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Folding of Tangle Hypergraph

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Abstract: ***In this paper will study folding of tangle hypergraph. We will introduce folding of hyper graph , and we will discuss folding of dual of tangle graph. The matrices of each case will be discussed.***

Keywords: Folding, tangle graph, hyper graph, dual of tangle graph

1. Introduction

Conway developed tangle theory and invented a system of notation for tabulating knots, nowadays known as Conway notation. Tangle theory can be considered analogous to knot theory except, instead of closed loop we use string whose end are nailed down. Tangles have been shown to be useful in studying DNA topology. Like in most fruitful mathematical theories, the theory of hyper graph has many applications. Hyper graphs model many practical problems in many different sciences. Hyper graphs have shown their power as a tool to understand problems in a wide variety of scientific field. Moreover it well known now that hyper graph theory is a very useful tool to resolve optimization problem such as scheduling problems, location problems and so on. The notion of isometric folding was introduced by S. A. Robertson who studied the stratification determined by the folds or singularities.

2. Definitions

Tangle graph: Let D be a unit cube, so D={ (x,y,z): 0< x,y,z<1} on the top face of cube place n points $a_1, a_2,\ldots,a_n$ similarly place on bottom face $b_1,b_2,\ldots,b_n$,now join the points $a_1,a_2,\ldots,a_n$ with $b_1,b_2,\ldots,b_n$ by arcs $d_1,d_2,\ldots,d_n$ these arcs are disjoint and each d_i connects some a_j to b_k not connect a_i to a_k or b_j to b_k this called tangle[1].

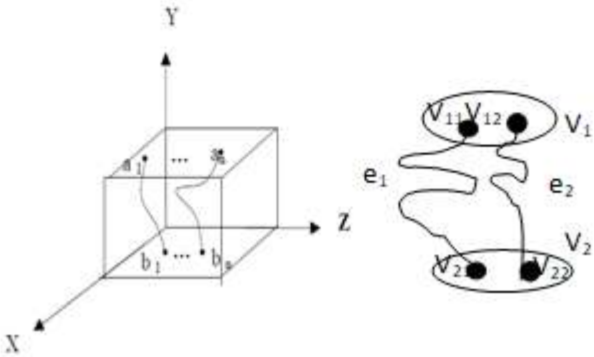

Figure 1

Mathematics Subject Classification, 51H10, 57H10
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Hyper graph
A hyper graph is a graph which an edge can connect any number of vertices. Formally a hyper graph ***H*** is a pair of ***H*** =(X,E) where X is asset of elements called nodes or vertices, and E is asset of non- empty subset of X called hyper edge or edge [2].

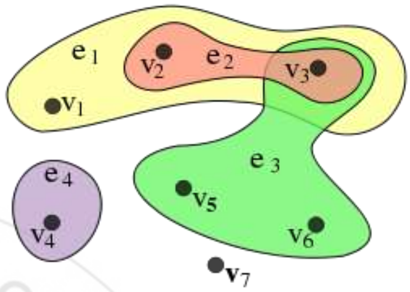

Figure 2

$X = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$ And $E = \{e_1, e_2, e_3, e_4\} = \{\{v_1, v_2, v_3\}, \{v_2, v_3\}, \{v_3, v_5, v_6\}, \{v_4\}\}$.

Tangle hyper graph
Tangle hyper graph is a graph whose vertices consists of inner and outer vertices ,the outer vertices looks the hyper edge of hyper graph and set of edges consists of more than two vertices[3].

Dual of tangle graph:
The tangle graph(T*) is called the dual tangle graph of (T). A tangle graph T= (V; E_1 ,$E_2,\ldots,E_m$); V=($v_1,v_2,\ldots,v_n$){ where (v) outer vertex in tangle graph} can be mapped to tangle graph T^* =(ε; $v_1,v_2,\ldots,v_n$) whose vertices are the points $e_1,e_2,\ldots,e_m$,and edges are $v_1,v_2,\ldots,v_n$ [4].

Folding:
Let M and N be two. Riemannian manifolds (not necessarily of the same dimension), a map f: M→N is said to be an **i**sometric folding of M into N if, for piecewise geodesic path β: I→ M (I= [0, 1] $\subseteq$R), the induced path $f0\beta$: I→N is piecewise geodesic and of the same length as β. If f not preserve lengths then f is topological folding[5].

3. Main Results

Folding of hyper graph:
Case 1:
Conversation of hyper graph into simple graph:

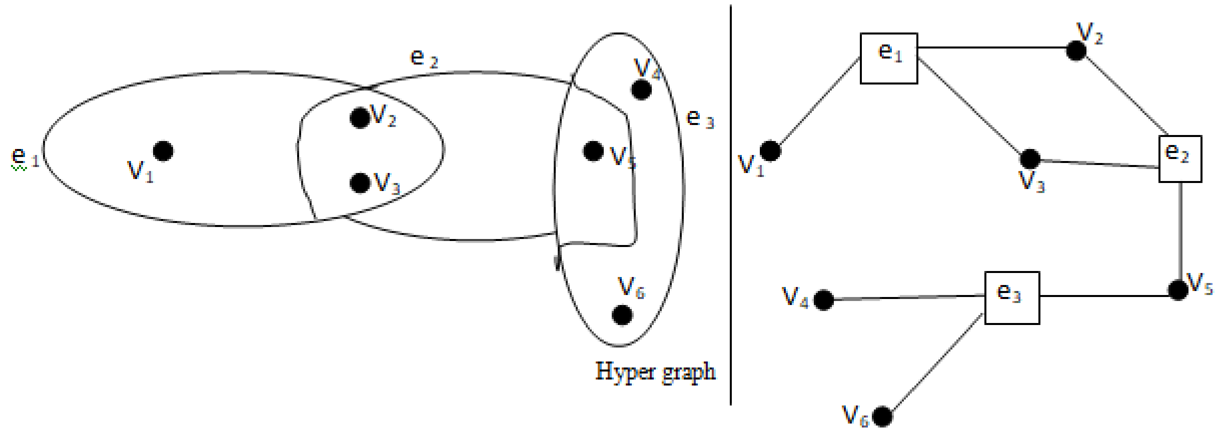

Figure 3

Hyper graph

Adjacent matrices:

$$A=\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array}\begin{bmatrix} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

The adjacent matrix in hyper graph is the same in simplification of it into simple graph.

Case1.1:
Folding of vertices:
Let F_1 be a function such that f_1 folding v_2,v_3 gives me one vertex say v_2

Simplification Hyper graph

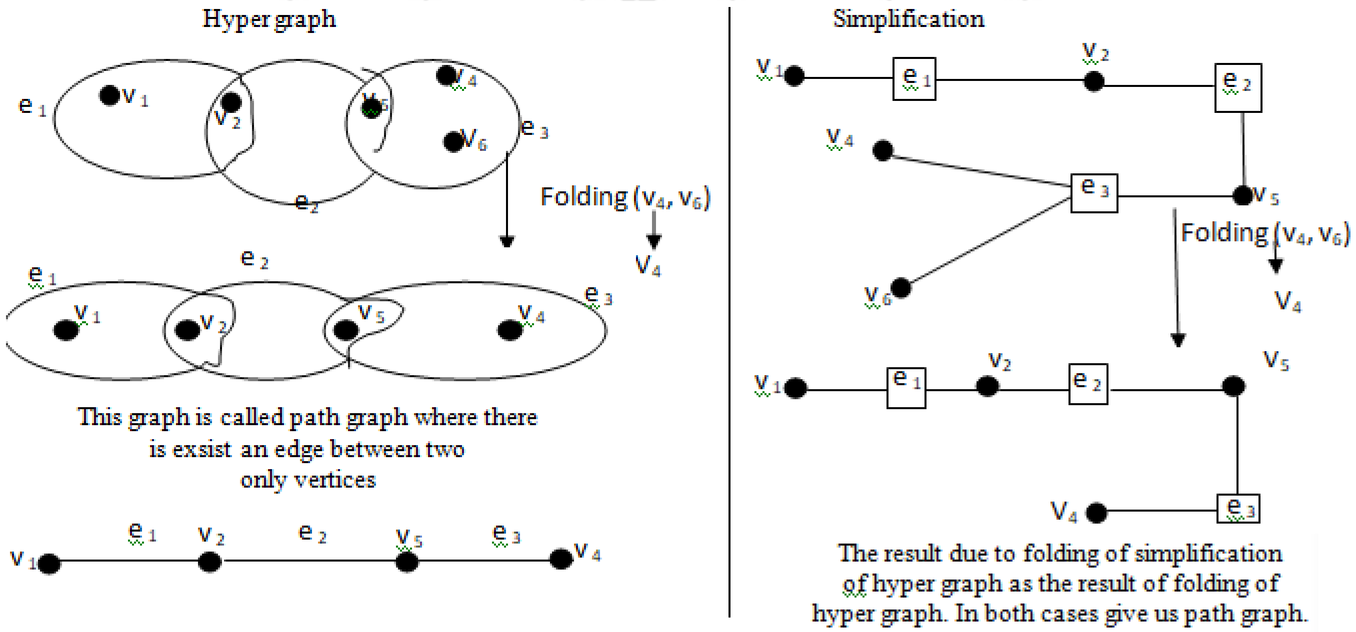

Figure 4

Case1.2:
Folding of edges:
The function (f) in this case folding the edges in hyper graph as shown in Fig (3). Let f_2 folding the edges (e_1, e_2) the result is e_2

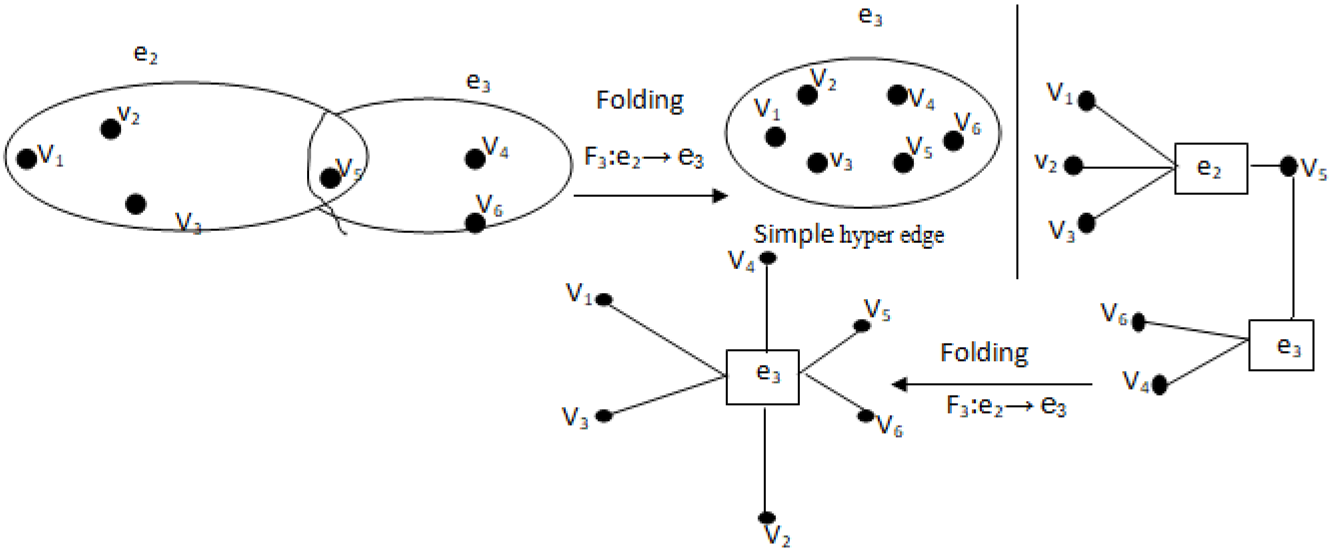

Figure 5

Proposition:
Limit of folding of hyper edge in hyper graph lead to Star of hyper graph looks like Star in graph.

$$\lim_{n\to\infty} f_n \rightarrow \textit{star hyper graph}$$

Case 2:
Folding of dual Tangle graph:

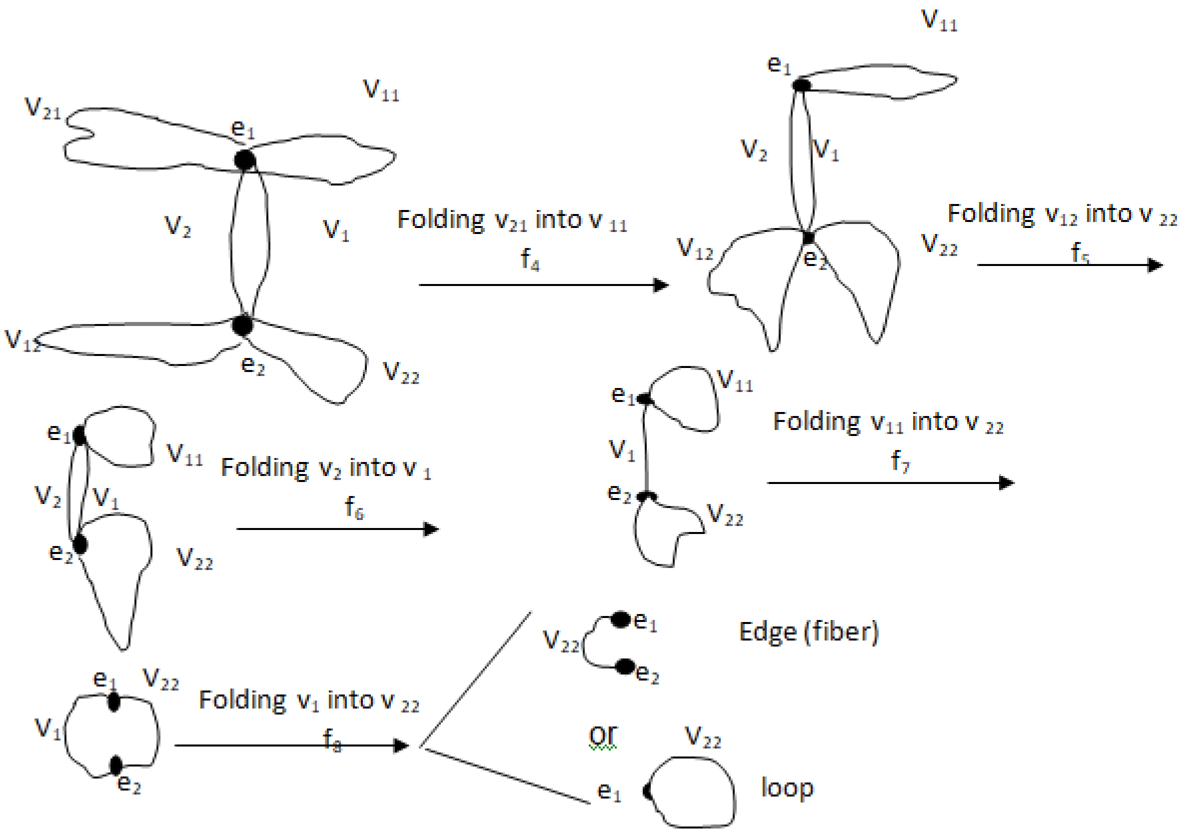

Figure 6

In Fig.(6) the folding of two edges (v_{21}, v_{11}) of dual tangle graph give us one edge as f_4. F_5 as f_4 give us one edge..The folding of v_2 into v_1 gives an edge v_1 as f_6. The folding of v_{11} into v_{22} gives a multiple edge as f_7 . The folding of $v_1$into v_{22} gives two cases:

1- Edge. 2-loop

Case 3: Folding of tangle hyper graph:
(Inner vertices) top face case 3.1:

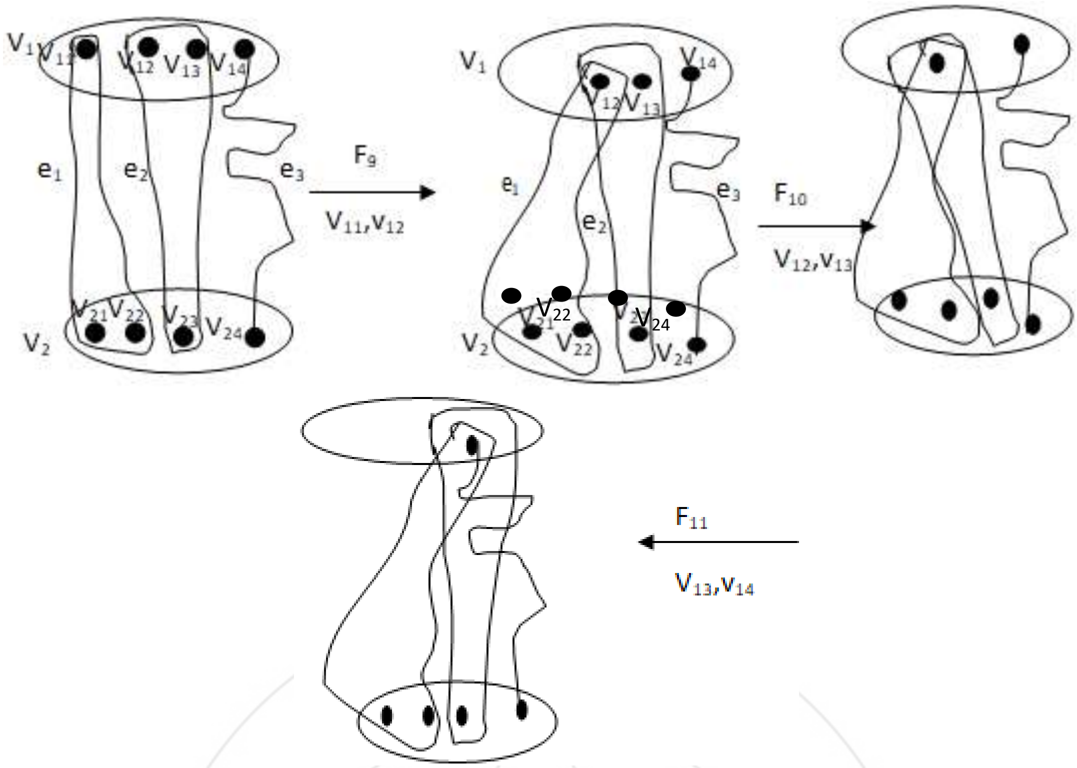

Figure 7

Bottom faces similarly top face. Finally due to folding more than one time we reach to simple edge (between two vertices) as fig.(8) and we obtain tangle graph or braid(if we deal with hyper edges not tangle edge).

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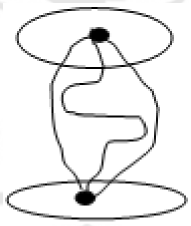


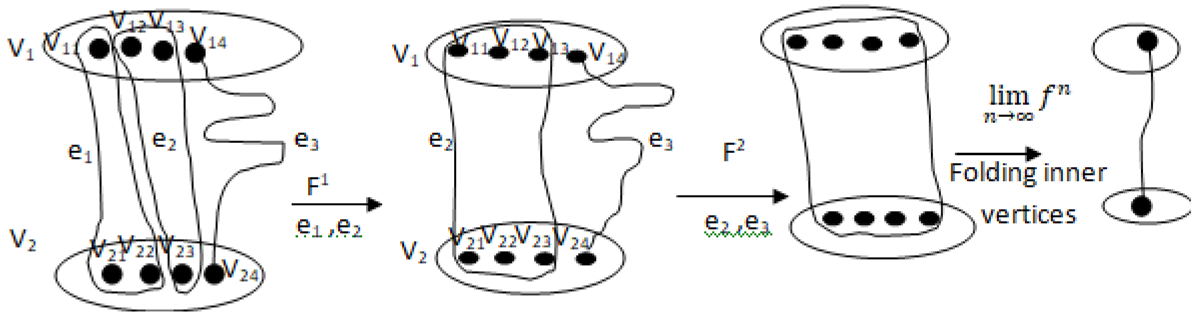
Figure 8

Matrices of fig.7:

$$A=\begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1^{1}_{30} \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1^{1}_{30} & 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{F_9} \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1^{1}_{30} \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1^{1}_{30} \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 1 & 1 & 1^{1}_{30} \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1^{1}_{30} \end{bmatrix} \xleftarrow{F_{11}} \begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1^{1}_{30} \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1^{1}_{30} \end{bmatrix} \xleftarrow{F_{10}}$$

Case 3.2: folding of edges:



In fig.9 the folding of two edges (e_1 ,e_2) gives one edge ,but vertices not fold as (F^1). (F^2) similarly (F^1). The limit of folding lead to hyper edge. The limit of folding of inner vertices of hyper edge gives simple edge and the final shape called braid (special case of tangle graph).

4. Corollary

We can obtain braid from tangle hyper graph by two steps:
1) Folding of edges of tangle hyper graph until we reach tangle of one edge (hyper edge).
2) Folding inner vertices of hyper edge more than one time until we reach simple edge (between two points). Then the final shape called braid as fig.9.

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