n,m)	$\hat{R}_{(s,k)_{mle}}$	$\hat{R}_{(s,k)_{sh1}}$	$\hat{R}_{(s,k)_{sh2}}$	$\hat{R}_{(s,k)_{th}}$	$\hat{R}_{(s,k)_{ls}}$	Best
(10,10)	0.0109793	0.0000362	0.0001201	0.00000001	0.0088413	Th
(10,40)	0.0075841	0.0000333	0.0004139	0.00000006	0.0358979	Th
(10,60)	0.0067435	0.0000186	0.0004672	0.00000007	0.0366204	Th
(10,100)	0.0063514	0.0000191	0.0005387	0.00000008	0.0143588	Th
(40,10)	0.0058389	0.0013779	0.0035632	0.0000004	0.0689790	Th
(40,40)	0.0025070	0.0000009	0.0000259	0.000000006	0.0024967	Th
(40,60)	0.0020322	0.0000007	0.0000795	0.00000002	0.0232003	Th
(40,100)	0.0017415	0.0000010	0.0002155	0.00000004	0.0073939	Th
(60,10)	0.0054219	0.0029055	0.0067569	0.00000114	0.3264228	Th
(60,40)	0.0022080	0.0000051	0.0001488	0.000000004	0.0071084	Th
(60,60)	0.0017119	0.00000005	0.0000178	0.000000006	0.0048690	Th
(60,100)	0.0012248	0.0000003	0.0000973	0.000000026	0.0070715	Th
(100,10)	0.0049296	0.0065899	0.0131079	0.0000039	0.4542944	Th
(100,40)	0.0016932	0.0000394	0.0009319	0.00000007	0.0810216	Th
(100,60)	0.0013413	0.0000005	0.0002196	0.000000007	0.1083833	Th
(100,100)	0.0009975	0.00000003	0.0000099	0.000000006	0.0303388	Th

Table 5: $R_{(s,k)}$ and $\hat{R}_{(s,k)}$ when (s,k)=(2,4) , α=2, β=6 ,α_0=2.001and β_0=6.001.

(n,m)	$R_{(s,k)}$	$\hat{R}_{(s,k)_{mle}}$	$\hat{R}_{(s,k)_{sh1}}$	$\hat{R}_{(s,k)_{sh2}}$	$\hat{R}_{(s,k)_{th}}$	$\hat{R}_{(s,k)_{ls}}$
(10,10)	0.28571	0.29300	0.28563	0.28554	0.28580	0.31743
(10,40)	0.28571	0.30489	0.28985	0.30769	0.28601	0.04798
(10,60)	0.28571	0.30609	0.28703	0.30983	0.28603	0.11897
(10,100)	0.28571	0.30207	0.28694	0.31133	0.28604	0.00543
(40,10)	0.28571	0.27901	0.24332	0.21549	0.2851	0.69275
(40,40)	0.28571	0.28764	0.28578	0.28571	0.28580	0.25259

(40,60)	0.28571	0.28996	0.28625	0.29506	0.28589	0.10684
(40,100)	0.28571	0.28827	0.28658	0.30240	0.28596	0.35259
(60,10)	0.28571	0.27880	0.22139	0.18395	0.28455	0.89983
(60,40)	0.28571	0.28541	0.28327	0.27259	0.28569	0.62013
(60,60)	0.28571	0.28552	0.28579	0.28560	0.28580	0.23436
(60,100)	0.28571	0.28968	0.28635	0.29697	0.28591	0.14905
(100,10)	0.28571	0.27664	0.18415	0.13741	0.28348	0.91523
(100,40)	0.28571	0.28679	0.27835	0.24928	0.28542	0.60351
(100,60)	0.28571	0.28509	0.28488	0.26856	0.28564	0.53373
(100,100)	0.28571	0.28406	0.28579	0.28554	0.28580	0.63633

Table 6: MSE for $\hat{R}_{(s,k)}$ when (s,k)=(2,4), α=2,β=6,α_0=2.001,β_0=6.001 and$R_{(s,k)}$=0.28571.

(n,m)	$\hat{R}_{(s,k)_{mle}}$	$\hat{R}_{(s,k)_{sh1}}$	$\hat{R}_{(s,k)_{sh2}}$	$\hat{R}_{(s,k)_{th}}$	$\hat{R}_{(s,k)_{ls}}$	Best
(10,10)	0.0140669	0.0000542	0.0001787	0.00000002	0.0087414	Th
(10,40)	0.0090126	0.0000454	0.0005802	0.00000009	0.0570207	Th
(10,60)	0.0092001	0.0000296	0.0006798	0.00000010	0.0384908	Th
(10,100)	0.0086515	0.0000278	0.0007493	0.00000011	0.0785783	Th
(40,10)	0.0080649	0.0020685	0.0055323	0.00000056	0.1752889	Th
(40,40)	0.0036029	0.0000013	0.0000382	0.000000009	0.0101616	Th
(40,60)	0.0030112	0.0000009	0.0001122	0.00000003	0.0368959	Th
(40,100)	0.0024498	0.0000014	0.0002987	0.00000006	0.0416443	Th
(60,10)	0.0071781	0.0045539	0.0110968	0.00000165	0.3802489	Th
(60,40)	0.0029411	0.0000074	0.0002205	0.000000005	0.1494233	Th
(60,60)	0.0024595	0.00000007	0.0000254	0.000000009	0.0036300	Th
(60,100)	0.0018772	0.0000004	0.0001419	0.00000004	0.0195684	Th
(100,10)	0.0071444	0.0109863	0.0228464	0.00000589	0.3965745	Th
(100,40)	0.0024753	0.0000581	0.0014119	0.000000097	0.1090419	Th
(100,60)	0.0017110	0.0000008	0.0003264	0.000000009	0.0668164	Th
(100,100)	0.0013549	0.00000004	0.0000139	0.000000008	0.1457821	Th

Table 7: $R_{(s,k)}$and $\hat{R}_{(s,k)}$when (s,k)=(1,3), α=6, β=2 ,α_0=6.001 and β_0=2.001.

(n,m)	$R_{(s,k)}$	$\hat{R}_{(s,k)_{mle}}$	$\hat{R}_{(s,k)_{sh1}}$	$\hat{R}_{(s,k)_{sh2}}$	$\hat{R}_{(s,k)_{th}}$	$\hat{R}_{(s,k)_{ls}}$
(10,10)	0.90000	0.89203	0.89992	0.89985	0.89997	0.92531
(10,40)	0.90000	0.89899	0.90126	0.90690	0.90004	0.60473
(10,60)	0.90000	0.89978	0.90034	0.90759	0.90004	0.54408
(10,100)	0.90000	0.90057	0.90036	0.90815	0.90005	0.62826
(40,10)	0.90000	0.89171	0.88405	0.87118	0.89972	0.97945
(40,40)	0.90000	0.89713	0.89995	0.89984	0.89997	0.90714
(40,60)	0.90000	0.89944	0.90013	0.90307	0.89999	0.64079
(40,100)	0.90000	0.90026	0.90026	0.90552	0.90002	0.79188
(60,10)	0.90000	0.89216	0.87395	0.85341	0.89955	0.99842
(60,40)	0.90000	0.89768	0.89909	0.89531	0.89993	0.90514
(60,60)	0.90000	0.89925	0.89997	0.90000	0.89997	0.90694
(60,100)	0.90000	0.89878	0.90015	0.90356	0.90000	0.75152
(100,10)	0.90000	0.89226	0.85431	0.81984	0.89919	0.99529
(100,40)	0.90000	0.89789	0.89742	0.88639	0.89984	0.95717
(100,60)	0.90000	0.89832	0.89966	0.89384	0.89991	0.99009
(100,100)	0.90000	0.89981	0.89997	0.90002	0.89997	0.91628

Table 8: MSE for $\hat{R}_{(s,k)}$when(s,k)=(1,3), α_1=6,α_2=2,α_0=2.001,β_0=6.001 , and $R_{(s,k)}$=0.90000

(n,m)	$\hat{R}_{(s,k)_{mle}}$	$\hat{R}_{(s,k)_{sh1}}$	$\hat{R}_{(s,k)_{sh2}}$	$\hat{R}_{(s,k)_{th}}$	$\hat{R}_{(s,k)_{ls}}$	Best
(10,10)	0.0021796	0.0000059	0.0000198	0.000000002	0.0010591	Th
(10,40)	0.0010598	0.0000046	0.0000566	0.0000000019	0.0956631	Th
(10,60)	0.0009845	0.0000029	0.0000658	0.000000002	0.1275450	Th
(10,100)	0.0008354	0.0000028	0.0000740	0.000000003	0.0951112	Th
(40,10)	0.0013879	0.0003052	0.0009932	0.00000009	0.0063902	Th
(40,40)	0.0004369	0.0000002	0.0000043	0.000000001	0.0008989	Th
(40,60)	0.0003637	0.00000009	0.0000122	0.0000000002	0.0906922	Th
(40,100)	0.0002818	0.0000001	0.0000325	0.0000000007	0.0220467	Th
(60,10)	0.0012763	0.0007802	0.0024829	0.00000024	0.0096921	Th
(60,40)	0.0003917	0.000001009	0.0000288	0.000000005	0.0001788	Th
(60,60)	0.0002739	0.000000008	0.0000027	0.000000001	0.0002299	Th
(60,100)	0.0002217	0.00000003	0.0000142	0.0000000001	0.0244871	Th
(100,10)	0.0012620	0.0023737	0.0072083	0.00000075	0.0091195	Th
(100,40)	0.0003169	0.0000071	0.0001995	0.000000026	0.0033628	Th

(100,60)	0.0002201	0.00000013	0.0000426	0.000000008	0.0084538	Th
(100,100)	0.0001716	0.000000005	0.0000017	0.0000000009	0.0006927	Th

Table 9: $R_{(s,k)}$ and $\hat{R}_{(s,k)}$ when (s,k)=(2,3), α=6, β=2 ,α_0=6.001 and β_0=2.001.

(n,m)	$R_{(s,k)}$	$\hat{R}_{(s,k)_{mle}}$	$\hat{R}_{(s,k)_{sh1}}$	$\hat{R}_{(s,k)_{sh2}}$	$\hat{R}_{(s,k)_{th}}$	$\hat{R}_{(s,k)_{ls}}$
(10,10)	0.77143	0.75940	0.77132	0.77122	0.77137	0.83789
(10,40)	0.77143	0.77376	0.77421	0.78616	0.77151	0.42869
(10,60)	0.77143	0.77309	0.77221	0.78747	0.77152	0.28881
(10,100)	0.77143	0.77132	0.77202	0.78825	0.77153	0.14981
(40,10)	0.77143	0.75922	0.73925	0.71411	0.77086	0.99535
(40,40)	0.77143	0.76559	0.77132	0.77107	0.77136	0.73058
(40,60)	0.77143	0.77104	0.77171	0.77785	0.77143	0.66923
(40,100)	0.77143	0.77082	0.77193	0.78279	0.77148	0.44456
(60,10)	0.77143	0.76315	0.72003	0.68149	0.77051	0.97242
(60,40)	0.77143	0.77082	0.76964	0.76221	0.77129	0.85980
(60,60)	0.77143	0.76805	0.77136	0.77120	0.77136	0.89409
(60,100)	0.77143	0.77146	0.77175	0.77904	0.77144	0.45909
(100,10)	0.77143	0.76012	0.68164	0.61926	0.76978	0.88369
(100,40)	0.77143	0.76926	0.76608	0.74369	0.77110	0.87654
(100,60)	0.77143	0.76885	0.77071	0.75869	0.77125	0.88080
(100,100)	0.77143	0.77077	0.77137	0.77140	0.77137	0.73696

Table 10: MSE for $\hat{R}_{(s,k)}$ when (s,k)=(2,3), α=6, β=2 ,α_0=6.001 and β_0=2.001, and $R_{(s,k)}$=0.77143.

(n,m)	$\hat{R}_{(s,k)_{mle}}$	$\hat{R}_{(s,k)_{sh1}}$	$\hat{R}_{(s,k)_{sh2}}$	$\hat{R}_{(s,k)_{th}}$	$\hat{R}_{(s,k)_{ls}}$	Best
(10,10)	0.0073852	0.0000239	0.0000793	0.000000007	0.0052276	Th
(10,40)	0.0043566	0.0000211	0.0002574	0.000000009	0.1305148	Th
(10,60)	0.0041100	0.0000142	0.0002973	0.000000011	0.2385390	Th
(10,100)	0.0036847	0.0000120	0.0003171	0.000000012	0.3913909	Th
(40,10)	0.0053620	0.0012421	0.0039058	0.00000038	0.0503749	Th
(40,40)	0.0018213	0.0000006	0.0000181	0.000000005	0.0046412	Th
(40,60)	0.0015134	0.0000004	0.0000531	0.0000000008	0.0151349	Th
(40,100)	0.0011405	0.0000006	0.0001379	0.000000003	0.1230323	Th
(60,10)	0.0044948	0.0030244	0.0091401	0.00000097	0.0404436	Th
(60,40)	0.0014517	0.0000039	0.0001112	0.000000021	0.0101099	Th
(60,60)	0.0011811	0.00000004	0.0000117	0.000000005	0.0179154	Th
(60,100)	0.0009163	0.00000012	0.0000647	0.0000000005	0.1133133	Th
(100,10)	0.0044207	0.0089331	0.0252554	0.00000309	0.0173309	Th
(100,40)	0.0013144	0.0000306	0.0008277	0.000000114	0.0120944	Th
(100,60)	0.0009309	0.0000006	0.0001817	0.000000034	0.0309155	Th
(100,100)	0.0007041	0.00000002	0.0000072	0.000000004	0.0024155	Th

Table 11: $R_{(s,k)}$ and $\hat{R}_{(s,k)}$ when (s,k)=(2,4), α=6, β=2 ,α_0=6.001 and β_0=2.001.

(n,m)	$R_{(s,k)}$	$\hat{R}_{(s,k)_{mle}}$	$\hat{R}_{(s,k)_{sh1}}$	$\hat{R}_{(s,k)_{sh2}}$	$\hat{R}_{(s,k)_{th}}$	$\hat{R}_{(s,k)_{ls}}$
(10,10)	0.83077	0.81714	0.83046	0.83019	0.83072	0.71255
(10,40)	0.83077	0.83051	0.83288	0.84215	0.83083	0.46729
(10,60)	0.83077	0.83127	0.83139	0.84330	0.83084	0.23549
(10,100)	0.83077	0.83344	0.83144	0.84426	0.83085	0.23258
(40,10)	0.83077	0.81565	0.80419	0.78313	0.83031	0.93028
(40,40)	0.83077	0.82804	0.83072	0.83071	0.83072	0.94066
(40,60)	0.83077	0.82868	0.83097	0.83565	0.83077	0.85408
(40,100)	0.83077	0.82935	0.83116	0.83962	0.83081	0.59875
(60,10)	0.83077	0.81969	0.78901	0.75671	0.83004	0.97401
(60,40)	0.83077	0.82991	0.82937	0.82353	0.83066	0.89224
(60,60)	0.83077	0.83054	0.83073	0.83086	0.83072	0.70533
(60,100)	0.83077	0.83041	0.83102	0.83672	0.83078	0.77805
(100,10)	0.83077	0.81782	0.75691	0.70328	0.82944	0.98995
(100,40)	0.83077	0.82677	0.82648	0.80834	0.83051	0.88708
(100,60)	0.83077	0.82823	0.83021	0.82077	0.83063	0.90482
(100,100)	0.83077	0.82997	0.83072	0.83073	0.83072	0.26678

Table 12: MSE for $\hat{R}_{(s,k)}$ when (s,k)=(2,4) ,α=6, β=2 ,α_0=6.001, β_0=2.001, and $R_{(s,k)}$=0.83077.

(n,m)	$\hat{R}_{(s,k)_{mle}}$	$\hat{R}_{(s,k)_{sh1}}$	$\hat{R}_{(s,k)_{sh2}}$	$\hat{R}_{(s,k)_{th}}$	$\hat{R}_{(s,k)_{ls}}$	Best
(10,10)	0.0051207	0.0000154	0.0000511	0.000000005	0.0235218	Th
(10,40)	0.0027206	0.0000123	0.0001529	0.000000005	0.1334375	Th
(10,60)	0.0024825	0.0000079	0.0001788	0.000000007	0.3594812	Th
(10,100)	0.0023287	0.0000078	0.0002031	0.000000008	0.3631790	Th
(40,10)	0.0032663	0.0008284	0.0026465	0.00000025	0.0131703	Th
(40,40)	0.0011795	0.0000004	0.0000115	0.000000003	0.0165995	Th
(40,60)	0.0008828	0.0000002	0.0000304	0.0000000004	0.0117696	Th
(40,100)	0.0007847	0.0000003	0.0000837	0.000000002	0.0916246	Th
(60,10)	0.0032689	0.0020181	0.0062807	0.000000625	0.0205437	Th
(60,40)	0.0009344	0.0000024	0.0000686	0.000000013	0.0045250	Th
(60,60)	0.0007021	0.00000002	0.0000072	0.000000003	0.0237874	Th
(60,100)	0.0005722	0.00000008	0.0000395	0.0000000003	0.0032211	Th
(100,10)	0.0033354	0.0061780	0.0181158	0.00000203	0.0253611	Th
(100,40)	0.0008304	0.0000197	0.0005401	0.00000007	0.0042854	Th
(100,60)	0.0006368	0.0000003	0.0001137	0.00000002	0.0064072	Th
(100,100)	0.0004179	0.00000001	0.0000041	0.000000003	0.0033539	Th

4. Discussion Numerical Simulation Results

From the tables above , for all n=(10,40,60,100) and m=(10,40,60,100) we obtained that, the minimum (MSE) for reliability estimation of $R_{(s,k)}$ model for the Generalized Rayleigh Distribution GRD(α,1),held for shrinkage estimator ($\hat{R}_{(s,k)_{th}}$)by usingmodified Thompson type shrinkage weight factor(Th).This implies that, the shrinkage for reliability estimator ($\hat{R}_{(s,k)_{th}}$) is the best and follows by($\hat{R}_{(s,k)_{sh1}}$) using shrinkage weight function and then by($\hat{R}_{(s,k)_{sh2}}$) for any n and m.

5. Conclusion

From the numerical simulation results, the shrinkage estimator method remainedappropriate for estimation especially when use modified Thompson type shrinkage weight factor (Th) as a linear combination between unbiased estimator, and prior estimate. The result estimator ($\hat{R}_{(s,k)_{th}}$) for the reliability system of multicomponent in stress-strength model perform well and will be the best estimator than the others in the sense of MSE.

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