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On (3, k)-Regular Fuzzy Graphs

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Abstract: ***In this paper, we defined d_3-degree of a vertex in fuzzy graphs, total d_3 -degree of a vertex in fuzzy graphs, (3, k)-regular fuzzy graphs and totally (3, k) - regular fuzzy graphs. (3, k)-regular fuzzy graphs and totally (3, k)-regular fuzzy graphs are compared through various examples. A necessary and sufficient condition under which they are equivalent is provided. Also, (3, k)-regularity on some fuzzy graphs whose underlying crisp graphs are a path on six vertices P_6, a Corona graph $C_n \circ K_1$ (n≥5), a Wagner graph and a cycle C_n (n≥5) is studied with some membership function. Some properties of (3, k)-regular fuzzy graphs studied and they are examined for totally (3, k)-regular fuzzy graphs.***

Keywords: d_3- degree of a vertex in fuzzy graphs, total d_3- degree of a vertex in fuzzy graphs, (3, k)-regular fuzzy graphs, totally (3, k)-regular fuzzy graphs.

1. Introduction

Azriel Rosenfeld introduced fuzzy graphs in 1975. It has been growing fast and has numerous applications in various fields. Nagoor Gani and Radha introduced regular fuzzy graphs, total degree and totally regular fuzzy graphs, we call it as (3, k)-regular graphs and studied some properties on (3, k)-regular graphs. N.R. Santhi Maheswari and C. Sekar introduced d_2 of a vertex in graph and also discussed some properties on d_2 of a vertex in graphs. In this paper, we define d_3 -degree of a vertex in fuzzy graphs and total d_3 -degree of a vertex in fuzzy graphs and introduce (3, k)-regular fuzzy graphs, totally (3, k) -regular fuzzy graphs. We make comparative study between (3, k) -regular fuzzy graphs and totally (3, k) - regular fuzzy graphs. We provide a necessary and sufficient condition under which they become equivalent. A characterization of (3, k)-regular graphs on a path on six vertices P_6, a Corona graph $C_n \circ K_1$ (n≥5), a Wagner graph and a cycle C_n (n≥5) is provided.

We present some known definitions and results for a ready reference to go through the work presented in this paper.

2. Preliminaries

Definition 2.1: For a given graph G, the d_3-degree of a vertex v in G, denoted by $d_3(v)$ means number of vertices at a distance three away from v.

Definition 2.2: A graph G is said to be (3, k)-regular (d_3 – regular) if $d_3(v)$ = k, for all v in G.
We observe that (3, k)-regular and d_3-regular graphs are same.

Definition 2.3: A fuzzy graph G is a pair of functions G: (σ, μ) where σ: V → [0, 1] is a fuzzy subset of a non empty set V and μ:V x V → [0,1] is symmetric fuzzy relation on σ such that for all u, v in V the relation $\mu(uv) \le \sigma(u) \wedge \sigma(v)$ is satisfied.
A fuzzy graph G is complete if $\mu(uv) = \sigma(u) \wedge \sigma(u)$ for all u,v∈V where uv denotes the edge between u and v.

Definition 2.4: Let G: (σ, μ) be fuzzy graph. The degree of a vertex u is $d_G(u)= \sum_{u\neq v} \mu(uv)$ for uv ∈ E and μ(uv)=0 for uv not in E, this equivalent to $d_G(u)= \sum_{uv\in E} \mu(uv)$

The minimum degree of G is $\delta(G) = \wedge \{d(v): v\in V\}$.
The maximum degree of G is $\Delta(G) = \vee \{d(v): v\in V\}$.
The order of a fuzzy graph is $O(G) = \sum \sigma(u)$.

Definition 2.5: The strength of connectedness between two vertices u and v is $\mu^{\infty}(uv)=\sup\{\mu^k(uv)\ /k = 1,2,\ldots\}$ where (uv)= $\sup\{\mu(uu_1)\wedge\mu(u_1u_2)\wedge\ldots\wedge\mu(u_{k-1}v) | u_1,u_2,\ldots u_{k-1} \in V\}$.

Definition 2.6: Let G: (σ, μ) be fuzzy graph on G^*:(V, E). If d(v) = k for all v∈V, then G is said to be regular fuzzy graph of degree k.

Definition 2.7: Let G: (σ, μ) be fuzzy graph on G^*:(V, E). If d(v) ≠ k for all v∈V, then G is said to be irregular fuzzy graph .

Definition 2.8: Let G: (σ,μ) be fuzzy graph on G^*:(V,E). The total degree of a vertex u is defined as $td(u)= \sum \mu(uv)+ \sigma(u) = d(u)+ \sigma(u)$, uv∈E.

If each vertex of G has the same total degree k, then G is said to be totally regular fuzzy graph of degree k or k-totally regular fuzzy graph.

If each vertex of G has not the same total degree k, then G is said to be totally irregular fuzzy graph.

Definition 2.9: The Wagner graph is a 3-regular with 8 vertices and 12 edges.

Definition 2.10: The Corona of two graphs G_1 and G_2 is the graph $G = G_1 \circ G_2$ formed from one copy of G_1 and $|v(G_1)|$ copies of G_2 where i^{th} vertex of G_1 is adjacent to every vertex in the i^{th} copy of G_2.

[Note that G_1 is a cycle of length ≥ 5 and G_2 is K_1].

Example 2.11:

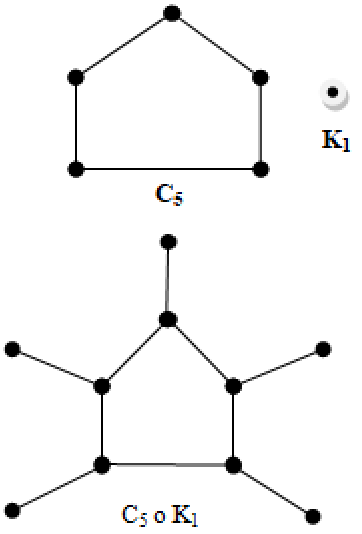

Figure 1

Remark 2.12: Let G_1: (σ_1, μ_1) and G_2: (σ_2, μ_2) denote two fuzzy graphs. Let G^*_1: (V_1,E_1) and G^*_2: (V_2, E_2) be underlying crisp graph such that $|V_i|=p_i$, i=1,2. Also $d_{G_i*}(u_i)$ denote degree of u_i in G_i^*.

3. d_3-degree and total d_3-degree of a vertex in fuzzy graphs

Definition 3.1: Let G: (σ, μ) be a fuzzy graph. The d_3-degree of a vertex u is $d_3(u) = \sum \mu^3(uv)$ where $\mu^3(uv)$ = $\{\ \mu(uu_1) \wedge \mu(u_1u_2) \wedge \mu(u_2v) |\ u, u_1, u_2, v\}$ is the shortest path connecting u and v of length 3. Also μ(uv)=0, for uv not in E.

The minimum d_3 degree of G is δ_3 (G) = ∧ {d_3 (v): v∈V}.
The maximum d_3 degree of G is Δ_3 (G) = ∨ {d_3 (v): v∈V}.

Example 3.2: Consider G*: (V, E) where V = { v_1, v_2, v_3, v_4, v_5} and E = { v_1v_2,v_2v_3,v_3v_4, v_4v_5,v_5v_1}. Define G : (σ, μ) by σ(v_1) = 0.3, σ (v_2) = 0.4, σ (v_3) =0.7, σ (v_4) = 0.2, σ (v_5) = 0.6 and μ(v_1v_2) = 0.3, μ(v_2v_3) = 0.4, μ(v_3v_4) = 0.1, μ(v_4v_5) =0.2, μ(v_5v_1) =0.3.

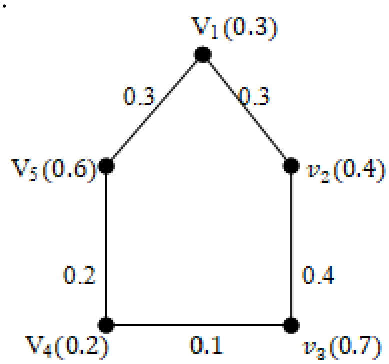

Figure 2

Here, $d_3(v_1)$ = {0.3 ∧ 0.2 ∧ 0.1} + {0.3 ∧ 0.4 ∧ 0.1 }
= 0.1 + 0.1 = 0.2.
$d_3(v_2)$ = {0.3 ∧ 0.3 ∧ 0.2} + {0.4 ∧ 0.1 ∧ 0.2 }
= 0.2 + 0.1 = 0.3.
$d_3(v_3)$ = {0.4 ∧ 0.3 ∧ 0.3} + {0.1 ∧ 0.2 ∧ 0.3 }
= 0.3 + 0.1 = 0.4.
$d_3(v_4)$ = {0.1 ∧ 0.4 ∧ 0.3} + {0.2 ∧ 0.3 ∧ 0.3 }
= 0.1 + 0.2 = 0.3.
$d_3(v_5)$ = {0.2 ∧ 0.1 ∧ 0.4} + {0.3 ∧ 0.3 ∧ 0.4 }
= 0.1 + 0.3 = 0.4.

Example 3.3: Consider G*: (V, E) where V = { v_1, v_2, v_3, v_4, v_5,v_6} and E = { v_1v_2,v_2v_3,v_3v_4, v_4v_5,v_5v_6,v_6v_1}. Define G : (σ, μ) by σ(v_1) = 0.3, σ (v_2) = 0.4, σ (v_3) =0.7, σ (v_4) = 0.5, σ (v_5) = 0.2, σ (v_6) = 0.6 and μ(v_1v_2) = 0.3, μ(v_2v_3) = 0.4, μ(v_3v_4) = 0.4, μ(v_4v_5) =0.2, μ(v_5v_6) =0.2, μ(v_6v_1) =0.3.

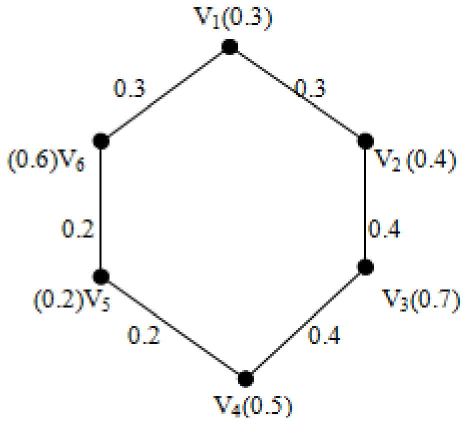

Figure 3

Here, $d_3(v_1)$ = {0.3 ∧ 0.2 ∧ 0.2} + {0.3 ∧ 0.4 ∧ 0.4 }
= 0.3 + 0.2 = 0.5.
$d_3(v_2)$ = {0.3 ∧ 0.3 ∧ 0.2} + {0.4 ∧ 0.4 ∧ 0.2 }
= 0.2 + 0.2 = 0.4.
$d_3(v_3)$ = {0.4 ∧ 0.3 ∧ 0.3} + {0.4 ∧ 0.2 ∧ 0.2 }
= 0.3 + 0.2 = 0.5.
$d_3(v_4)$ = {0.4 ∧ 0.4 ∧ 0.3} + {0.2 ∧ 0.2∧ 0.3 }
= 0.3 + 0.2 = 0.5.
$d_3(v_5)$ = {0.2 ∧ 0.4 ∧ 0.4} + {0.2 ∧ 0.3 ∧ 0.3 }
= 0.2 + 0.2 = 0.4.
$d_3(v_6)$ = {0.2 ∧ 0.2 ∧ 0.4} + {0.3 ∧ 0.3 ∧ 0.4 }
= 0.2 + 0.3 = 0.5.

Definition 3.4: Let G: (σ, μ) be fuzzy graph on G*:(V, E). The total d_3 -degree of a vertex u ∈V is defined as $td_3(u)$ = $\sum \mu^3(uv) + \sigma(u)$=$d_3$(u)+ σ(u).

The minimum td_3-degree of G is $t\delta_3$ (G) = ∧ {td_3(v): v∈V}.
The maximum td_3-degree of G is $t\Delta_3$ (G) = ∨ {td_3 (v): v∈V}.

Example 3.5: Consider G*: (V, E) where V = {v_1, v_2, v_3, v_4, v_5} and E = { v_1v_2, v_2v_3, v_3v_4, v_4v_5,v_5v_1}. Define G : (σ, μ) by σ(v_1) = 0.5, σ (v_2) = 0.6, σ (v_3) =0.5, σ (v_4) = 0.6, σ (v_5) = 0.5 and μ(v_1v_2) = 0.2, μ(v_2v_3) = 0.2, μ(v_3v_4) = 0.3, μ(v_4v_5) =0.4, μ(v_5v_1) =0.4.

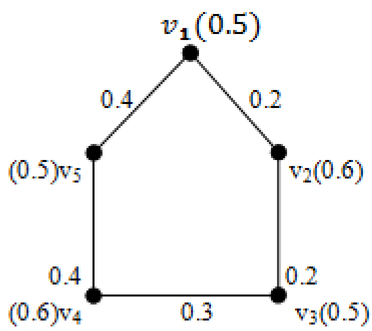

Figure 4

Here, $d_3(v_1)$ = {0.4 ∧ 0.4 ∧ 0.3} + {0.2 ∧ 0.2 ∧ 0.3 }

$= 0.2 + 0.3 = 0.5$

$d_3(v_2) = \{0.2 \wedge 0.4 \wedge 0.4\} + \{0.2 \wedge 0.3 \wedge 0.4\}$
$= 0.2 + 0.2 = 0.4.$

$d_3(v_3) = \{0.2 \wedge 0.2 \wedge 0.4\} + \{0.3 \wedge 0.4 \wedge 0.4\}$
$= 0.3 + 0.2 = 0.5.$

$d_3(v_4) = \{0.3 \wedge 0.2 \wedge 0.2\} + \{0.4 \wedge 0.4 \wedge 0.2\}$
$= 0.2 + 0.2 = 0.4.$

$d_3(v_5) = \{0.4 \wedge 0.3 \wedge 0.2\} + \{0.4 \wedge 0.2 \wedge 0.2\}$
$= 0.2 + 0.2 = 0.4.$

$t\,d_3(v_1) = d_3(v_1) + \sigma(v_1) = 0.5 + 0.5 = 1.0.$
$td_3(v_2) = d_3(v_2) + \sigma(v_2) = 0.4 + 0.6 = 1.0.$
$td_3(v_3) = d_3(v_3) + \sigma(v_3) = 0.5 + 0.5 = 1.0.$
$td_3(v_4) = d_3(v_4) + \sigma(v_4) = 0.4 + 0.6 = 1.0$
$td_3(v_5) = d_3(v_5) + \sigma(v_5) = 0.5 + 0.5 = 1.0.$

4. (3, K)-regular and totally (3, K)-regular fuzzy graphs

Definition 4.1: Let G: (σ, μ) be fuzzy graph on G*:(V, E). If $d_3(v)=k$ for all u∈V, then G is said to be (3,K) – regular fuzzy graph.

Example 4.2: Consider G*:(V, E), where V = $\{v_1, v_2, v_3, v_4, v_5, v_6\}$ and E = $\{v_1v_2, v_2v_3, v_3v_4, v_4v_5, v_5v_6\}$. Define G: (σ, μ) by $\sigma(v_1) = 0.3$, $\sigma(v_2) = 0.4$, $\sigma(v_3) = 0.5$, $\sigma(v_4) = 0.6$, $\sigma(v_5) = 0.7$, $\sigma(v_6) = 0.8$ and $\mu(v_1v_2) = 0.3$, $\mu(v_2v_3) = 0.3$, $\mu(v_3v_4) = 0.3$, $\mu(v_4v_5) = 0.3$, $\mu(v_5v_6) = 0.3$.

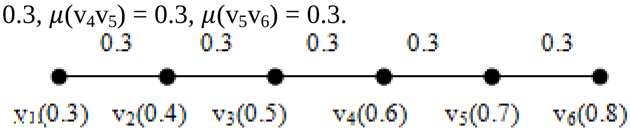

Figure 5

Here, $d_3(v_1) = 0.3$, $d_3(v_2) = 0.3$, $d_3(v_3) = 0.3$, $d_3(v_4) = 0.3$, $d_3(v_5) = 0.3$, $d_3(v_6) = 0.3$. G is (3, 0.3)- regular fuzzy graph.

Example 4.3: Consider G*:(V, E), where V = $\{v_1, v_2, v_3, v_4, v_5\}$ and E = $\{v_1v_2, v_2v_3, v_3v_4, v_4v_5, v_5v_1\}$. Define G: (σ, μ) by $\sigma(v_1) = 0.3$, $\sigma(v_2) = 0.4$, $\sigma(v_3) = 0.5$, $\sigma(v_4) = 0.6$, $\sigma(v_5) = 0.7$ and $\mu(v_1v_2) = 0.3$, $\mu(v_2v_3) = 0.4$, $\mu(v_3v_4) = 0.3$, $\mu(v_4v_5) = 0.6$, $\mu(v_5v_1) = 0.3$.

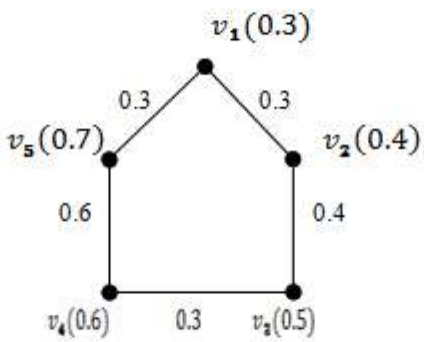

Figure 6

Here, $d_3(v_1) = 0.6$, $d_3(v_2) = 0.6$, $d_3(v_3) = 0.6$, $d_3(v_4) = 0.6$, $d_3(v_5) = 0.6$, $d_3(v_6) = 0.6$. So G is (3, 0.6)- regular fuzzy graph.

Example 4.4: Consider G* :(V, E), where V = $\{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8\}$ and E = $\{v_1v_2, v_2v_3, v_3v_4, v_4v_5, v_5v_6, v_6v_7, v_7v_8, v_8v_1\}$. Define G: (σ, μ) by $\sigma(v_1) = 0.2$, $\sigma(v_2) = 0.3$, $\sigma(v_3) = 0.4$, $\sigma(v_4) = 0.5$, $\sigma(v_5) = 0.6$, $\sigma(v_6) = 0.7$, $\sigma(v_7) = 0.8$, $\sigma(v_8) = 0.9$ and $\mu(v_1v_2) = 0.2$, $\mu(v_2v_3) = 0.3$, $\mu(v_3v_4) = 0.2$, $\mu(v_4v_5) = 0.3$, $\mu(v_5v_6) = 0.2$, $\mu(v_6v_7) = 0.3$, $\mu(v_7v_8) = 0.2$

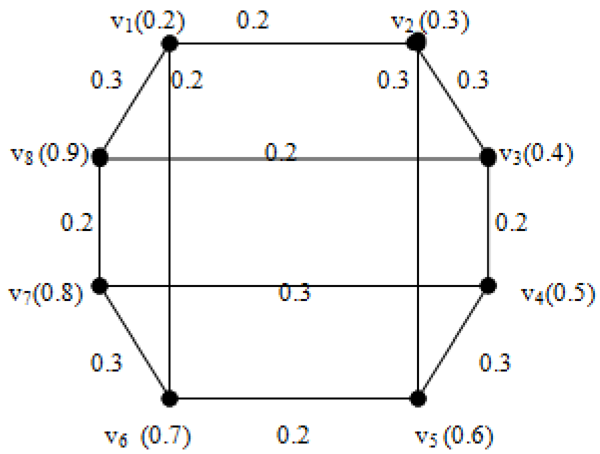

Figure 7

Here, $d_3(v_1) = \{\mu(v_1v_8)\wedge\mu(v_8v_7)\wedge\mu(v_7v_4)\} + \{\mu(v_1v_8)\wedge\mu(v_8v_3)\wedge\mu(v_3v_4)\} + \{\mu(v_1v_2)\wedge\mu(v_2v_3)\wedge\mu(v_3v_4)\} + \{\mu(v_1v_2)\wedge\mu(v_2v_5)\wedge\mu(v_5v_4)\} + \{\mu(v_1v_6)\wedge\mu(v_6v_5)\wedge\mu(v_5v_4)\}$.

$d_3(v_1) = \{0.3\wedge 0.2 \wedge 0.3\} + \{0.3 \wedge 0.2 \wedge 0.2\} + \{0.2\wedge + \{0.2 \wedge 0.3 \wedge 0.3\} + \{0.2 \wedge 0.2 \wedge 0.3\}$
$= 0.2 + 0.2 + 0.2 + 0.2 + 0.2$
$= 1.0$

$d_3(v_2) = \{0.2 \wedge 0.3 \wedge 0.2\} + \{0.2 \wedge 0.2 \wedge 0.3\} + \{0.3 \wedge 0.2 \wedge 0.2\} + \{0.3 \wedge 0.2 \wedge 0.3\} + \{0.3 \wedge 0.2 \wedge 0.3\}$
$= 0.2 + 0.2 + 0.2 + 0.2 + 0.2 = 1.0$

$d_3(v_3) = \{0.3 \wedge 0.2 \wedge 0.2\} + \{0.3 \wedge 0.3 \wedge 0.2\} + \{0.2 \wedge 0.3 \wedge 0.2\} + \{0.2 \wedge 0.3 \wedge 0.3\} + \{0.2 \wedge 0.2 \wedge 0.3\}$
$= 0.2 + 0.2 + 0.2 + 0.2 + 0.2 = 1.0$

$d_3(v_4) = \{0.2 \wedge 0.3 \wedge 0.2\} + \{0.2 \wedge 0.2 \wedge 0.3\} + \{0.3 \wedge 0.3 \wedge 0.2\} + \{0.3 \wedge 0.2 \wedge 0.2\} + \{0.3 \wedge 0.2 \wedge 0.3\}$
$= 0.2 + 0.2 + 0.2 + 0.2 + 0.2 = 1.0$

$d_3(v_5) = \{0.3 \wedge 0.3 \wedge 0.2\} + \{0.3 \wedge 0.2 \wedge 0.2\} + \{0.2 \wedge 0.3 \wedge 0.2\} + \{0.2 \wedge 0.2 \wedge 0.3\} + \{0.3 \wedge 0.2 \wedge 0.3\}$
$= 0.2 + 0.2 + 0.2 + 0.2 + 0.2 = 1.0$

$d_3(v_6) = \{0.2 \wedge 0.3 \wedge 0.2\} + \{0.2 \wedge 0.3 \wedge 0.3\} + \{0.3 \wedge 0.2 \wedge 0.2\} + \{0.3 \wedge 0.3 \wedge 0.2\} + \{0.2 \wedge 0.2 \wedge 0.3\}$
$= 0.2 + 0.2 + 0.2 + 0.2 + 0.2$
$= 1.0$

$d_3(v_7) = \{0.3 \wedge 0.2 \wedge 0.3\} + \{0.3 \wedge 0.2 \wedge 0.2\} + \{0.2 \wedge 0.3 \wedge 0.2\} + \{0.2 \wedge 0.2 \wedge 0.3\} + \{0.3 \wedge 0.2 \wedge 0.3\}$
$= 0.2 + 0.2 + 0.2 + 0.2 + 0.2 = 1.0$

$d_3(v_8) = \{0.2 \wedge 0.3 \wedge 0.2\} + \{0.2 \wedge 0.3 \wedge 0.3\} + \{0.3 \wedge 0.2 \wedge 0.2\} + \{0.3 \wedge 0.2 \wedge 0.3\} + \{0.2 \wedge 0.2 \wedge 0.3\}$
$= 0.2 + 0.2 + 0.2 + 0.2 + 0.2$
$= 1.0.$

So G is (3, 1.0)- regular fuzzy graph.

Example 4.5: Consider G* :(V, E), where V = $\{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}\}$ and E = $\{v_1v_2, v_2v_3, v_3v_4, v_3v_5, v_5v_6, v_5v_7, v_7v_8, v_7v_9, v_9v_{10}, v_9v_2\}$. Define G: (σ, μ) by $\sigma(v_1) = 0.2$, $\sigma(v_2) = 0.3$, $\sigma(v_3) = 0.4$, $\sigma(v_4) = 0.5$, $\sigma(v_5) = 0.6$, $\sigma(v_6) = 0.7$, $\sigma(v_7) = 0.8$, $\sigma(v_8) = 0.9$, $\sigma(v_9) = 0.4$, $\sigma(v_{10}) = 0.6$ and $\mu(v_1v_2) = 0.2$, $\mu(v_2v_3) = 0.2$, $\mu(v_3v_4) = 0.3$, $\mu(v_3v_5) = 0.3$, $\mu(v_5v_6) = 0.2$, $\mu(v_5v_7) = 0.2$, $\mu(v_7v_8) = 0.3$, $\mu(v_7v_9) = 0.3$, $\mu(v_9v_{10}) = 0.2$, $\mu(v_9v_2) = 0.2$.

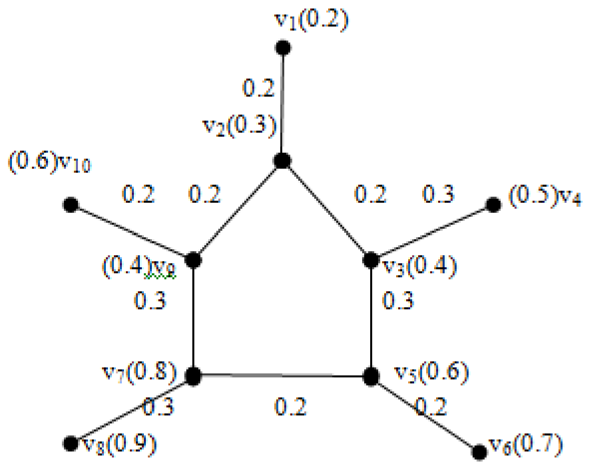

Figure 8

Here, $d_3(v_1) = \{\mu(v_1v_2)\wedge\mu(v_2v_3)\wedge\mu(v_3v_4)\} + \{\mu(v_1v_2)\wedge \mu(v_2v_3)\wedge\mu(v_3v_5)\} + \{\mu(v_1v_2)\wedge \mu(v_2v_9)\wedge \mu(v_9v_7)\} + \{\mu(v_1v_2)\wedge\mu(v_2v_9)\wedge\mu(v_9v_{10})\}$.

$d_3(v_1) = \{0.2\wedge0.2\wedge0.3\} + \{0.2\wedge0.2\wedge0.3\} + \{0.2\wedge0.2\wedge0.3\} + \{0.2\wedge0.2\wedge0.2\} = 0.2 + 0.2 + 0.2 + 0.2 = 0.8.$

$d_3(v_2) = \{0.2\wedge0.3\wedge0.2\} + \{0.2\wedge0.3\wedge0.2\} + \{0.2\wedge0.3\wedge 0.2\} + \{0.2 \wedge0.3\wedge0.3\} = 0.2 + 0.2 + 0.2 + 0.2 = 0.8.$

$d_3(v_3) = \{0.3 \wedge 0.2 \wedge 0.3\} + \{0.3 \wedge 0.2 \wedge 0.3\} + \{0.2\wedge0.2\wedge 0.2\} + \{0.2\wedge0.2 \wedge0.3\} = 0.2 + 0.2 + 0.2 + 0.2 = 0.8.$

$d_3(v_4) = \{0.3 \wedge0.2\wedge0.2\} + \{0.3\wedge0.2\wedge0.2\} + \{0.3\wedge0.3 \wedge 0.2\} + \{0.3 \wedge 0.3 \wedge 0.2\} = 0.2 + 0.2 + 0.2 + 0.2 = 0.8.$

$d_3(v_5) = \{0.3 \wedge0.2\wedge0.2\} + \{0.3\wedge0.2\wedge0.2\} + \{0.2\wedge0.3\wedge 0.2\} + \{0.2 \wedge 0.3 \wedge 0.2\} = 0.2 + 0.2 + 0.2 + 0.2 = 0.8.$

$d_3(v_6) = \{0.2\wedge0.3\wedge0.2\} + \{0.2\wedge0.3\wedge0.3\} + \{0.2\wedge0.2 \wedge 0.3\} + \{0.2 \wedge 0.2 \wedge 0.3\} = 0.2 + 0.2 + 0.2 + 0.2 = 0.8.$

$d_3(v_7) = \{0.3\wedge0.2\wedge0.2\} + \{0.3\wedge0.2\wedge0.2\} + \{0.2\wedge0.3\wedge 0.2\} + \{0.2 \wedge 0.3 \wedge 0.3\} = 0.2 + 0.2 + 0.2 + 0.2 = 0.8.$

$d_3(v_8) = \{0.3\wedge0.3\wedge0.2\} + \{0.3\wedge0.2\wedge0.3\} + \{0.3\wedge0.2\wedge 0.2\} + \{0.3\wedge0.3\wedge0.2\} = 0.2 + 0.2 + 0.2 + 0.2 = 0.8.$

$d_3(v_9) = \{0.2\wedge0.2\wedge0.3\} + \{0.3\wedge0.2\wedge0.3\} + \{0.2\wedge 0.2\wedge0.3\} + \{0.3 \wedge 0.2 \wedge 0.2\} = 0.2 + 0.2 + 0.2 + 0.2 = 0.8.$

$d_3(v_{10}) = \{0.2\wedge0.2\wedge0.2\} + \{0.2\wedge0.3\wedge0.2\} + \{0.2\wedge0.3\wedge 0.3\} + \{0.2 \wedge 0.2 \wedge 0.2\} = 0.2 + 0.2 + 0.2 + 0.2 = 0.8.$

So G is (3, 0.8)- regular fuzzy graph.

Definition 4.6: If each vertex of G has the same total d_3 -degree k, then G is said to be totally (3, k)-regular fuzzy graph.

Example 4.7: 1. A totally (3, k)-regular fuzzy graph need not be a (3, k)-regular fuzzy graph. Consider G^*: (V, E) where V = $\{v_1, v_2, v_3, v_4, v_5\}$ and E = $\{v_1v_2, v_2v_3, v_3v_4, v_4v_5, v_5v_1\}$.

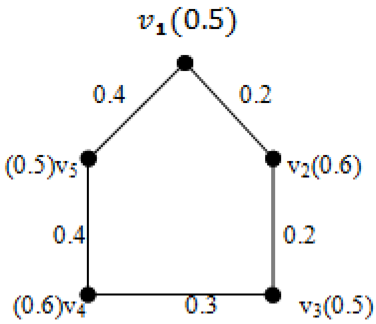

Figure 9

Here, $d_3(v_1) = 0.5$, $d_3(v_2) = 0.4$, $d_3(v_3) = 0.5$, $d_3(v_4) = 0.4$, $d_3(v_5) = 0.4$ and $td_3(v_1) = 1.0$, and $td_3(v_2) = 1.0$, $td_3(v_3) = 1.0$, $td_3(v_4) = 1.0$, $td_3(v_5) = 1.0$. Each vertex has same total d_3-degree 1. Hence G is totally (3, 1)-regular fuzzy graph. But G is not (3, k)-regular fuzzy graph.

2. A (3, k)-regular fuzzy graph need not be a totally (3, k)-regular fuzzy graph.
Consider G^*: (V, E) where V = $\{v_1, v_2, v_3, v_4, v_5, v_6\}$ and E = $\{v_1v_2, v_2v_3, v_3v_4, v_4v_5, v_5v_6, v_6v_1\}$.

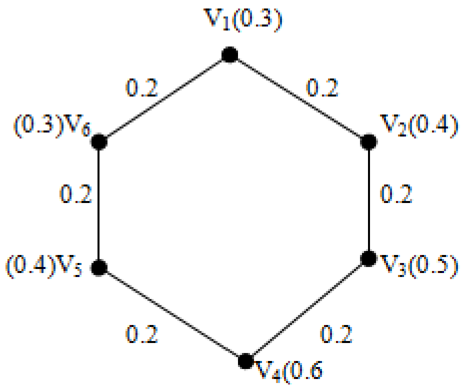

Figure 10

Here, $d_3(v_1) = 0.4$, $d_3(v_2) = 0.4$, $d_3(v_3) = 0.4$, $d_3(v_4) = 0.4$, $d_3(v_5) = 0.4$, $d_3(v_6) = 0.4$, and $t\,d_3(v_1) = 0.7$, $t\,d_3(v_2) = 0.8$, $t\,d_3(v_3) = 0.9$, $t\,d_3(v_4) = 1.0$, $t\,d_3(v_5) = 0.8$, $t\,d_3(v_6) = 0.7$. Each vertex has same d_3-degree 0.4. So, G is (3, 0.4)-regular fuzzy graph. But G is not a totally (3, k)-regular fuzzy graph.

3. A (3, k)-regular fuzzy graph which is totallt (3, k)-regular fuzzy graph.
Consider G^*: (V, E) where V = $\{v_1, v_2, v_3, v_4, v_5, v_6\}$ and E = $\{v_1v_2, v_2v_3, v_3v_4, v_4v_5, v_5v_6, v_6v_1\}$. Define G : (σ, μ) by $\sigma(v_1) = 0.5$, $\sigma(v_2) = 0.5$, $\sigma(v_3) = 0.5$, $\sigma(v_4) = 0.5$, $\sigma(v_5) = 0.5$, $\sigma(v_6) = 0.5$ and $\mu(v_1v_2) = 0.4$, $\mu(v_2v_3) = 0.4$, $\mu(v_3v_4) = 0.4$, $\mu(v_4v_5) = 0.4$, $\mu(v_5v_6) = 0.4$, $\mu(v_6v_1) = 0.4$.

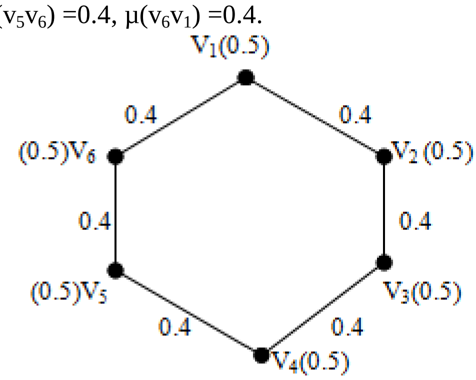

Figure 11

Here, $d_3(v_1) = 0.8$, $d_3(v_2) = 0.8$, $d_3(v_3) = 0.8$, $d_3(v_4) = 0.8$, $d_3(v_5) = 0.8$, $d_3(v_6) = 0.8$ and $td_3(v_1) = 1.3$, $td_3(v_2) = 1.3$, $td_3(v_3) = 1.3$, $td_3(v_4) = 1.3$, $td_3(v_5) = 1.3$, $td_3(v_6) = 1.3$. Each vertex has the same d_3-degree 0.8. So, G is a (3, 0.8)-regular fuzzy graph. Each vertex has the same total d_3-degree 1.3. Hence G is a totally (3, 1.3)-regular fuzzy graph.

Theorem 4.8: Let G : (σ, μ) be fuzzy graph on G^*: (V, E). Then $\sigma(u) = c$, for all $u \in V$ if and only if the following conditions are equivalent.
1. G : (σ, μ) is (3, k)-regular fuzzy graph.
2. G : (σ, μ) is totally (3, k + c)-regular fuzzy graph.

Proof: Suppose that $\sigma(u) = c$, for all $u \in V$.
It is assumed that G : (σ, μ) is a (3, k)-regular fuzzy graph.
Then $d_3(u) = k$, for all $u \in V$.
So $td_3(u) = d_3(u) + \sigma(u)$, for all $u \in V$
$\Rightarrow t\, d_3(u) = k + c$, for all $u \in V$
Hence G : (σ, μ) is a totally (3, k + c)-regular fuzzy graph.
Thus, (1) $\Rightarrow$ (2) is proved.
Suppose G : (σ, μ) is a totally (3, k + c)-regular fuzzy graph.
Then, $t\, d_3(u) = k + c$, for all $u \in V$.
$\Rightarrow d_3(u) + \sigma(u) = k + c$, for all $u \in V$.
$\Rightarrow d_3(u) + c = k + c$, for all $u \in V$.
$\Rightarrow d_3(u) = k$, for all $u \in V$.
Hence G : (σ, μ) is a (3, k)-regular fuzzy graphs.
Thus, (2) $\Rightarrow$ (1) is proved.
Hence (1) and (2) are equivalent.
Conversely, it is assumed that (1) and (2) are equivalent.

Suppose that σ is not a constant function. Then $\sigma(u) \neq \sigma(v)$, for atleast one pair $u,v \in V$. Let G be a (3,k)-regular fuzzy graph. Then $d_3(u) = d_3(v) = k$. So $td_3(u) = d_3(u) + \sigma(u) = k + \sigma(u)$ and $td_3(v) = d_3(v) + \sigma(v) = k + \sigma(v)$.
Since $\sigma(u) \neq \sigma(v)$, $\Rightarrow k + \sigma(u) \neq k + \sigma(v)$.
$\Rightarrow td_3(u) \neq td_3(v)$.
So,G is not totally (3,k)-regular fuzzy graph. Which is contradiction to our assumption.
Let G be a totally (3,k)-regular fuzzy graph.
Then $td_3(u) = td_3(v)$. $\Rightarrow d_3(u) + \sigma(u) = d_3(v) + \sigma(v)$.
$\Rightarrow d_3(u) - d_3(v) = \sigma(v) - \sigma(u) \neq 0$.
$\Rightarrow d_3(u) \neq d_3(v)$.
So,G is not (3, k)-regular fuzzy graph.Which is contradiction to our assumption.
Hence σ is a constant function.

Theorem 4.9: If a fuzzy graph G is both (3, k)-regular and totally (3, k)-regular then σ is constant function.
Proof: Let G be (3, k_1) -regular and totally (3, k_2) –regular fuzzy graph.
Then $d_3(u) = k_1$ and $td_3(u) = k_2$, for all $u \in V$.
Now $td_3(u) = k_2$, for all $u \in V$.
$\Rightarrow d_3(u) + \sigma(u) = k_2$, for all $u \in V$.
$\Rightarrow k_1 + \sigma(u) = k_2$, for all $u \in V$.
$\Rightarrow \sigma(u) = k_2 - k_1$, for all $u \in V$.
Hence σ is a constant function.

Remark 4.10: The converse of theorem 4.9 is not true. Consider G*: (V,E) where $V = \{v_1, v_2, v_3, v_4, v_5\}$ and $E = \{v_1v_2, v_2v_3, v_3v_4, v_4v_5, v_5v_1\}$. Define G : (σ, μ) by $\sigma(v_1) = 0.5$, $\sigma(v_2) = 0.5$, $\sigma(v_3) = 0.5$, $\sigma(v_4) = 0.5$, $\sigma(v_5) = 0.5$ and $\mu(v_1v_2) = 0.2$, $\mu(v_2v_3) = 0.2$, $\mu(v_3v_4) = 0.3$, $\mu(v_4v_5) = 0.4$, $\mu(v_5v_1) = 0.4$.

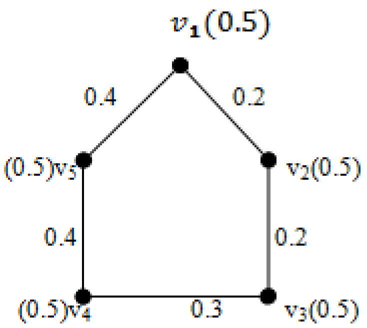

Figure 12

Here, $d_3(v_1) = 0.5$, $d_3(v_2) = 0.4$, $d_3(v_3) = 0.5$, $d_3(v_4) = 0.4$, $d_3(v_5) = 0.4$ and $td_3(v_1) = 1.0$, $td_3(v_2) = 0.9$, $td_3(v_3) = 1.0$, $td_3(v_4) = 0.9$, $td_3(v_5) = 0.9$. Here, σ is a constant function. But G is neither (3, k)-regular fuzzy graph nor totally (3, k)-regular fuzzy graph.

5. (3, k)- regular fuzzy graphs on a path on 6 vertices with some specific membership functions

In this section (3, k)-regularity and totally (3, k)-regularity on fuzzy graph whose underlying crisp graph is a path on 6 vertices is studied with some specific membership functions.

Theorem 5.1: Let G: (σ, μ) be a fuzzy graph such that G^*: (V, E) a path on 6 vertices. If μ is constant function, then G is (3, c)-regular fuzzy graph.

Proof: Suppose that μ is a constant function, say $\mu(uv) = c$, for all $uv \in E$. Then $d_3(v) = c, \forall v \in V$.
Hence G is a (3, c)- regular fuzzy graph.

Remark 5.2: Converse the theorem 5.1 need not be true. For example, Consider G:(σ, μ) be a fuzzy graph on G^*: (V, E) is a path on 6 vertices.

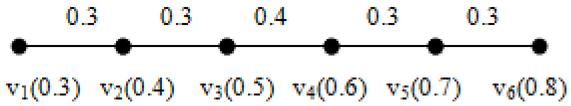

Figure 13

Here, $d_3(v_1) = 0.3$, $d_3(v_2) = 0.3$, $d_3(v_3) = 0.3$, $d_3(v_4) = 0.3$, $d_3(v_5) = 0.3$, $d_3(v_6) = 0.3$. Hence G is (3, 0.3) - regular fuzzy graph. But μ is not a constant function.

Theorem 5.3: Let G: (σ, μ) be fuzzy graph such that G*: (V, E) is a path on six vertices. If alternate edges have same membership values, then G is (3, k)-regular fuzzy graph, where $k = \min\{c_1, c_2\}$.

Proof If alternate edges have same membership values, then
$$\mu(e_i) = \begin{cases} c_1, & if\ i\ is\ odd \\ c_2 & if\ i\ is\ even \end{cases}.$$
If $c_1 = c_2$, then μ is constant function. So G is (3, c_1)-regular fuzzy graph.
If $c_1 < c_2$, then $d_3(v) = c_1$, for all $v \in V$. So G is (3, c_1)-regular fuzzy graph.
If $c_1 > c_2$, then $d_3(v) = c_2$, for all $v \in V$. So G is (3, c_2)-regular fuzzy graph.

Theorem 5.4: Let G: (σ, μ) be a fuzzy graph such that G^*: (V, E) is a path on 6 vertices. If middle edge have membership value less than membership value of the remaining edges, then G is (3, k)- regular fuzzy graph, where k= membership value of the middle edge.
For example, Consider G:(σ, μ) be a fuzzy graph such that G^*: (V, E) a path on 6 vertices.

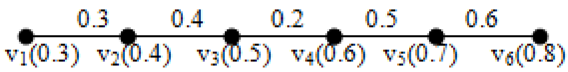

Figure 14

Here, $d_3(v_1) = 0.2$, $d_3(v_2) = 0.2$, $d_3(v_3) = 0.2$, $d_3(v_4) = 0.2$, $d_3(v_5) = 0.2$, $d_3(v_6) = 0.2$. Hence G is (3, 0.2) - regular fuzzy graph.

Remark 5.5: If σ is not a constant function, then the (3, k)-regular fuzzy graphs in the above theorems 5.1, 5.3 and 5.4 are not totally (3, k)- regular fuzzy graphs.
For example, Consider $G:(\sigma, \mu)$ be a fuzzy graph such that G^*: (V, E) is a path on 6 vertices.

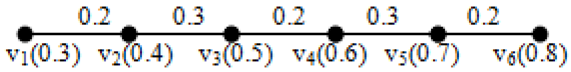

Figure 15

Here, $d_3(v_1) = 0.2$, $d_3(v_2) = 0.2$, $d_3(v_3) = 0.2$, $d_3(v_4) = 0.2$, $d_3(v_5) = 0.2$, $d_3(v_6) = 0.2$ and $td_3(v_1) = 0.5$, $td_3(v_2) = 0.6$, $td_3(v_3) = 0.7$, $td_3(v_4) = 0.8$, $td_3(v_5) = 0.9$, $td_3(v_6) = 1.0$. Hence G is (3, 0.2) - regular fuzzy graph but not totally (3, k)-regular fuzzy graph.

6. (3, k) – regular fuzzy graphs on Wagner graph with some specific membership function

In this section (3, k)-regularity and totally (3, k)-regularity on fuzzy graph whose underlying crisp graph is a Wagner graph is studied with some specific membership function

Theorem 6.1: Let G: (σ, μ) be fuzzy graph such that G^*: (V, E) is Wagner graph. If μ is constant function, then G is (3, k) - regular fuzzy graph.

Proof: Suppose that, μ is constant function say μ(uv) = c for u v ∈ E. Then $d_3(v) = 5c$, for all v ∈ V.
Hence G is **a** (3, 5c) – regular fuzzy graph.

Remark 6.2: Converse of the theorem 6.2 need not be true.
For example, Consider G:(σ,μ) be fuzzy graph such that G^*:(V,E) is wagner graph.

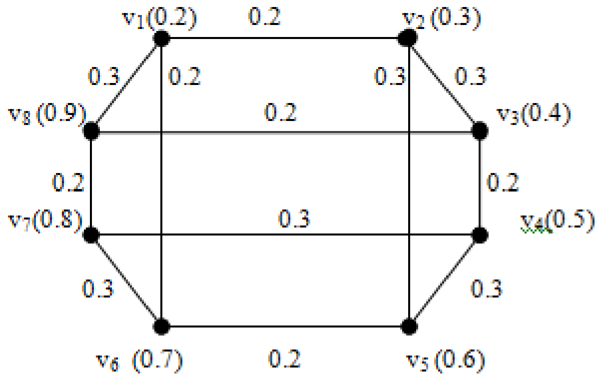

Figure 16

Here, $d_3(v_1) = 1.0$, $d_3(v_2) = 1.0$, $d_3(v_3) = 1.0$, $d_3(v_4) = 1.0$, $d_3(v_5) = 1.0$, $d_3(v_6) = 1.0$, $d_3(v_7) = 1.0$, $d_3(v_8) = 1.0$. So, G is (3, 1.0)-regular fuzzy graph. But μ is not a constant function.

Remark 6.3: The theorem 6.1 does not hold for totally (3,k) – regular fuzzy graphs.

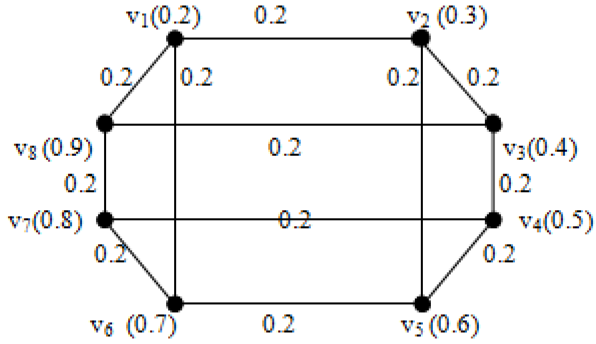

Figure 17

Here, $d_3(v_1) = 1.0$, $d_3(v_2) = 1.0$, $d_3(v_3) = 1.0$, $d_3(v_4) = 1.0$, $d_3(v_5) = 1.0$, $d_3(v_6) = 1.0$, $d_3(v_7) = 1.0$, $d_3(v_8) = 1.0$ and $td_3(v_1) = 1.2$, $td_3(v_2) = 1.3$, $td_3(v_3) = 1.4$, $td_3(v_4) = 1.5$, $td_3(v_5) = 1.6$, $td_3(v_6) = 1.7$, $td_3(v_7) = 1.8$, $td_3(v_8) = 1.9$. Hence G is (3, 1.0) - regular fuzzy graph but not totally (3, k)-regular fuzzy graph. But μ is a constant function.

7. (3, k) - regular fuzzy graphs on Corona graph with some specific membership function

In this section (3, k)-regularity and totally (3, k)-regularity on fuzzy graph whose underlying crisp graph is a Corona graph is studied with some specific membership function.

Theorem 7.1: Let G: (σ, μ) be fuzzy graph such that G^*: (V, E) is Corona graph $C_n \circ K_1$ where $n \geq 5$. If μ is a constant function, then G is (3, 4c) - regular fuzzy graph.

Proof: Suppose that, μ is a constant function say μ(uv) = c for all u v ∈ E , then $d_3(v) = 4c$, for all v ∈ V.
Hence G is (3, 4c) – regular fuzzy graph.

Remark 7.2: Converse of the theorem 7.1 need not be true.
For example, Consider G: (σ,μ) be fuzzy graph such that G^*: (V,E) is corona graph.

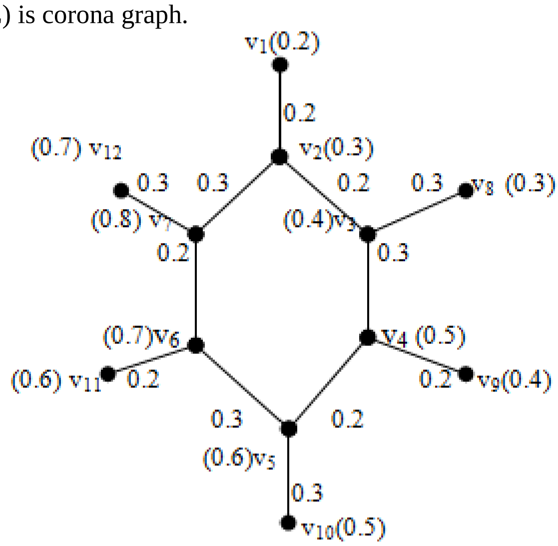

Figure 18

Here, $d_3(v_1) = 0.8$, $d_3(v_2) = 0.8$, $d_3(v_3) = 0.8$, $d_3(v_4) = 0.8$, $d_3(v_5) = 0.8$, $d_3(v_6) = 0.8$, $d_3(v_7) = 0.8$, $d_3(v_9) = 0.8$, $d_3(v_{10}) = 0.8$, $d_3(v_{11}) = 0.8$, $d_3(v_{12}) = 0.8$. So G is (3, 0.8)- regular fuzzy graph. But μ is not a constant function.

Remark 7.3: The theorem 7.1 does not hold for totally (3, k)-regular fuzzy graphs.

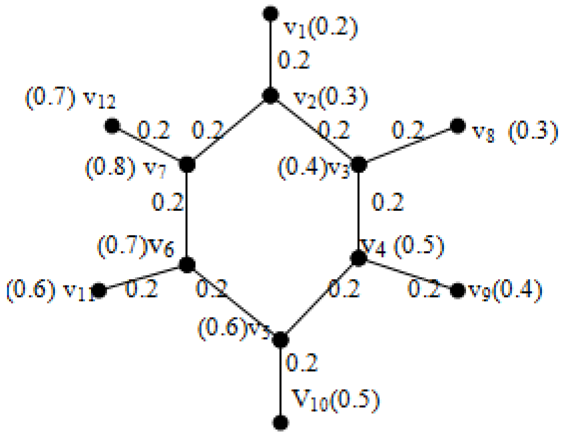

Figure 19

Here, $d_3(v_1) = 0.8$, $d_3(v_2) = 0.8$, $d_3(v_3) = 0.8$, $d_3(v_4) = 0.8$, $d_3(v_5) = 0.8$, $d_3(v_6) = 0.8$, $d_3(v_7) = 0.8$, $d_3(v_9) = 0.8$, $d_3(v_{10}) = 0.8$ 0.2, $d_3(v_{11}) = 0.8$, $d_3(v_{12}) = 0.8$ and $td_3(v_1) = 1.0$, $td_3(v_2) = 1.1$, $td_3(v_3) = 1.2$, $td_3(v_4) = 1.3$, $td_3(v_5) = 1.4$, $td_3(v_6) = 1.5$, $td_3(v_7) = 1.6$, $td_3(v_8) = 1.1$, $td_3(v_9) = 1.2$, $td_3(v_{10}) = 1.3$, $td_3(v_{11}) = 1.4$, $td_3(v_{12}) = 1.5$. Hence G is (3, 0.8) - regular fuzzy graph but not totally (3, k)-regular fuzzy graph. But μ is a constant function.

8. (3, k)-regular fuzzy graphs on a Cycle of length ≥ 5 with some specific membership function

In this section (3, k)-regularity and totally (3, k)-regularity on fuzzy graph whose underlying crisp graph is a Cycle of length ≥ 5 is studied with some specific membership function.

Theorem 8.1: Let G: (σ, μ) be fuzzy graph such that G^*: (V, E) is cycle of length≥ 5. If μ is constant function, then G is (3, 2c) - regular fuzzy graph.

Proof: Suppose that, μ is constant function say μ(uv) = c for all uv∈ E , then $d_3(v) = 2c$, for all v∈ V.
Hence G is (3, 2c) – regular fuzzy graph.

Remark 8.2: Converse of the theorem 8.1 need not be true.
For example, Consider G:(σ,μ) be fuzzy graph such that G^*:(V,E) is odd cycle of length seven.

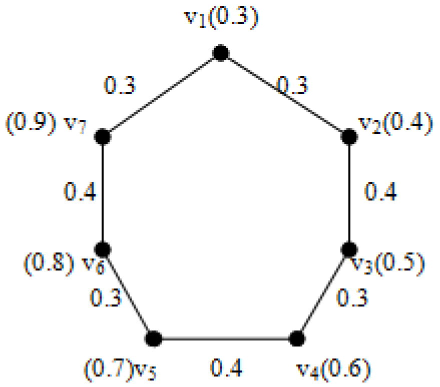

Figure 20

Here, $d_3(v_1) = 0.6$, $d_3(v_2) = 0.6$, $d_3(v_3) = 0.6$, $d_3(v_4) = 0.6$, $d_3(v_5) = 0.6$, $d_3(v_6) = 0.6$, $d_3(v_7) = 0.6$. So G is (3, 0.6) – regular fuzzy graph. But μ is not a constant function.

Remark 8.3: The theorem 8.1 does not hold for totally (3, k)-regular fuzzy graphs**.**

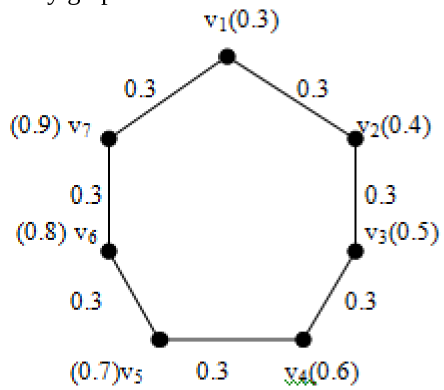

Figure 21

Here, $d_3(v_1) = 0.6$, $d_3(v_2) = 0.6$, $d_3(v_3) = 0.6$, $d_3(v_4) = 0.6$, $d_3(v_5) = 0.6$, $d_3(v_6) = 0.6$, $d_3(v_7) = 0.6$ and $td_3(v_1) = 0.9$, $td_3(v_2) = 1.0$, $td_3(v_3) = 1.1$, $td_3(v_4) = 1.2$, $td_3(v_5) = 1.3$, $td_3(v_6) = 1.4$, $td_3(v_7)$ =1.5.So G is (3, 0.6)-regular fuzzy graph, but not totally(3, k)-regular fuzzy graph. But μ is a constant function.

Theorem 8.4: Let G: (σ, μ) be fuzzy graph such that G*: (V, E) is even cycle of length ≥ 5. If alternate edges have same membership values, then G is (3, k)-regular fuzzy graph.

Proof If alternate edges have same membership values, then
$\mu(e_i) = \begin{cases} c_1, if\ i\ is\ odd \\ c_2\ if\ i\ is\ even \end{cases}$.
If $c_1 = c_2$, then μ is constant function. So G is (3, $2c_1$)-regular fuzzy graph.
If $c_1 < c_2$, then $d_3(v) = 2c_1$, for all v∈ V. So G is (3, $2c_1$)-regular fuzzy graph.
If $c_1 > c_2$, then $d_3(v) = 2c_2$, for all v∈ V. So G is (3, $2c_2$)-regular fuzzy graph.
Remark 8.5: The theorem 8.4 does not hold for totally (3,k)-regular fuzzy graphs
For example, Consider G:(σ,μ) be fuzzy graph such that G^*:(V,E) is even cycle of length ≥ 5.

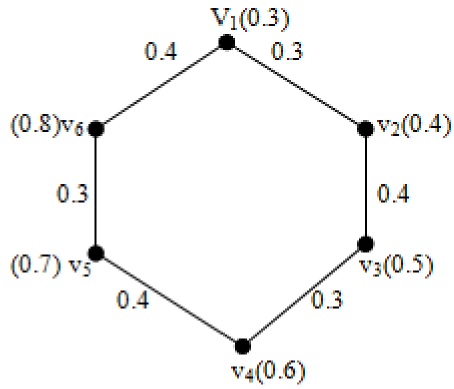

Figure 22

Here, $d_3(v_1) = 0.6$, $d_3(v_2) = 0.6$, $d_3(v_3) = 0.6$, $d_3(v_4) = 0.6$, $d_3(v_5) = 0.6$, $d_3(v_6) = 0.6$ and $td_3(v_1) = 0.9$, $td_3(v_2) = 1.0$, $td_3(v_3) = 1.1$, $td_3(v_4) = 1.2$, $td_3(v_5) = 1.3$, $td_3(v_6) = 1.4$. So G is not totally (3, k)-regular fuzzy graph.

Remark 8.6: Let $G:(\sigma,\mu)$ be fuzzy graph such that $G^*:(V,E)$ is odd cycle of length ≥ 5. If alternate edges have same membership values, then G is (3, k)-regular fuzzy graph.
For example, Consider $G:(\sigma,\mu)$ be fuzzy graph such that $G^*:(V,E)$ is odd cycle of length 5.

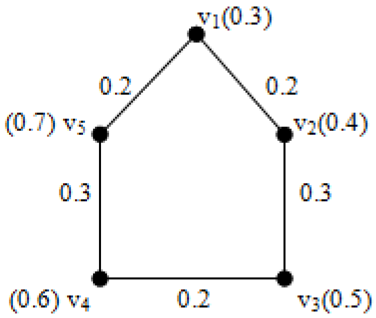

Figure 23

Here $d_3(v_1) = 0.4$, $d_3(v_2) = 0.4$, $d_3(v_3) = 0.4$, $d_3(v_4) = 0.4$, $d_3(v_5) = 0.4$. G is (3, k)-regular fuzzy graph.

Remark 8.7: The remark 8.6 does not hold for totally (3, k)-regular fuzzy graphs. For example, Consider $G:(\sigma,\mu)$ be fuzzy graph such that $G^*:(V,E)$ is odd cycle of length 5.

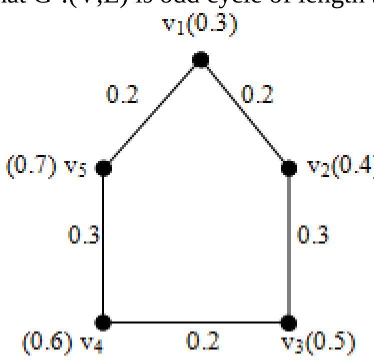

Figure 24

Here $d_3(v_1) = 0.4$, $d_3(v_2) = 0.4$, $d_3(v_3) = 0.4$, $d_3(v_4) = 0.4$, $d_3(v_5) = 0.4$ and $td_3(v_1) = 0.7$, $td_3(v_2) = 0.8$, $td_3(v_3) = 0.9$, $td_3(v_4) = 1.0$, $td_3(v_5) = 1.1$.

Theorem 8.8: Let $G:(\sigma,\mu)$ be fuzzy graph such that $G^*:(V,E)$ is any odd cycle of length ≥ 5.
Let $\mu(e_i) = \begin{cases} c_1, & \text{if } i \text{ is odd.} \\ c_2 \geq c_1, & \text{if } i \text{ is even} \end{cases}$, then G is $(3, 2c_1)$-regular fuzzy graph.

Proof: Let $\mu(e_i) = \begin{cases} c_1, & \text{if } i \text{ is odd.} \\ c_2 \geq c_1, & \text{if } i \text{ is even} \end{cases}$

Case 1: Let $G:(\sigma,\mu)$ be fuzzy graph such that $G^*:(V,E)$ is even cycle of length ≥ 5.
$d_3(v_i) = \{c_1 \wedge c_2 \wedge c_1\} + \{c_2 \wedge c_1 \wedge c_2\} = c_1 + c_1 = 2c_1$, for all $v \in V$. So, G is $(3, 2c_1)$-regular fuzzy graph.
Case 2: Let $G:(\sigma,\mu)$ be fuzzy graph such that $G^*:(V,E)$ is an odd cycle of length ≥ 5. Let $e_1, e_2, e_3 \ldots e_{2n+1}$ be the edges of the odd cycle of G^* in that order.

$d_3(v_1) = \{\mu(e_{2n+1}) \wedge \mu(e_{2n}) \wedge \mu(e_{2n-1})\} + \{\mu(e_1) \wedge \mu(e_2) \wedge \mu(e_3)\}$
$= \{c_1 \wedge c_2 \wedge c_1\} + \{c_1 \wedge c_2 \wedge c_1\} = c_1 + c_1 = 2c_1.$
$d_3(v_2) = \{\mu(e_1) \wedge \mu(e_{2n+1}) \wedge \mu(e_{2n})\} + \{\mu(e_2) \wedge \mu(e_3) \wedge \mu(e_4)\}$
$= \{c_1 \wedge c_1 \wedge c_2\} + \{c_2 \wedge c_1 \wedge c_2\} = c_1 + c_1 = 2c_1.$
$d_3(v_3) = \{\mu(e_2) \wedge \mu(e_1) \wedge \mu(e_{2n+1})\} + \{\mu(e_3) \wedge \mu(e_4) \wedge \mu(e_5)\}$
$= \{c_2 \wedge c_1 \wedge c_1\} + \{c_1 \wedge c_2 \wedge c_1\} = c_1 + c_1 = 2c_1.$
For $i = 4, 5, 6, \ldots, 2n$.
$d_3(v_i) = \{\mu(e_{i-1}) \wedge \mu(e_{i-2}) \wedge \mu(e_{i-3})\} + \{\mu(e_i) \wedge \mu(e_{i+1})\ \mu(e_{i+2})\}$
$= \{c_1 \wedge c_2 \wedge c_1\} + \{c_2 \wedge c_1 \wedge c_2\} = c_1 + c_1 = 2c_1$
$d_3(v_{2n+1}) = \{\mu(e_{2n+1}) \wedge \mu(e_1) \wedge \mu(e_2)\} + \{\mu(e_{2n}) \wedge \mu(e_{2n-1}) \wedge \mu(e_{2n-2})\}$
$= \{c_1 \wedge c_2 \wedge c_1\} + \{c_2 \wedge c_1 \wedge c_2\} = c_1 + c_1 = 2c_1.$
Hence, $d_3(v_i) = 2c_1$, for all $v \in V$.
So G is $(3, 2c_1)$ - regular fuzzy graph.

Remark 8.9: The theorem 8.8 does not hold for totally (3, k) - regular fuzzy graph.

1.Consider $G:(\sigma,\mu)$ be fuzzy graph such that $G^*:(V,E)$ is even cycle of length six.

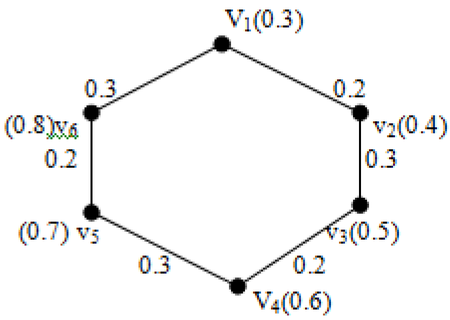

Figure 25

Here, $d_3(v_1) = 0.4$, $d_3(v_2) = 0.4$, $d_3(v_3) = 0.4$, $d_3(v_4) = 0.4$, $d_3(v_5) = 0.4$, $d_3(v_6) = 0.4$. So G is (3, k)-regular fuzzy graph. But $td_3(v_1) = 0.7$, $td_3(v_2) = 0.8$, $td_3(v_3) = 0.9$, $td_3(v_4) = 1.0$, $td_3(v_5) = 1.1$, $td_3(v_6) = 1.2$. So G is not totally (3, k)-regular fuzzy graph.

2. Consider $G:(\sigma,\mu)$ be fuzzy graph such that $G^*:(V,E)$ is an odd cycle of length five.

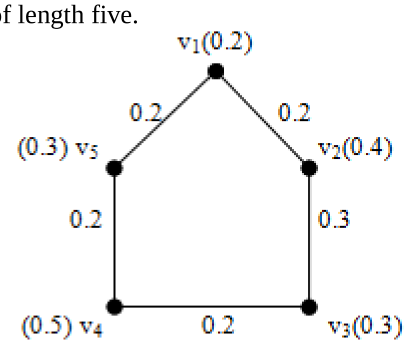

Figure 26

Here, $d_3(v_1) = 0.4$, $d_3(v_2) = 0.4$, $d_3(v_3) = 0.4$, $d_3(v_4) = 0.4$, $d_3(v_5) = 0.4$. So G is (3, 0.4))-regular fuzzy graph. But $td_3(v_1) = 0.6$, $td_3(v_2) = 0.8$, $td_3(v_3) = 0.7$, $td_3(v_4) = 0.9$, $td_3(v_5) = 0.7$. So G is not totally (3, k)-regular fuzzy graph.

9. Conclusion and Future Studies

In this paper (3, k) - regular fuzzy graphs and totally (3, k)-regular fuzzy graphs are compared through various examples. A necessary and sufficient condition under which they are equivalent is provided. Also we provide (3, k)-regular fuzzy graphs and totally (3, k)-regular fuzzy graphs in which underlying crisp graphs are a path on six vertices, a corona graph, a wagner graph and a cycle of length ≥ 5 is studied with some specific membership function. Some properties of (3, k)-regular fuzzy graphs studied and they are examined for totally (3, k)-regular fuzzy graphs. The results discussed may be used to study about various fuzzy graphs invariants. For further invetigation, the following open problem is suggested.

"(r, m)-regular fuzzy graph and totally (r, m)-regular fuzzy graph, for m > 3 may be investigated".

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