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Comparison between Inverse Distance Weighted (IDW) and Kriging

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Abstract: ***In general, ground survey techniques provide the most accurate data and sampling for Digital Terrain Model (DTM) generation. The main limitations of these methods however, is that they are only practical and economical to implement over relatively small areas of limited areal extent. The choice of method and its parameters can be critical if one is to avoid misleading results. Variogram functions have, generally, been used for prior analysis of data sets during interpolation by geostatistical analysis in Geographical Information Systems (GIS). The interpolation methods are being effectively used as tools to predict and estimate values for unknown points by using points with known values (sample points). In this paper, two methods are to be compared; Namely IDW and kriging. From the results obtained, kriging method is found to be more accurate then the IDW interpolation method.***

Keywords: Inverse Distance Weighting, KRIGING, Geostatistic, variogram

1. Introduction

In general, ground survey techniques whether using traditional methods or the **G**lobal **P**osition **S**ystem (**GPS**), provide the most accurate data and sampling for **D**igital **T**errain **M**odel (**DTM**) generation, but they are usually the most time consuming and thus considered to be the most costly. The main limitations of these methods however, is that they are only practical and economic at to implement over relatively small areas of limited areal extent such as planning of building areas or new roads constructions. Over large areas, it will be economical to obtain the data for the generation of DTMs either using photogrammetric methods, remote sensing techniques or by digitizing contours on existing topographic maps. While the terrain plays a fundamental role in modulating earth surface processes; so strong is that understanding of the nature of terrain can confer understanding of the nature of these processes directly, in both subjective and analytical terms.

In a variable and mountainous terrain, however, the assumptions made about the underlying variation that has been sampled and the choice of method and its parameters can be critical if one is to avoid misleading results. Variogram functions have, generally, been used for prior analysis of data sets during interpolation by geostatistical analysis in **G**eographical **I**nformation **S**ystems (**GIS**).

The interpolation methods are being effectively used as tools to predict and estimate values for unknown points by using points with known values (sample points), such as elevation, rainfall, noise levels, and so on. All points on the surface of the earth are correlated, but disability appears on mountains and hard layers. The solution of this problem is GIS that enables us to find the correlation between points by the variogram model and subsequently the production of DTM with high accuracy. The pervasiveness of computers has participated to considerable change in the way survey and map data are collected, processed and stored.

1.1 Semivariograms and variograms

Geostatistics deals with spatial and temporal data from natural phenomena and it involves estimating values of variables. The basic concept of geostatistics is the study of spatial correlation of data measured at various points in a three dimensional space. The first step in a geostatistical analysis is to investigate the spatial structure, which is a function of local data variability with distance between observations. The most common structure function is the semivariogram function. Semivariogram functions are used to determine the existence of autocorrelation between data points.

A variogram can be defined as a geostatistical technique that evaluates autocorrelation commonly observed in spatial data. (Wallace et al.2000). The term autocorrelation is meant to be correlation between elements of a series and others from the same series separated from them by a given interval. (Oxford Dictionary of English (2010, 2012)).

The spatial autocorrelation structure can be depicted by the variogram, which is the semi-variance plotted against distance under the intrinsic hypothesis. (Bailey & Gatrell, 1995). The semivariogram depicts the spatial autocorrelation of the measured sample points. Once each pair of locations is plotted, a model is fitted through them. There are certain characteristics that are commonly used to describe these models. Mathematically, the variogram is given by equation (1) below:

$$\gamma(h) = \frac{1}{2N(h)} \sum_{\alpha=1}^{N(h)} [z(u_\alpha + h) - z(u_\alpha)]^2 \;\ldots\ldots (1)$$

γ(h): predicted value.
u: vector of spatial coordinates (with components x, y or "easting" and "northing").
z(u): variable under consideration as a function of spatial location.
h: lag vector representing separation between two spatial locations.
z(u+h): lagged version of variable under consideration and N is the number of data pairs at distance h.

The parameters in Fig (1) are used to describe variograms model.

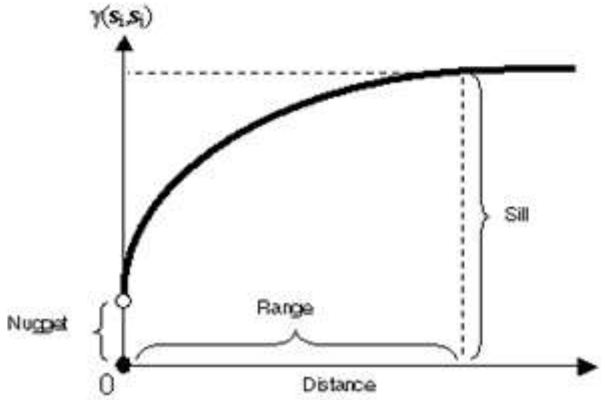

Figure 1: features of a variogram model

The forms of the variogram can be quite revealing about the kinds of spatial variation present in an area and can help to decide how to proceed. Semivariogram has been used in different fields and known by different names. One of the most commonly used model is the spherical model this model ,given in equation (2), shows a progressive decrease of spatial autocorrelation(equivalently, an increase of semi-variance) until some distance, beyond which autocorrelation is zero.

$$\gamma(h) = c_0 + c_1\left[\frac{3}{2}\left(\frac{h}{a}\right) - \frac{1}{2}\left(\frac{h}{a}\right)^3\right] \quad \ldots\ldots(2)$$

for $0 < h \leq a$

$\gamma(h) = c_0 + c_1$ for $h > a$

where:

a: is the range.

h: is the lag.

c_0: is the nugget variance.

$c_0 + c_1$: is equal to sill.

2. Interpolation Methods

There are two main groupings of interpolation techniques: deterministic and geostatistical. Deterministic interpolation techniques create surfaces from measured points (e.g. IDW). Geostatistical interpolation techniques (e.g. kriging) utilize the statistical properties of the measured points.

2.1 Inverse Distance Weighting

In this interpolation method, observation points are weighted during interpolation such that the influence of one point relative to another declines with distance from the new point. Weights are assigned to observation points through the use of a weighting power that controls how weighting factors drop off as the distance from a new point increases. The greater the weighting powers, the less effects points far from the new point have during interpolation. As the power increases, the value of the new point approaches the value of the nearest observational point. IDW assumes a value for an attribute z at any un-sampled point as a distance weighted average of sampled points lying within a defined neighborhood around that un-sampled point.

The general formula for IDW is:

$$Z(X_0) = \sum_{i=1}^{N} \lambda_i . Z(X_i) \quad \ldots\ldots(3)$$

N: is the number of scattered observation points in the set.

$Z(X_i)$: are values of the scattered observation points; i=1, 2…N

λ_i : are the weights assigned to observation points.

The common form of weighting function is given by

$$\lambda_i = \frac{d_{ij}^{-p}}{\sum_{i=1}^{n} d_{ij}^{-p}} \quad \ldots\ldots(4)$$

P: is the weight power.

d_{ij}: is the distance between grid node i and the neighboring point j; as the power increases, the generated surface becomes polygonal.

The polygons represent the nearest observation to the interpolation grid nodes. The accepted power values usually take values of one, two or three.

2.2 Kriging

Kriging assumes that the distance or direction between sample points reflects a spatial correlation that can be used to explain the variation in the surface. Kriging fits a mathematical function to a specified number of points, or all points, within a specified radius, to determine the output value for each new location.

Kriging is a multistep process; it includes exploratory statistical analysis of the data, variogram modeling, creating the surface, and (optionally) exploring a variance surface. Kriging is most appropriate when we know, a priori, there is a spatially correlated distance or directional bias in the data. Kriging is similar to IDW in that it weights the surrounding measured values to derive a prediction for an unmeasured location. The most general and widely used form of kriging is the ordinary kriging.

The interpolation in ordinary kriging starts with the construction of a variogram from the scatter point set to be interpolated. Once the model variogram is determined, it is used to compute the weights used in kriging.

The basic equation used in ordinary kriging is:

$$Z(X_0) = \sum_{i=1}^{N} \lambda_i . Z(X_i) \quad \ldots\ldots(5)$$

$Z(X_0)$: interpolated point.

N: is the number of scattered observation points in the set.

$Z(X_i)$: are values of scattered observation points.

λ_i : are the weights except that rather than using weights based on an arbitrary function of distance, the weights used in kriging are based on a model variogram.

3. Objective of the study

The objective of this study is to compare between deterministic inverse distance weighting method to geostatistical kriging methods.

4. The Study Area Data and Data Processing

The area of study falls within an area bounded by 866336.847mE to 869136.847mE and 2026890.409mN to 2026890.409Mn. The minimum and maximum elevations of the area are 666.003 and 695.416 respectively. The heights were obtained using lidar techniques. 811 points were used in this study of which 20 are used as check points. These are shown in Figure (3).

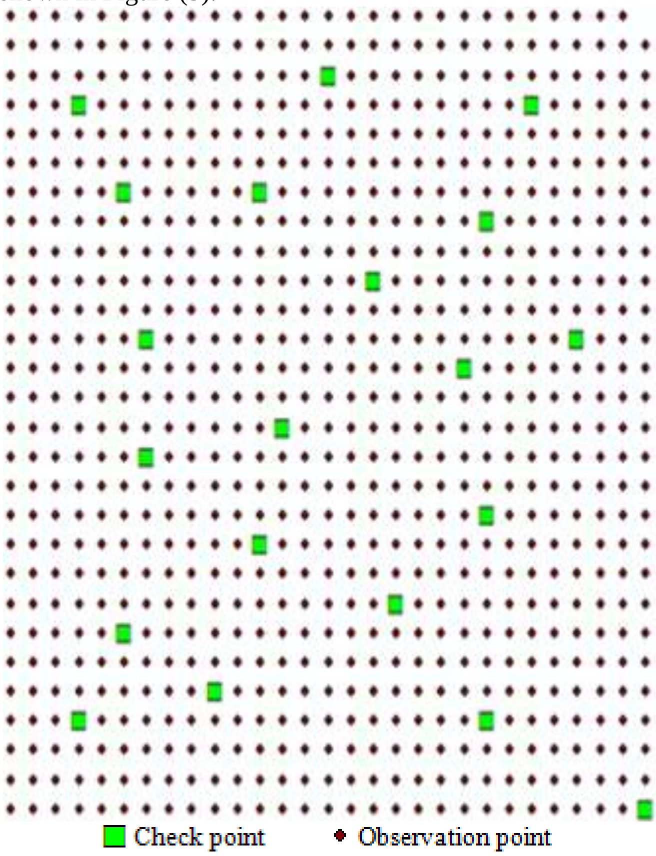

Figure 3: observation and check points

5. Methodology

The methodology adopted is based on the following steps.

- Abase map of the study area a map showing the spatial distribution of the DTM is to be prepared using ArcGIS 10.3 software.
- Investigation of the statistical properties of the data set to ensure that the data is distributed normally.
- After ensuring the normality of the data, a variogram for the data is determined.
- Using the computed variogram, the experimental variogram is plotted.
- The best variogram model is fitted through the experimental variogram.
- Using this model, the spatial autocorrelation between sample points is examined.
- Define the radius of search is defined.
- Use the appropriate method of interpolation (IDW or kriging) to determine the values of test points. These steps are shown in Figure (2).

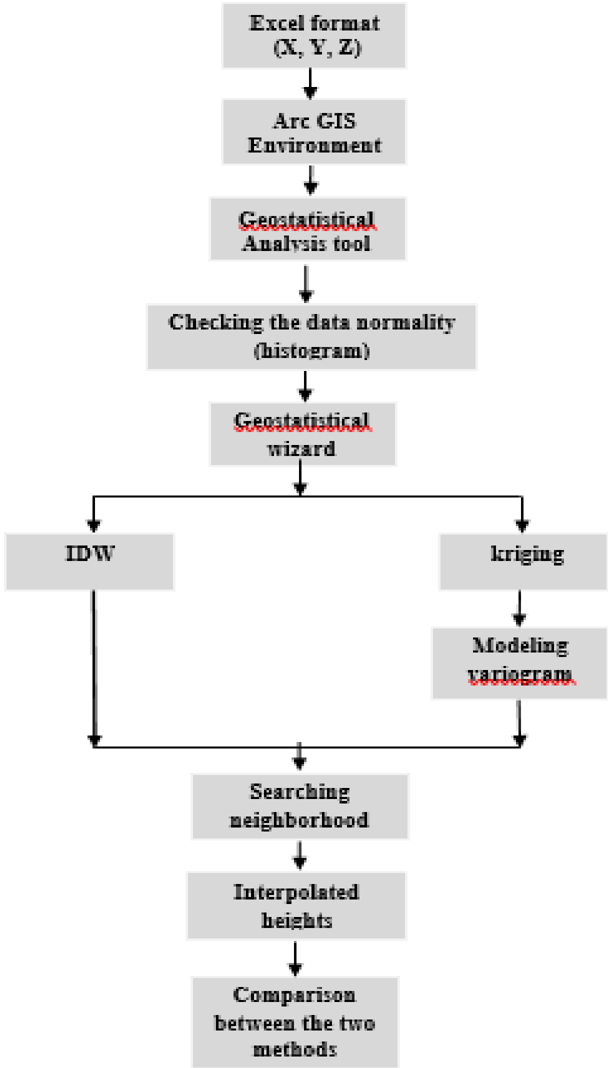

Figure 2: Methodology

5.1 Testing for normality

Generally, the interpolation methods that are used to generate a surface give the best results if the data is normally distributed (a bell shaped curve). The histogram tool allows the examination of the shape of the distribution by direct observations.

Statistically, the data is normally distributed if the mean and median are close to each other. The data sample of our study area resulted in the histogram shown in Figure (4). From the figure it can be seen that the distribution of the data is reasonably normally.

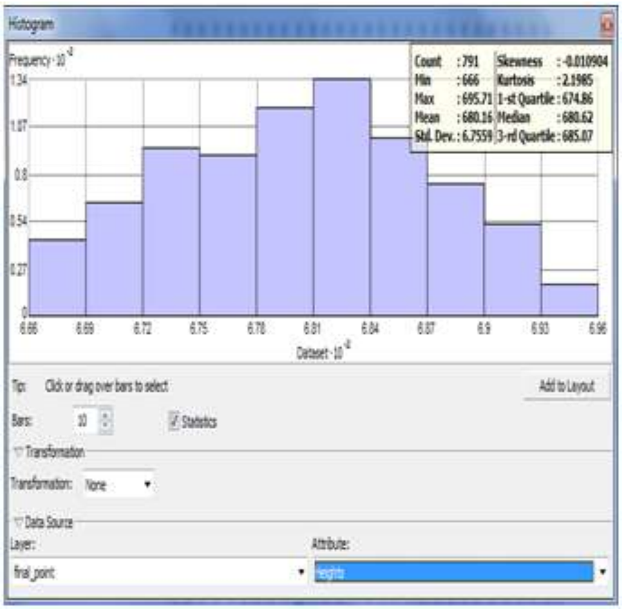

Figure 4: Histogram of the data sample of the study area

5.2 Semivariogram modeling

The goal of semivariogram modeling is to determine the best fit for a model that will pass through the points in the semivariogram. In this study, the value of the nugget is zero (the curve passes through the origin) and the model that best fits the experimental data semivariogram is the spherical model as can be seen in Figure (5).

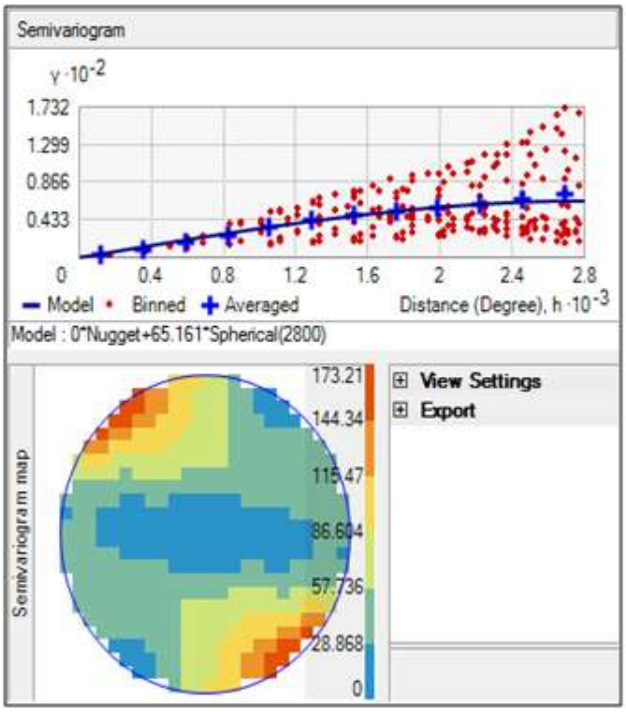

Figure 5: Semivariogram modeling

5.3 Searching neighborhood

It can be assumed that as the locations get farther from the prediction location, the measured values will have less spatial autocorrelation with the prediction location. Thus, it is possible to eliminate locations that are farther away that demonstrate little influence using search neighborhoods. Not only is there less relationship with locations that are farther away, but it is possible that the locations that are farther away may have a detrimental influence if they are located in an area much different than the prediction location (see equations (4) and (5)). In this study, the shape is considered to be standard (circle) with maximum neighbors of 5 and a minimum of 2 (by default) in 4 sectors. Using the data configuration within the specified neighborhood, in conjunction with the fitted semivariogram model, the weights for the measured locations are determined from the weights and values, the predicted values for all tests and check points are determined. The predicted values of all 20 check points are presented in tables (1) and (2). A surface representing all interpolated points is shown in Figure (6).

As in kriging, the neighborhood was chosen to be the standard type (circle).The maximum and minimum numbers of neighborhoods are taken to be 15 and 10 respectively. The power used in determining the weights was taken to be 2 (i.e. weight is the inverse of the square of the distance from the observation (interpolated) point. All predicted point values are then determined from which a surface is drawn as in Figure (7). The predicted heights are shown in Tables (1) and (2).

6. Results

The results obtained, using the outlined two methods are used to determine root mean square errors for the results of the two methods. These are found to be 0.726 meters and 1.1386 meters for kriging and IDW methods respectively. The mean of the difference between observation point values and predicted values are also determined and found to be 0.064 meters for kriging and -0.027 meters for IDW. The variances of these differences area calculated and found to be 0.551and 1.364 for kriging and IDW respectively.

Table 1: Interpolated values for the krigingmethod

No	Know Height	Interpolated values (kriging)		
		Prediction	D	D^2
1	671.012	670.756	0.256	0.065536
2	678.482	678.023	0.459	0.210681
3	676.454	674.766	1.688	2.849344
4	681.786	681.378	0.408	0.166464
5	680.975	680.997	-0.022	0.000484
6	689.905	689.755	0.15	0.0225
7	678.460	678.648	-0.188	0.035344
8	675.125	676.003	-0.878	0.770884
9	682.132	682.213	-0.081	0.006561
10	682.038	681.671	0.367	0.134689
11	670.794	670.343	0.451	0.203401
12	674.331	673.904	0.427	0.182329
13	682.578	683.170	-0.592	0.350464
14	680.759	681.432	-0.673	0.452929
15	677.240	677.171	0.069	0.004761
16	688.446	689.512	-1.066	1.136356
17	688.177	687.517	0.66	0.4356
18	674.196	673.450	0.746	0.556516
19	686.049	685.363	0.686	0.470596
20	690.191	691.770	-1.579	2.493241
			$\sum D = 1.288$ $\overline{D} = 0.064$	$\sum D^2 = 10.549$ RMSE=0.726

Table 2: Interpolated values for the IDWmethod

No	Know Height	Interpolated values (IDW)		
		Prediction	D	D^2
1	671.012	670.746	0.266	0.070756
2	678.482	678.233	0.249	0.062001
3	676.454	674.969	1.485	2.205225
4	681.786	680.970	0.816	0.665856
5	680.975	682.652	-1.677	2.812329
6	689.905	688.052	1.853	3.433609
7	678.460	678.696	-0.236	0.055696
8	675.125	675.702	-0.577	0.332929
9	682.132	682.512	-0.38	0.1444
10	682.038	682.296	-0.258	0.066564
11	670.794	670.341	0.453	0.205209
12	674.331	674.910	-0.579	0.335241
13	682.578	683.540	-0.962	0.925444
14	680.759	682.757	-1.998	3.992004
15	677.240	676.697	0.543	0.294849
16	688.446	689.334	-0.888	0.788544
17	688.177	687.398	0.779	0.606841
18	674.196	673.275	0.921	0.848241
19	686.049	684.217	1.832	3.356224
20	690.191	692.365	-2.174	4.726276
			$\sum D =$ -0.532 $\bar{D} = -0.0266$	$\sum D^2 = 25.93$ RMSE=1.1386

6.1 Contour Surface

Contour maps produced from the data using the two methods of interpolation are almost similar.

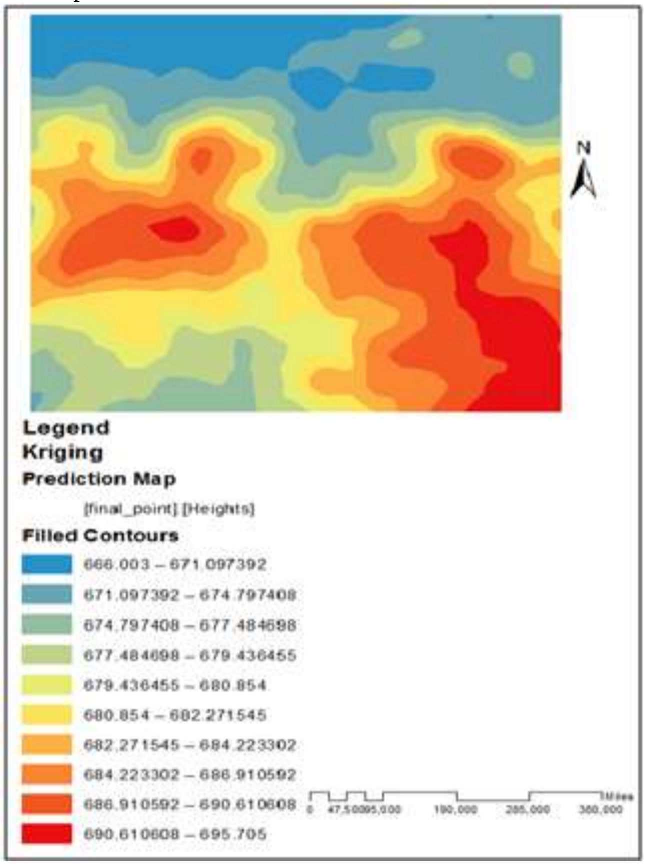

Figure 6: contour map from kriging

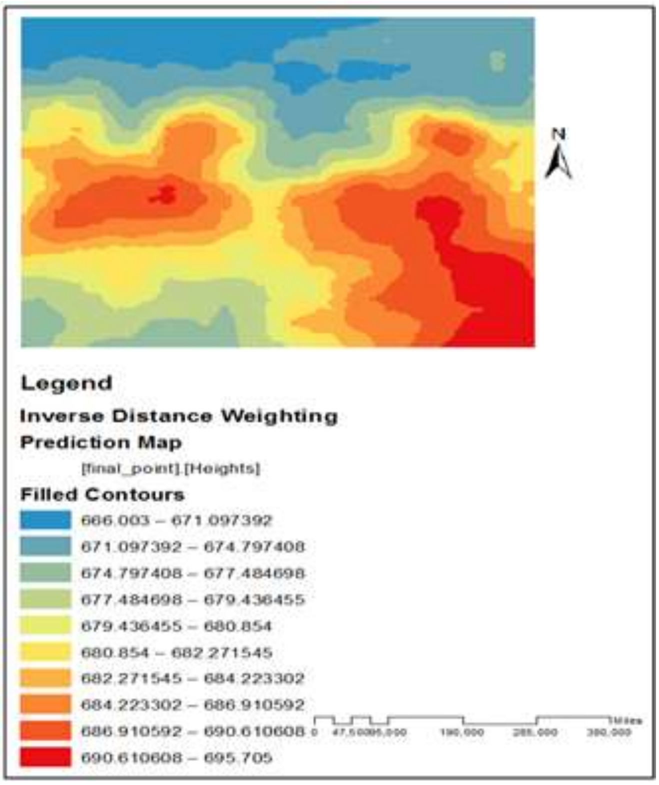

Figure 7: Contour map from IDW

6.2 Geostatistical testing of the results

Using a significance level of 5% tests were carried out to see whether both methods give acceptable results or not. To do so, the mean of the deviations of interpolated values from their respective sample point values were tested to see whether they are equal to their theoretical values. The test is framed as below:

$$\left.\begin{aligned} H_0 &: \bar{D}_{k,id} = 0 \\ H_A &: \bar{D}_{k,id} \neq 0 \end{aligned}\right\} \quad \dots\dots (6)$$

It should be noted that this statistic ($\bar{D}$) follow a t distribution with (ν) degrees of freedom (i.e.$\bar{D} \sim t_{1-\alpha/2,\nu}$) the calculated values for the statistic resulted in:

$$t_k = 0.386 \text{And} t_{i,d} = -0.102$$

The critical value for ($\alpha = 0.05 = 2.093$).Since the calculated values of the t distribution are smaller in magnitude than the critical value then the null hypotheses is accepted indicating that both methods give interpolated values which are equal in 95% of the cases.

To confirm this conclusion, another test was carried out. The difference between the interpolated values resulting from the two methods is to be tested. This statistic follows a paired-T test. The test is set out as follows:

$$\left.\begin{aligned} H_0 &: \bar{d} = 0 \\ H_A &: \bar{d} \neq 0 \end{aligned}\right\} \quad \dots\dots (7)$$

The statistic to be determined is given by equation (8) below

$$t = \frac{\bar{d} - 0}{S_{\bar{d}} / \sqrt{N}} \quad \dots\dots (8)$$

where:
$\bar{d}$: The mean of the differences. It can be calculated from

$$\bar{d} = \frac{\sum_{I=1}^{N} d_i}{N} \quad \dots\dots (9)$$

$S_{\bar{d}}$: The variance of the mean. It can be calculated from

$$S_{\bar{d}}^2 = \frac{N \sum_{i=1}^{N} d_i^2 - (\sum_{i=1}^{N} d_i)^2}{N(N-1)} \quad \dots\dots (10)$$

And the standard deviation of the mean is

$$S_{\bar{d}} = \sqrt{S_{\bar{d}}^2} \quad \dots\dots (11)$$

The calculated t statistic was found to be -0.531 which is, again points to the acceptance of the null hypotheses. This means that the two methods give the same results. The following table shows the calculated statistics.

Table 3: Calculated statistics

γ	$\bar{D}_{k\,[\mathrm{m}]}$	$\bar{D}_{id\,[\mathrm{m}]}$	$\bar{d}_{[\mathrm{m}]}$	$\bar{t}_k$	$\bar{t}_{id}$	$t_{\bar{d}}$	Critical t value
19	0.064	-0.0266	-0.091	0.386	-0.109	-0.527	2.093

6.3 Discussion

From Table (3), it can be noticed that the kriging method is better than IDW method. The RMSE for IDW is 1.139m and that of kriging is 0.726m. Also, kriging underestimates the interpolated values while IDW overestimates the interpolated values. In other words, IDW gives interpolated values, which are smaller than their respective true values while kriging gives values which are larger than their true values. To see whether the two methods give identical results, the tests carried out revealed that they do so. The calculated means of both methods when tested for their equality to zero, the null hypotheses of equation (6) was accepted. That means the mean of the deviations of the interpolated heights from their respective known heights is zero. To confirm the other test equation (7) related to the difference between interpolated heights using the two methods also lead to the acceptance of the null hypotheses.

The calculated mean of the difference between interpolated values using the two methods statistically equals zero indicating that the two methods give identical (the same) results in 95% of the cases.

7. Conclusion

From the data used and the tests carried out the following conclusion can be drawn:-

1) The kriging method of interpolation gives better results than IDW method in terms of precession.
2) Both methods give acceptable results. The predicted height values are statistically equal to their respective known heights.
3) The IDW method overestimates the interpolated values while kriging method underestimates the interpolated height values.

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