International Journal of Science and Research (IJSR)
ISSN (Online): 2319-7064
Index Copernicus Value (2015): 78.96 | Impact Factor (2015): 6.391

Sharan's Equations for Velocity and Energy at Variable Fractional Heights

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Abstract: ***In the following pages, I put forth formulae which hold good for finding velocity, kinetic energy, total energy of a body which is projected vertically upwards or vertically downwards with some initial velocity "u" in one dimension motion, where the body has covered fraction (λ/β)th of the total displacement "h". Here the fraction (λ/β) in my derivation is represented by χ for the sake of convenience. For example: If a body has covered (1/2)th of total height then $\lambda = 1$, $\beta = 2$ and $\chi = (1/2)$***

1. Introduction

These are the terms which will be used in the following pages

- **Initial velocity (u):** The initial velocity is the velocity of the object before acceleration causes a change
  e.g.: If a body starts with rest then its initial velocity is 0.

- **Final velocity (v):** Final Velocity is defined as the Velocity attained by a body after completing or reaching certain distance or a given or certain time interval**.**
  eg: If after acceleration the velocity of a body becomes 12 m/s, then the final velocity is 12 m/s

- **Acceleration due to gravity (g):** Acceleration due to gravity is the acceleration on an object caused by the force of gravitation. Neglecting friction such as air resistance, all small bodies accelerate in a gravitational field at the same rate relative to the center of mass. This equality is true regardless of the masses or compositions of the bodies. For Earth, the value of acceleration due to gravity is normally taken as 9.8 ms^{-2}.

- **Mass (m):** Mass is a property of a physical body. It is the measure of an object's resistance to acceleration (a change in its state of motion) when a net force is applied. It also determines the strength of its mutual gravitational attraction to other bodies. The basic SI unit of mass is the kilogram (kg).

- **Kinetic energy (KE):** Kinetic energy of an object is the energy that it possesses due to its motion. It is defined as the work needed to accelerate a body of a given mass from rest to its stated velocity.

- **Potential energy (PE):** Potential Energy is the energy possessed by an object because of its position relative to other objects, stresses within itself, its electric charge, or other factors.

- **Total energy (TE):** Total energy of a system is defined as sum total of all energy possessed by a body. For mechanical energy, the total energy is sum total of Kinetic energy and Potential energy.

2. Existing Formulae

There are formulae existing for finding Initial velocity, Final velocity, Acceleration, Displacement, Kinetic energy, Potential energy.

1)For Initial velocity, Final velocity, Acceleration, Displacement:

- $v = u + at$
- $s = ut + (at^2)/2$
- $v^2 - u^2 = 2as$

2)For Kinetic energy:

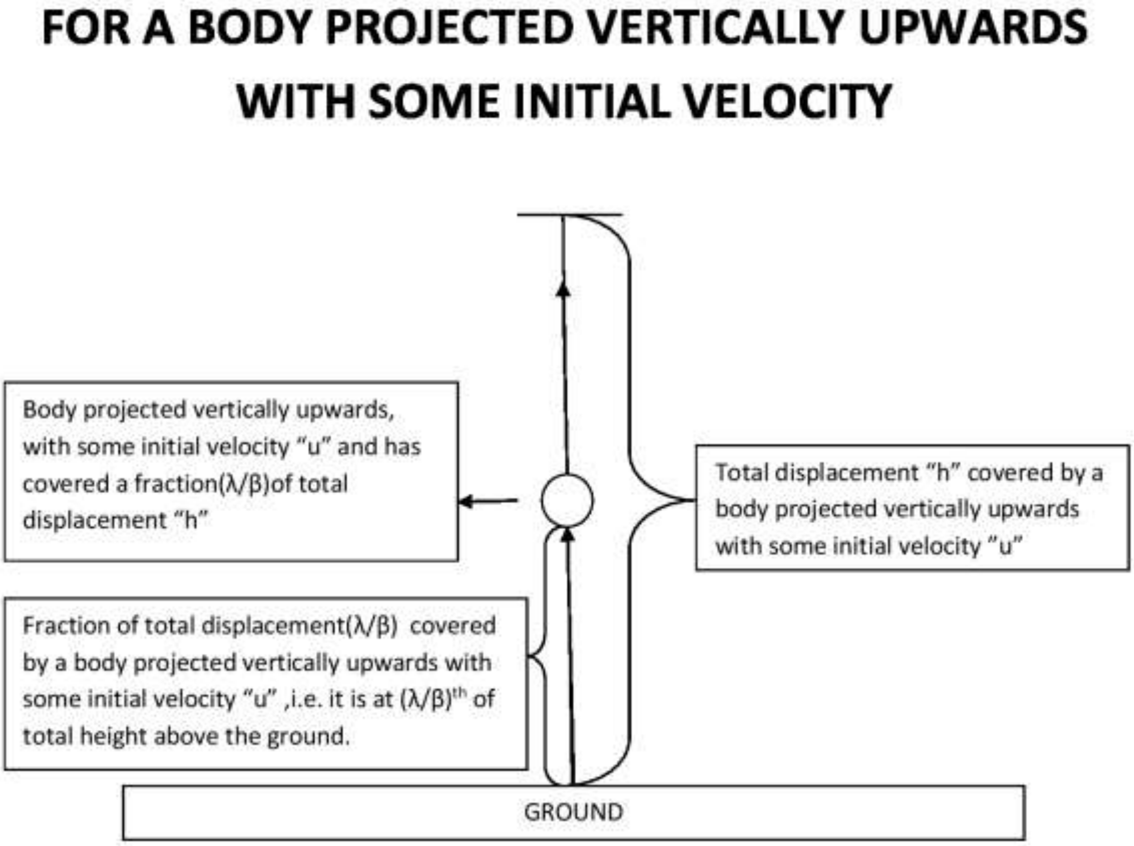
- $KE = mv^2/2$

3)For Potential energy:

- $PE = mgh$

3. Proposed Methodology

For a body projected vertically upwards with some initial velocity



For everybody projected up with some initial velocity "u", it covers fractional displacement "χ" of total height "h", which is represented by (λ/β)h in one dimension motion, "g" is acceleration due to gravity and negative sign indicates negative acceleration.

$v^2 - u^2 = 2(-g)(\chi h)$

$v^2 = u^2 - 2g\chi h$

$v^2 = u^2 - 2g\chi \frac{u^2}{2g}$ (h = u^2 / 2g in one dimension motion)

$v^2 = u^2 - \chi u^2$

$v = \sqrt{u^2 - \chi u^2}$

$v = u\sqrt{1 - \chi}$

$v = u\sqrt{1 - \frac{\lambda}{\beta}}$ (χ is a fraction in form λ/β)

$v = u\sqrt{\frac{\beta - \lambda}{\beta}}$ -- 1

We know that Kinetic energy (KE) = $mv^2/2$

KE = $\frac{1}{2}\, m\,(u\sqrt{1-\chi}\,)^2$ (Substituting eqn 1 in this eqn)

KE = $\frac{1}{2}\, mu^2\,(1 - \chi)$ --2

KE = $\frac{1}{2}\, mu^2 \left(\frac{\beta - \lambda}{\beta}\right)$

We know that Potential energy (PE) = mgh. Since the body covers the displacement of χ, so it is at height χ of total height h.

PE = mg(χ h) --3

We know that Total energy (TE) = Potential energy (PE) + Kinetic enrgy (KE)

$TE = mg\chi h + \frac{1}{2}\, mu^2\,(1 - \chi)$

(Substituting eqn 2 and eqn 3 in this eqn)

$TE = mg\chi h + \frac{1}{2}\, mu^2 - \frac{1}{2}\, mu^2\chi$

$TE = \frac{1}{2}(2mg\chi h + mu^2 - mu^2\chi)$

$TE = m\left(\frac{\frac{2g\chi u^2}{2g} + u^2 - u^2\chi}{2}\right)$

(h = u^2 / 2g in one dimension motion)

$TE = \frac{1}{2}\, mu^2 \quad (OR) \quad \frac{mgu^2}{2g}$

(By multiplying by g both in numerator and denominator)

$TE = \frac{1}{2}\, mu^2 \quad (OR) \quad mgh$

(u^2 / 2g = h in one dimension motion)

4. Result of the Findings

1) When a body of mass 5 kg is projected vertically upwards with initial velocity 10 m/s, and it has covered $(4/5)^{th}$ of its total displacement.

Here: m = 5 kg, u = 10 m/s, λ = 4, β = 5

A) Velocity of the body at $(4/5)^{th}$ of total displacement is

$v = u\sqrt{\frac{\beta - \lambda}{\beta}}$

$v = 10\sqrt{\frac{5-4}{5}}\ m/s$

$v = 10\sqrt{\frac{1}{5}}\ m/s$

$v = 2\sqrt{5}\ m/s$

B) Kinetic energy of the body at $(4/5)^{th}$ of the total displacement is

KE $= \frac{1}{2}\, mu^2 \left(\frac{\beta - \lambda}{\beta}\right)$

$KE = \frac{1}{2}(5)\,10^2 \left(\frac{5-4}{5}\right) J$

$$KE = \frac{1}{2}(5)(100)\left(\frac{1}{5}\right)J$$
$$KE = 50\,J$$

C) Total energy of the body at $(4/5)^{th}$ of the total displacement is

$$TE = \frac{1}{2}mu^2$$
$$TE = \frac{1}{2}(5)10^2\,J$$
$$TE = \frac{1}{2}(5)(100)\,J$$
$$TE = 250\,J$$

2) When a body of mass 100 kg is projected vertically upwards with initial velocity 100 m/s, and it has covered $(1/100)^{th}$ of its total displacement.

Here: m = 100 kg, u = 100 m/s, λ = 1, β = 100

A) Velocity of the body at $(1/100)^{th}$ of total displacement is

$$v = u\sqrt{\frac{\beta-\lambda}{\beta}}$$
$$v = 100\sqrt{\frac{100-1}{100}}\ m/s$$
$$v = 100\sqrt{\frac{99}{100}}\ m/s$$
$$v = 30\sqrt{11}\ m/s$$

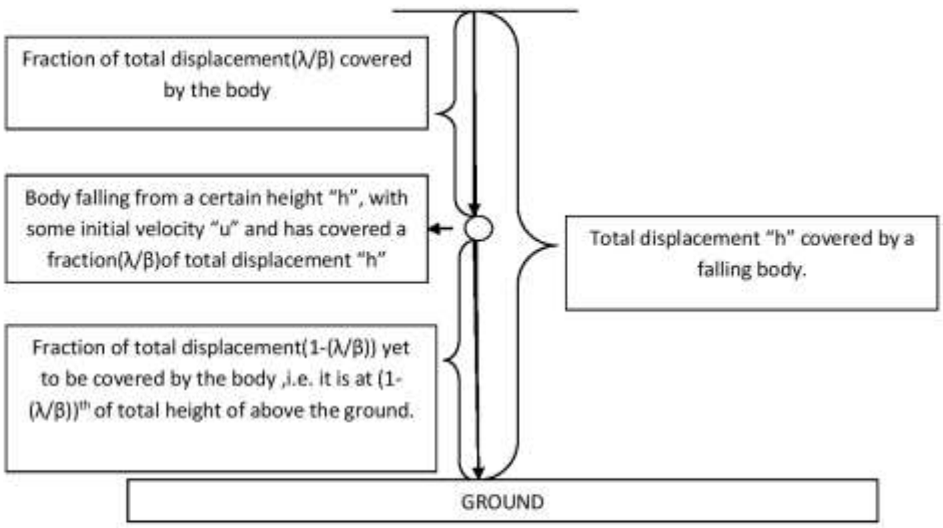
B) Kinetic energy of the body at $(1/100)^{th}$ of the total displacement is

$$KE = \frac{1}{2}mu^2\left(\frac{\beta-\lambda}{\beta}\right)$$
$$KE = \frac{1}{2}(100)100^2\left(\frac{100-1}{100}\right)J$$
$$KE = \frac{1}{2}(100)100^2\left(\frac{99}{100}\right)J$$
$$KE = 495000\,J$$

C) Total energy of the body is

$$TE = \frac{1}{2}mu^2$$
$$TE = \frac{1}{2}(100)100^2\,J$$
$$TE = \frac{1}{2}(100)(10000)\,J$$
$$TE = 500000\,J$$

FOR A BODY FALLING WITH SOME INITIAL VELOCITY



For everybody falling with some initial velocity "u", it covers fractional displacement "χ" of total height "h", which is represented by (λ/β)h in one dimension motion, "g" is acceleration due to gravity.

$$v^2 - u^2 = 2\,(g)(\chi h)$$
$$v^2 = u^2 + 2g\chi h$$
$$v^2 = u^2 + 2g\chi\frac{u^2}{2g} \quad \text{(h = } u^2/2g \text{ in one dimension motion)}$$
$$v^2 = u^2 + \chi u^2$$
$$v = \sqrt{u^2 + \chi u^2}$$
$$v = u\sqrt{1+\chi}$$
$$v = u\sqrt{1+\frac{\lambda}{\beta}} \qquad (\chi \text{ is a fraction in form } \lambda/\beta)$$
$$v = u\sqrt{\frac{\beta+\lambda}{\beta}} \qquad \text{--1}$$

We know that Kinetic energy (KE) = $mv^2/2$

$$KE = \frac{1}{2}\,m\,(u\sqrt{1+\chi}\,)^2 \quad \text{(Substituting eqn 1 in this eqn)}$$
$$KE = \frac{1}{2}\,mu^2\,(1+\chi) \qquad \text{--2}$$
$$KE = \frac{1}{2}\,mu^2\left(\frac{\beta+\lambda}{\beta}\right)$$

We know that Potential energy (PE) = mgh. But the body covers the displacement of χ, so it is at height (1-χ) of total height h from the ground.

PE = mg(1-χ)h --3

We know that Total energy (TE) = Potential energy (PE) + Kinetic energy (KE)

$$TE = mg(1-\chi)h + \frac{1}{2}\,mu^2\,(1+\chi)$$ (Substituting eqn 2 and eqn 3 in this eqn)

$$TE = mgh - mg\chi h + \frac{1}{2}\,mu^2 + \frac{1}{2}\,mu^2\chi$$
$$TE = \frac{1}{2}(2mgh - 2mg\chi h + mu^2 + mu^2\chi)$$

$$TE = \frac{1}{2}(2mgh(1-\chi) + mu^2(1+\chi))$$

$$TE = \frac{1}{2}\left(2mg\frac{u^2}{2g}(1-\chi) + mu^2(1+\chi)\right)$$ (h = u^2 / 2g in one dimension motion)

$$TE = \frac{1}{2}(mu^2(1-\chi) + mu^2(1+\chi))$$

$$TE = \frac{1}{2}mu^2(1-\chi+1+\chi)$$

$$TE = \frac{1}{2}mu^2(2)$$

$$TE = mu^2 \quad (OR) \quad \frac{2mg\,u^2}{2g}$$ (By multiplying by 2g both in numerator and denominator)

$$TE = mu^2 \quad (OR) \quad 2mgh$$

(u^2 / 2g = h in one dimension motion)

If,

TE_1 = Total energy of a body when it is projected upwards with some initial velocity "u".

TE_2 = Total energy of a body when it falling with some initial velocity "u"

Then,

$$\frac{TE_1}{TE_2} = \frac{mgh}{2mgh} = \frac{1}{2}$$

$$TE_1 : TE_2 :: 1:2$$

From equations:

KE α m

KE α u^2

TE α m

TE α u^2

m α $1/u^2$

Where

KE = Kinetic energy

TE = Total energy

m = Mass

u = Initial velocity

5. Result of the Findings

1) When a body of mass 20 kg is falling with initial velocity 2 m/s, and it has covered $(1/2)^{th}$ of its total displacement.

Here: m = 20 kg, u = 2 m/s, λ = 1, β = 2

A) Velocity of the body at $(1/2)^{th}$ of total displacement is

$$v = u\sqrt{\frac{\beta+\lambda}{\beta}}$$

$$v = 2\sqrt{\frac{2+1}{2}}\ m/s$$

$$v = 2\sqrt{\frac{3}{2}}\ m/s$$

$$v = \sqrt{6}\ m/s$$

B) Kinetic energy of the body at $(1/2)^{th}$ of the total displacement is

$$KE = \frac{1}{2}mu^2\left(\frac{\beta+\lambda}{\beta}\right)$$

$$KE = \frac{1}{2}(20)2^2\left(\frac{2+1}{2}\right) J$$

$$KE = \frac{1}{2}(20)2^2\left(\frac{3}{2}\right) J$$

$$KE = 60\ J$$

C) Total energy of the body is

$$TE = mu^2$$

$$TE = (20)2^2\ J$$

$$TE = (20)(4)\ J$$

$$TE = 80\ J$$

D) Thus we can find height from which ball is falling

TE = 2mgh (Where h = maximum height)

80 = 2(20)(10)h (Taking g=10 m/s) So, h = 0.2 m

2) When a body of mass 1 kg is falling with initial velocity 100 m/s, and it has covered $(2/5)^{th}$ of its total displacement.

Here: m = 1 kg, u = 100 m/s, λ = 2, β = 5

A) Velocity of the body at $(2/5)^{th}$ of total displacement is

$$v = u\sqrt{\frac{\beta+\lambda}{\beta}}$$

$$v = 100\sqrt{\frac{2+5}{5}}\ m/s$$

$$v = 100\sqrt{\frac{7}{5}}\ m/s$$

$$v = 20\sqrt{35}\ m/s$$

B) Kinetic energy of the body at $(2/5)^{th}$ of the total displacement is

$$KE = \frac{1}{2}mu^2\left(\frac{\beta+\lambda}{\beta}\right)$$

$$KE = \frac{1}{2}(1)100^2\left(\frac{2+5}{5}\right) J$$

$$KE = \frac{1}{2}(1)100^2\left(\frac{7}{5}\right) J$$

$$KE = 7000\ J$$

C) Total energy of the body is

$$TE = mu^2$$

$$TE = (1)100^2\ J$$

$$TE = (1)(10000)\ J$$

$$TE = 10000\ J$$

D) Thus we can find height from which ball is falling

TE = 2mgh (Where h = maximum height)

10000 = 2(1) (10)h (Taking g=10 m/s)

h = 500 m

References

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The author of this article is Sharanbasaweshwar M. Patil who studies in class XI of Vidyaniketan Pre University College, Vidyanagar, Hubli, Dharwad District, Karnataka, India. He has already published an article regarding the Math's formula with the title "Relationship of LCM and HCF of 3 numbers in AP" in the "International Journal of Mathematical Archive". The article was published in Vol 8 No 3 issue of the Journal.