Study on Quark Confinement Lattice QCD Calculations Using Simulation Techniques

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Abstract: ***To observe the phenomenon of quark confinement it is needed to investigate whether the linear term in QCD potential actually exists. For this purpose perturbative techniques were insufficient. But numerical simulations of lattice gauge theory on a space time lattice is an important tool for lattice QCD calculations. In this paper we review the Wilson loop techniques for the study of quark confinement using the gauge invariance.***

Keywords: Lattice QCD calculations (Wilson loop), Lattice gauge theories

1. Introduction to Wilson Loops

Quantum Chromodynamics (QCD) is the theory that describes the interaction between quarks and gluons. Although QCD has been established as the fundamental theory of strong interaction, but manifestation of QCD are of great complexity .Confinement is up to now not satisfactory understood by the theoretical physicist. The Wilson loop which corresponds to a phase factor along a closed contour is used to set up a criterion for the recognition of the quarks in QCD. [1, 2, 3]The aim of our study to introduce a brief information about the Wilson loop, how it is used and what are the fields of application.

2. Wilson loops

A. Definition of Wilson loop

The Wilson loop is defined as a mathematical framework given by the trace of the path ordered exponential of gauge field A_μ, transported along a closed path C. It can be formulated as–

$$W_C = Tr\,[\,P\, e^{i} \int_C A_\mu(z) dz^\mu] \qquad (1)$$

Where, P represent path ordering operator, and Tr is the trace over the matrix.

Wilson loops are phase factor in Abelian and Non Abelian gauge theories. Until 1970s all the predictions of Quantum Chromodynamics (QCD) were restricted to the perturbative regime. In 1974 K.G. Wilson [1] used lattice regularization for the study of non perturbative phenomenon of confinement of quark. For the lattice formulation they introduced a phase factor for the simplest closed contour on the lattice is called Wilson loop. Thus Wilson loops play a central role in the lattice formulation of gauge theories. QCD can be reformulated through the Wilson loops in a manifest gauge invariant way. Wilson loop contains the holonomy of the gauge connection around a given loop; can be used for the study of confinement in QCD via the static quark antiquark potential, as well as used for solving various matrix models.

The phase factor in gauge theories is associated with the parallel transport in external gauge field. In general parallel transport is a vector around a closed loop, used to compare the phases of a wave function at two different points. Parallel transport is supposed to be curved space generalization of the concept of keeping the vector constant as we move along the path.

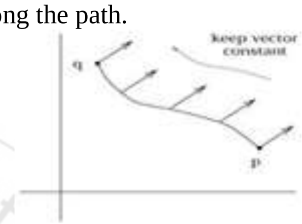

Figure 1: Parallel transport between two points

B. Aharonov-Bohm Effect

Wilson loops are essential phase factor in gauge fields; and are observable in quantum theory by Aharonov Bohm effect [4]. In quantum mechanics the phase differences can be observed rather than the phase itself. The phase difference depends on the value of phase factor along which the parallel transport is performed.

To observe the phase factor in Aharonov Bohm Effect scheme is depicted in figure 2.

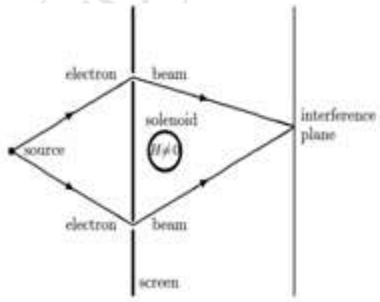

Figure 2: Scheme for Aharonov Bohm experiment

When the coherent beam of electron passes through a solenoid; it splits into two parts. Electrons do not pass inside the solenoid where the magnetic field is concentrated. Nevertheless a phase difference arises between the electron beam passing through the slits. The phase of the wave function changes along the curve because the dependent vector potential is non-zero. i.e.$\vec{A} \neq 0$;Then the wave function is-

$$\psi = \psi_0 \exp\left[\frac{ie}{\hbar c} \int A_\mu(z) dz^\mu\right] \qquad (2)$$

Where ψ_0= free case wave function, and $A_\mu$is the vector formulation of the potential and can be written as –

$$A_\mu(z) = \phi(\vec{z}, t) - \vec{A}(z, t) \qquad (3)$$

also, $\vec{A}(z) = A_0(z) + \vec{\nabla}\lambda$ (4)
where λ is arbitrary gauge function of z.
with z = (x, y, z, t) and $dz^{\mu} = (cdt, d\vec{z})$
$|A_0| = 0 \; i.e. \vec{A}(z) = \vec{\nabla}\lambda$

$$\vec{B} = \vec{\nabla} \times A = \vec{\nabla} \times (A_0(z) + \vec{\nabla}\lambda) \quad (5)$$

Hence the wave function cannot enter in the regime of the magnetic field. To compute interference pattern consider two wave functions through the path 1 and path 2 are given as-

$$\psi = \psi_{1,0} \exp\left[\frac{ie}{\hbar c}\int_1 A_\mu(z) dz^\mu\right] \quad (6)$$

$$\psi = \psi_{2,0} \exp\left[\frac{ie}{\hbar c}\int_2 A_\mu(z) dz^\mu\right] \quad (7)$$

The real part of the phase difference depends on

$$= \exp\left[\frac{ie}{\hbar c}\int_1 A_\mu(z) dz^\mu - \frac{ie}{\hbar c}\int_2 A_\mu(z) dz^\mu\right] \quad (8)$$

$$= \exp\frac{ie}{\hbar c}\oint_C dz^\mu A_\mu(z) \quad (9)$$

where C = path 1 – path 2
Using Stokes theorem the integral changes to

$$= \exp\frac{ie}{\hbar c}\sigma^{\mu\nu} F_{\mu\nu} \quad (10)$$

where $\sigma^{\mu\nu}$ is the area element of the enclosed loop, and $F_{\mu\nu}$ is field strength tensor.

$$= \exp\frac{ie}{\hbar c} HS \quad (11)$$

This shows the real part of the phase difference only depends on HS, the magnetic flux through solenoid and does not depends on the shape of the path 'C'.

C. Phase Factor in QED (Abelian Phase Factor)

An analog of the phase factor was first introduced by H Weyl in 1919.[5] In gauge theory, the transformation between the possible gauges, form a symmetry group G .The elements of the subset S of the group G are called group generators of the field represented by T_α . If these generators are commutative; as $[T_a T_b] = 0$; and gauge bosons are not self interacting, Then the field is abelian gauge field. In Quantum Electrodynamics (QED) photons play the role of mediators are not self interacting ; Also the group generators commute with each other i.e. $[T_a T_b] = if_{abc} T_c = 0$, where f_{abc} is structure constant and vanishes in case of QED. Hence QED is an abelian gauge theory with symmetry group U (1). For Quantum Chromodynamics (QCD) the mediators are quarks. Quarks are of self interacting nature, Also the group generators anti commute with each other i.e. $[T_a T_b] \neq 0$ here. So, QCD is non abelian gauge theory with symmetric group SU(3). Abelian phase factor is defined as

$$U(y,x) = \exp\left[ie\int_{\Gamma_{yx}} A_\mu(z) dz^\mu\right] \quad (12)$$

Under the gauge transformation

$$A_\mu(z) \xrightarrow{g.t.} A_\mu(z) + \frac{1}{e}\partial_\mu \alpha(z) \quad (13)$$

The abelian phase factor transforms as-

$$U(y,x) \xrightarrow{g.t.} e^{i\alpha(y)}\, U(y,x)\, e^{-i\alpha(x)} \quad (14)$$

A wave function at the point x is transformed as-

$$\phi(x) \xrightarrow{g.t.} e^{i\alpha(x)}\, \phi(x) \quad (15)$$

therefore the phase factor is transformed as the product $\phi(y)\, \phi^\dagger(x)$

$$U(y,x)\ \widetilde{g.t.}\ \phi(y)\, \phi^\dagger(x) \quad (16)$$

A wave function at the point x transforms like one at the point y after multiplication by the phase factor ;

$$U(y,x)\, \phi(x)\ \widetilde{g.t.}\ \phi(y) \quad (17)$$

And analogously-

$$\phi^\dagger(y)\, U(y,x)\ \widetilde{g.t.}\ \phi^\dagger(x) \quad (18)$$

The phase factor plays the role of a parallel transporter in an electromagnetic field, and to compare the phase of a wave function at points x and y, we should first make a parallel transport along some contour Γ_{yx}.The result is Γ-dependent except when $A_\mu(z)$ is a pure gauge (vanishing field strength $F_{\mu\nu}(z)$).Certain subtleties occur for not simply connected spaces (the Aharonov Bohm effect).

D. Non Abelian Gauge Invariance: Yang Mills Theory

An extension to non abelian gauge group was given by Yang and Mills in 1954.[6] Yang Mills theory explain the term 'gauge invariance' gauging literally means fixing a scale. Yang Mills theory describe the behaviour of elementary particle using the non abelian lie groups and explain the unification of weak and electromagnetic force i.e. [U(1)×SU(2)]. Thus Yang Mills theory provide a understanding about standard model in particle physics.

In the case of non abelian group SU(N) local gauge transformation given as

$$\Psi(x) \xrightarrow{g.t.} \psi'(x) = \Omega(x)\Psi(x) \quad (19)$$

Here $\Omega(x) \in G$ with G being a semisimple lie group which is called the gauge group.

The gauge transformation of field Ψ gives the transformation of non abelian gauge field

$$A_\mu(x) \xrightarrow{g.t.} A'_\mu(x) \quad (20)$$

$$= \Omega(x) A_\mu(x)\, \Omega^\dagger(x) + \frac{i}{g}\Omega(x)\partial_\mu\, \Omega^\dagger(x) \quad (21)$$

where g is coupling constant. A_μ is defined as-

$$A_\mu = A_\mu^a T_a = \sum_{a=1}^{N^2-1} A_\mu^a T_a \quad (22)$$

T_a introduce the generators of the group G(a=1 ... $N^2 - 1$) for SU(N),can be normalised such that

$$\mathrm{Tr}\, T_a T_b = \delta_{ab} \quad (23)$$

where Tr is the trace over the generators.
The covariant derivative is given as-

$$D_\mu = \partial_\mu - igA_\mu(x) \quad (24)$$

The QCD action in the matrix form may be given as-

$$S = \int d^4x \left[\frac{1}{2g^2}(F_{\mu\nu} F_{\mu\nu}) + \overline{\Psi}(\gamma_\mu D_\mu + m)\Psi\right] \quad (25)$$

$$\text{Where } F_{\mu\nu} = D_\mu A_\nu - D_\nu A_\mu \quad (26)$$

$$= \partial_\mu A_\nu - \partial_\mu A_\mu - ig\,[A_\mu, A_\nu] \quad (27)$$

is the Hermitian matrix of the non abelian field strength.

E. Non Abelian Phase Factors

The proper extension of the Abelian formula given in equation (12) is -

$$U[y,x] = P\exp\,[ie\int_{\Gamma_{yx}} A_\mu(z) dz^\mu \quad (28)$$

The symbol P refers for path ordering. If we represent the integral over dt such that

$$dz^\mu = dt\dot{z}^\mu \quad (29)$$

Then phase factor in equivalent form may be written as-

$$U[y,x] = P\exp\left[ie\int_{\Gamma_{yx}} dt\dot{z}^\mu(t) A_\mu(z)\right] \quad (30)$$

$$U[y,x]\ \prod_{t=0}^{\tau}[1 + i\, dt\dot{z}^\mu(t)\, A_\mu(z(t))] \quad (31)$$

Using Eq. (29) Eq. (31) may be written as-

$$\prod_{z\in\Gamma_{yx}}[1+idz^{\mu}\,(\mathrm{t})A_{\mu}(z) \quad (32)$$

If the contour is discretized, then the non-abelian phase factor within the limit $z_{i-1} \to z_i$ is approximated by

$$\mathrm{U[y,\,x]} = \lim_{M\to\infty}\prod_{i=1}^{M}[1+i[z_i - z_{i-1}]^{\mu}\, A^{\mu}\left(\frac{[z_i+z_{i-1}]}{2}\right)] \quad (33)$$

The non abelian phase factor (28) is an element of the gauge group G itself. while A_μ belongs to the Lie algebra of G.

3. Definition and Properties

A. Derivation of the Wilson Loop

Let us choose a path γ from $x \to y$.The explicit form of the finite parallel transport [7] is,

$$\mathrm{U_\gamma(y,x) = U(y\,,x_n)U(x_n\,,x_{n-1})\ldots\ldots U\,(\,x_1,x)} \quad (34)$$

We split the path into infinitesimal segments:

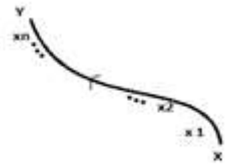

Figure 3: Dividing the path γ into infinitesimal segments.

for the infinitesimal segments, parallel transport is-

$$U(x_1,x) \approx \exp[ieA_\mu(x_1 - x)^\mu] \quad (35)$$

and hence $U_\gamma(y,x) = \exp[ie\int_\gamma A_\mu(x)\,A_\mu\, dx^\mu]$ (36)

$U_\gamma(y,x)$ is not necessarily path independent

$$U_{\gamma1}(y,x) \neq U_{\gamma2}(y,x) \quad (37)$$

$$U_\gamma(y,x) = \lim_{n\to\infty} U(y\,,x_n)U(x_n\,,x_{n-1})\ldots\ldots U\,(\,x_1,x) \quad (38)$$

$$= \lim_{\Delta x_j\to 0}\prod_{j=0}^{n}(1+igA_\mu\,(x_j)\Delta x_j^\mu\,) \quad (39)$$

with $\Delta x_j^\mu = x_{j+1}^\mu - x_j^\mu, x_0 = x\,, x_{n+1} = y.\ A_\mu$ is a matrix and so, $[A_\mu(x_j), A_\upsilon(x_k)] \neq 0$ in general.

$$U_\gamma(y,x) = 1 + ig\sum_{k=0}^{n}A_\mu(x_k)\Delta x_k^\mu + (ig)^2\sum_{k=0}^{n}\sum_{k=0}^{j-1}A_\mu\,(x_j)\Delta x_k^\mu A_\upsilon x_l \Delta x_l^\upsilon + \cdots + \quad (40)$$

Now we introduce $x^\mu\,(r)$ to parameterise γ with different segments $r_1, r_2 \ldots\ldots\ldots r_n$.

$$x^\mu(0) = 0, x^\mu(1) = y^\mu, r \in [0,1] \quad (41)$$

$$U_\gamma(y,x) = 1 + ig\int_0^1 dr_1\, A_\mu(x(r_1))\frac{dx^\mu}{dr_1} + (ig)^2\int_0^1 dr_1\int_0^{r_1} dr_2\, A_\mu(x(r_1))\frac{dx^\mu}{dr_1}A_\upsilon(x(r_2))\frac{dx^\upsilon}{dr_2} + \ldots \quad (42)$$

$$= \sum_{n=0}^{\infty}(ig)^n\int_0^1 dr_1\int_0^{r_1} dr_2 \ldots \int_0^{r_{n-1}} dr_n\, A_{\mu 1}(x(r_1))\frac{dx^{\mu 1}}{dr_1}\ldots\ldots A_{\mu n}(x(r_n))\frac{dx^{\mu n}}{dr_n} \quad (43)$$

$$= \sum_{n=0}^{\infty}(ig)^n\int_{r\geq r_1\geq r_2\ldots\geq r_n\geq 0} dr_1\ldots\ldots dr_n\, A_{\mu 1}(x(r_1))\frac{dx^{\mu 1}}{dr_1}\ldots\ldots A_{\mu n}(x(r_n))\frac{dx^{\mu n}}{dr_n} \quad (44)$$

It is observed that the factors are in path order i.e. larger values of r's stand to the left. So we may introduce the path ordered product

$$\mathrm{P}(A_{\mu 1}(x(r_1))\ldots.A_{\mu n}(x(r_n)) \quad (45)$$

So the path ordered exponential expressed as-

$$= \mathrm{P}\exp[\mathrm{ig}\int_0^1 dr\frac{dx^\mu}{dr}A_\mu(x(r))] \quad (46)$$

$$= \mathrm{P}\exp[\,\mathrm{ig}\int_\Gamma A_\mu(x)\,dx^\mu] \quad (47)$$

By construction, under a gauge transformation,

$$U_\Gamma(y,x) \to \Omega(y)U_\Gamma(y,x)\Omega^\dagger(x) \quad (48)$$

For a closed loop Γ, $U_\Gamma(x,x)$ is non trivial. The non abelian generalisation of Stokes theorem can be used to relate the parallel transport around the loop to flux passing through the loop. For an infinitesimal loop

$$U_\Gamma(x,x)\psi = \frac{1}{2}F_{\mu\upsilon}\sigma^{\mu\upsilon}\psi \quad (49)$$

where $\sigma^{\mu\upsilon}$ is the area element encircled by the loop. Under a gauge transformation,

$$U_\Gamma(x,x) \to \Omega(x)U_\Gamma(x,x)\Omega^\dagger(x) \quad (50)$$

and hence

$$W_\Gamma(\mathrm{x}) = \mathrm{Tr}\,(U_\Gamma(\mathrm{x,\,x}) = \mathrm{Tr}\,(\mathrm{P}\exp ig\oint_\Gamma A_\mu dx^\mu) \quad (51)$$

is gauge invariant. This is the non abelian Wilson loop.

B. Properties of Wilson Loop

- **Hermiticity –**It implies that the Hermitian conjugate of a Wilson line gives the same line in opposite direction. [8] Let γ is a Wilson line from a to b along the direction y then- $\gamma_x^\dagger[a,\mathrm{b}] = \gamma_{-x}[b\,,\mathrm{a}]$ (52)
- **Causality** - If we first have a Wilson line from a to b then a line along the same direction y from b to c, we can glue them together into the Wilson line from a to c- $\gamma_y[\mathrm{b,c}]\ \gamma_y[a,\mathrm{b}] = \gamma_y[a,\mathrm{c}]$ (53)
- **Unitarity- If** we have a Wilson line from a to b and then a line back from b to a in the opposite direction, they will give 1. $\gamma_y[a,\mathrm{b}]\ \gamma_y[a\,,\mathrm{b}] = 1$ (54)

Lattice gauge theory

4. Introduction to Lattice Gauge Theory

The lattice is defined as a set of points of d-dimensional Euclidean space with co-ordinates

$$x_\mu = n_\mu a \quad (55)$$

Where $n_\mu = n_1, n_2 \ldots n_d$ are integer number and a is the dimensional constant, which equals the distance between the neighbouring sites and is called the lattice spacing. Link is represented by l ={x, μ} connects two neighbouring site x and x+a$\hat{\mu}$ where $\hat{\mu}$ is a unit vector along the μ direction. Similarly a plaquette p={x; μ, υ} is the combination of links in the direction μ and υ. The set of four links which bound the plaquette p is denoted by ∂p. When the size of the lattice is taken infinitely large and its sites infinitesimally close to each other, the continuum gauge theory becomes applicable.

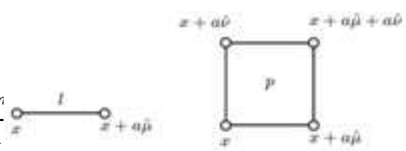

Figure 4: Representation of Link (left) and Plaquette(right)

A. Wilson loop on a lattice

Consider a rectangular Wilson loop [9] of size R×T. The Wilson loop average for T >> R is related to the interaction energy of the static (i.e. infinitely heavy) quarks, separated by a distance R, by the formula-

$$W(R,T) \propto e^{-E_0(R)T} \text{ (for T } >> R) \qquad (56)$$

It can be proved in the axial gauge A_4=0 where $U_4(x) = 1$ so that only the vertical segment contribute to U(R×T).

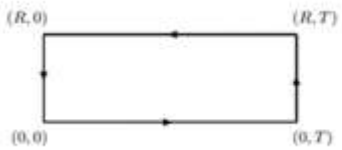

Figure 5: Rectangular Wilson Loop

Denoting- $\psi_{jk}(t) \equiv [Pe^{i\int_0^R dz_1 A_1(z_1,\dots t)}]_{jk}$

We have- $W(R\times T) = \langle \frac{1}{N} tr\, \psi(0)\psi^\dagger(T) \rangle \qquad (57)$

Inserting a sum over a complete set of intermediate states-

$$\sum_n |n\rangle\langle n| = 1 \qquad (58)$$

We obtain-

$$W(R\times T) = \sum_n \frac{1}{N} \langle \psi_{jk}(0)|n\rangle\langle n|\psi_{kj}^\dagger(T)\rangle = \sum_n \frac{1}{N} \left|\langle \psi_{jk}(0)|n\rangle\right|^2 e^{-E_n T} \qquad (59)$$

where E_n is the energy of the state $|n\rangle$ .As T→∞ only the ground state with lowest energy survive in the sum over states and finally we get

$$W(R\times T) \rightarrow e^{-E_0(R)T} \qquad (60)$$

Since nothing in the derivation relies on the lattice, it holds for a rectangular loop in the continuum theory as well.

B. Area Law and Confinement

The leading order in β corresponding to filling a minimal surface

$$W(C) = [W(\partial p)]^{A_{min}(c)} \qquad (61)$$

where $W(\partial p)$ is plaquette average and is given by equation $W(\partial p) = \frac{\beta}{2N^2}$ for SU(N)with N ≥ 3

$$= \frac{\beta}{4} \text{ for SU(2)} \qquad (62)$$

The exponential dependence of the Wilson loop average on the area of the minimal surface as in equation (61) is called the area law. It is assumed that if an area law holds for loops of large area in the pure SU(3) gauge theory then quarks are confined .In other words there are no physical $|in\rangle$ or $\langle out|$ quark states.This is the essence of Wilson's confinement criterion. The argument is that physical amplitudes do not have quark singularities when the Wilson criterion is satisfied.

C. Linear Potential

A justification for the Wilson criterion is based on the relationship (56) between the Wilson loop average and the potential energy of interaction between static quarks. When the area law

$$W(C) \xrightarrow{large\ C} e^{-KA_{\min}} \qquad (63)$$

holds for large loops, the potential energy is a linear function of the distance between the quarks

$$E(R)=Kr \qquad (64)$$

The coefficient K is called the string tension because the gluon field between quarks contracts to a tube or string, having energy proportional to its length. The value of K is the energy of the string per unit length. The string is stretched with the distance between quarks and prevents them from moving apart to macroscopic distances .Equation (61) then gives

$$K = \frac{1}{a^2} ln \frac{2N^2}{\beta} = \frac{1}{a^2} \ln(2Ng^2) \qquad (65)$$

for the string tension to the leading order of the strong coupling expansion. The next orders of the strong coupling expansion result in corrections in β to this formula. Confinement holds in the lattice gauge theory to any order of the strong coupling expansion.

D. Asymptotic Scaling

Equation (64) establishes the relationship between lattice spacing a and the coupling g^2.Let K equals its experimental value

$$K = (400\text{MeV})^2 \approx GeV/fm \qquad (66)$$

The renormalizability prescribes that variations of a, which plays the role of a lattice cut-off, and of the bare charge g^2should be made simultaneously in order that K does not change. Given eq. (65), this procedure calls for a→ ∞ as $g^2 \rightarrow \infty$.i.e. the lattice spacing is large in the strong coupling limit, compared with 1fm.Such a coarse lattice cannot describe the continuum limit and in particular the rotational symmetry. In order to pass to the continuum, the lattice spacing a should be decreased. Equation (65) shows that 'a' decreases with decreasing g^2.However this formula ceases to be applicable in the intermediate region of $g^2 \sim 1$ and, therefore, a $\sim 1fm$.To further decrease 'a', we further decrease g^2.While no analytic formulas are available at intermediate values of g^2.The expected relation between a and g^2 is predicted by the known two-loop Gell-Mann Low function of QCD. For pure SU(3) Yang Mills, eq.(65) is replaced at small g^2 by-

$$K = \text{const.} \frac{1}{a^2} \left(\frac{8\pi^2}{11\, g^2}\right)^{\frac{102}{121}} e^{-\frac{8\pi^2}{11g^2}} \qquad (67)$$

where the two loop Gell-Mann Low function is used. The exponential dependence of K on $1/g^2$ is called asymptotic scaling. Asymptotic scaling sets in for some value of $1/g^2$. The strong coupling formula (67) holds for small $1/g^2$.The asymptotic–scaling formula (65) sets in for large $1/g^2$.Both formulas are not applicable in the intermediate region $1/g^2 \sim 1$.For such values of g^2, where asymptotic scaling holds, the lattice gauge theory has a continuum limit.

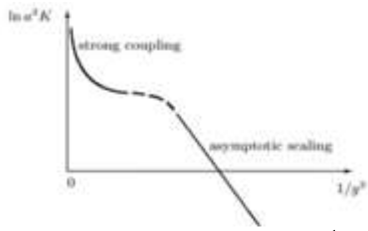

Figure 6: String tension versus $1/g^2$.

The knowledge of the two asymptotic behaviours says nothing about the behavior of a^2K in the intermediate region of $g^2{\sim}1$.There can be either a smooth transition between these two regimes or a phase transition.

5. Conclusion

Wilson loop is a non abelian path ordered phase factor. The QCD studies have been done using the gauge invariance of the Wilson loop. Continuum gauge theory provides the linear relationship between quarks potential and the distance between them. This linear relationship indicates the confined nature of quarks. The proportionality constant of this relation (string tension) shows strong coupling in large lattice spacing region and the asymptotic scaling in small spacing region. But the intermediate region does not show any analytic relationship. Confinement holds in the lattice gauge theory for any order of the strong coupling region.

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